Option Value of Cash*

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March 23, 2009

Abstract

Using a dynamic model of heterogeneous beliefs where investors agree to disagree, this paper shows: (i) beliefs may diverge, which prevents pessimists from providing liquidity. This prediction is supported by the analyst target price forecasts for ten major banks during the crisis since 2007; (ii) in the case that beliefs cross (i.e., pessimists become more optimistic than the initial optimists), liquidity provision occurs but can be delayed due to the option to sell cash higher (using the troubled asset as numeraire) if the downturn worsens. Government can reduce the option value of cash by acting as buyer of last resort.

*I thank Patrick Bolton, Martin Oehmke, Guillaume Plantin, Tano Santos, Suresh Sundaresan, Neng Wang, Wei Xiong, and seminar participants at Columbia Business School, Florida State University for helpful comments. Yang Chen provided excellent research assistance. Mathematica code to replicate the calculations in this paper can be found at http://www.columbia.edu/~jy2167.

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1 Introduction

During financial crises, distressed companies often hold the troubled assets for a long time. For example, years after the crisis started since 2007, there is little sign that major banks have offloaded much of the alphabet soup of debt that caused repeated write-downs, including the mortgage-backed securities (MBS), collateralized debt obligations (CDO), etc. The inability or unwillingness to sell via the market channel raises the concern with market liquidity. Curiously, the banks’ burdens of troubled assets coexist with large amount of outside capital that can be a potential source of liquidity yet, for some reasons, has not stepped in. For example, as of the first quarter of 2008, there are $3.4 trillion, $5.3 trillion, and $7.8 trillion of money market funds, Treasuries (excluding Agency and Government Sponsored Enterprise (GSE)), and time/saving deposits respectively, according to the Federal Reserve statistical release (2008). As of the second half of 2007, the notional amount of interest rate/foreign exchange derivatives and equity derivatives totaled $382 trillion and $10 trillion respectively (International Swaps and Derivatives Association (2007)). Although many of the derivatives positions are for hedge purposes, the data suggest neither a lack of capital nor a lack of appetite for risk. What reasons then delay the capital from flowing to the trouble assets?

This paper uses a dynamic model of heterogeneous beliefs to study the provision of market liquidity. In this model, investors have different priors at the start of a downturn and update their beliefs based on Bayesian learning from common information. In another word, the investors agree to disagree (Aumann (1976)). Trade (liquidity provision) opportunity arrives when beliefs cross, i.e., when an initial pessimist becomes more optimistic than an initial optimist that holds the troubled asset.

The model points out two reasons that can delay the provision of market liquidity. First, for some investors, beliefs may not cross and may instead diverge. i.e., the pessimists become more pessimistic relative to the optimists. This is seemingly contradictory to the prediction from Bayesian learning that beliefs converge when more data are available. The reason is that the convergence occurs for the belief parameter, which is the recovery probability in the model. However, asset value relates to the expected length of the bear market, which is the inverse of the recovery probability. In a prolonged bear market, Bayesian investors update their recovery probability towards zero. This, under certain conditions, leads to divergence in perceived asset values. This finding is unique
to prolonged bear markets or severe financial crises. Such divergence in valuations precludes some investors (pessimists) from providing liquidity. These pessimists buy few troubled assets and impose high financing cost to fund the optimists' purchases. Belief dynamics constructed from analyst target prices for ten major banks indicate that such predicted belief divergence occurs for some investors during 2007-08.

For other investors, belief crossing happens. This includes the notable case of an experienced pessimist who has a more precise prior than the optimist. When the bear market persists, both the pessimist and the optimist revise down their perceived asset values. However, the revision is slower if the pessimist has a sharper prior. In an extreme case, if the pessimist’s prior is infinitely precise (i.e., the pessimist knows the true recovery probability), the pessimist’s belief does not change and belief crossing occurs when the optimist becomes sufficiently pessimistic. This can capture the bottom fishing behavior by some sophisticated investors who hold cash and wait to buy other assets at distressed prices.

When beliefs cross, will the pessimists (who are the new optimists) immediately buy the troubled asset? The answer is no. This is the other reason that delays the provision of market liquidity. The delayed buying by pessimists is due to “heads I win” (when the bear market worsens, the pessimists can buy at bargain prices) and “tails I don’t lose” (when the bear market ends, holding cash does not incur an economic loss relative to buying immediately at perceived fair price). Such option value of cash is consistent with the anecdote that “cash is king” and provides an alternative perspective for the occurrence of cash hoarding during crises. Calibration shows that the option can result in substantial delays in liquidity provision, which increases the illiquidity of bank assets and leaves banks vulnerable to runs (Diamond and Dybvig (1983)). The belief dispersion also prevents competition (among pessimists) from eliminating the option value.\(^1\) The degree of competition is not solely determined by the amount of troubled asset relative to the amount of outside capital. Rather, belief dispersion plays an important role. Wider belief dispersion implies the outside capital is stretched thinner, which reduces competition per unit of belief dispersion and allows further delay in liquidity provision. Therefore, belief dispersion amplifies the “cash-in-the-market” pricing in Allen and Gale (1994). Increasing competition among potential liquidity providers helps

\(^1\)See Grenadier (2002), Lambrecht and Perraudin (2003), and Kondor (2007) for the effect of competition on option when beliefs are homogeneous
improve liquidity provision. Specifically, the model shows that if the government acts as the “buyer of last resort” and purchases a fraction of the troubled assets, the increased competition due to less supply of troubled assets reduces the delay in liquidity provision. This has implications for the US government’s Troubled Asset Relief Program (TARP). However, the effect from the buyer of last resort is limited. As long as there is belief dispersion, the option value to delay remains. Further, the improvement in liquidity should be weighed against other issues such as the taxes used to finance the intervention, distortions to incentives, and ex-ante efficiency (Allen and Gale (1998), Gorton and Huang (2004), and Bolton, Santos, and Scheinkman (2008)).

Interestingly, the option value of cash in crises relates to the speculative bubble seen in booms. Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003) show that an asset can be bid higher by speculators in anticipation of selling it to a “greater fool” in the future. Here, using the troubled asset as numeraire, cash is overvalued by liquidity providers in anticipation of selling cash higher in the future (i.e., when the bear market worsens, the troubled asset is worth less using cash as numeraire). The key ingredient shared by this paper and the speculative bubble literature is belief crossing and short-sales constraint of the overvalued asset. In the current paper, the overvalued asset is cash. Short-sales constraint implies that investors cannot borrow money unlimitedly to buy the troubled asset. This is not unrealistic in financial crises when investors are mainly concerned with deleveraging. The belief dispersion also implies high financing cost to borrow from pessimists. The result does not rely on short-sales constraint of the troubled asset. The option value of cash remains irrespective of whether the troubled asset can be shorted. However, whether the troubled asset can be shorted determines how the option value of cash manifests. When the trouble asset cannot be shorted, the option manifests as delay in liquidity provision. This finding is new to the speculative bubble literature and applies to many assets in the crisis since 2007. When the troubled asset can be shorted, the option is directly manifested in the price. In particular, some of the liquidity providers may even short at prices below their perceived fundamental, betting on further distress in the future. This resembles the predatory trading in Brunnermeier and Pedersen (2005), though the investors in the current paper are not strategic.

Heterogeneous investor models are useful for the study of liquidity provision and trading in gen-

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2 In October 2008, the U.S. signed into law a bill authorizing the Treasury department to purchase as much as $700 billion in troubled assets, $250 billion of which was made available to purchase equity in financial institutions.
eral. Heterogeneity is modeled in this paper by heterogeneity in beliefs, which allows tractable dynamics from Bayesian learning. Nonetheless, the option of delay can apply to other heterogeneities such as preferences, wealth fluctuations, or other constraints during crises, if such heterogeneities generate crossing of heterogeneous valuations.

Heterogeneous beliefs in this paper come from heterogeneous priors. Since a severe crisis like the one since 2007 is almost unprecedented and viewed by some as “a once-in-a-half-century, probably once-in-a-century type of event” (Greenspan (2008)), attributing belief dispersion regarding the crisis as mainly due to differences in priors is likely realistic. Investors have few experiences with similar events. This prevents learning from completely eliminating differences in priors. Heterogeneous beliefs can alternatively come from asymmetric information, which is also an important component in the market microstructure literature (e.g., Glosten and Milgrom (1985) and Kyle (1985)). To focus on its main contribution, this paper assumes away asymmetric information along with other important issues such as agency problem, maturity mismatch, mark to market, funding liquidity, solvency, liquidity spiral, complexity, specialized assets, and market segmentation among others. On the other hand, it shows that the option to delay liquidity provision does not rely on these other mechanisms.

Section 2 provides some empirical evidence regarding belief dynamics during 2007-08. Section 3 presents a model of heterogeneous beliefs. Section 4 shows the possibility of belief divergence. Section 5 shows the option to delay when beliefs cross and the effect of the buyer of last resort. Section 6 contains additional discussions and extensions of the model. Section 7 concludes. The appendix contains the proofs.

2 Belief dynamics during 2007-08

This section provides some empirical evidence regarding the belief dynamics during the crisis since 2007. Brunnermeier (2008) provides a detailed account of the crisis during 2007-08. Analyst target price forecast history is obtained from Bloomberg on Oct 28, 2008 for the five largest commercial banks and five largest investment banks according to equity market capitalization at the end of 2006. The ten companies include Bank of America, Citigroup, JPMorgan Chase, Wachovia, Wells

3See, for example, Bolton, Santos, and Scheinkman (2008), Brunnermeier and Oehmke (2009), Gorton (2009), He and Xiong (2008), and Oehmke (2008) for discussions on some of these issues.
Fargo, Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley. Target price forecasts are assumed to reveal analysts’ beliefs regarding stock valuations (target prices are shown to be informative by Brav and Lehavy (2003) and Asquith, Mikhail, and Au (2005)).

Bloomberg provides, for each stock, a list of analysts currently following the stock. However, this list does not include analysts who provided forecasts in the past but subsequently dropped coverage. This affects especially Bear Stearns (taken over by JPMorgan Chase in March 2008) and Lehman Brothers (filed for Chapter 11 bankruptcy protection in September 2008). To reduce the effect from dropped coverage, we search in Bloomberg for an analyst’s past coverage of all ten stocks as long as the analyst is currently covering at least one of the ten stocks. Bloomberg archives each analyst’s forecast history, including forecasts before coverage drop. This mitigates the effect of dropped coverage because an analyst covering one financial company likely covers some other financial companies, too. This results in a total of 51 unique analysts and 213 unique analyst-company pairs. Each analyst on average covers 4 companies and each financial stock on average has forecasts from 21 analysts. The data contain 4,824 target price forecasts during the sample period of 2003-2008 (1,298 forecasts in 2007 and 1,374 forecasts in 2008). Bloomberg provides a variable “Period” indicating the horizon of the target price forecasts. Among the 1,982 non-missing horizon indicators, 94% (1,864 observations) are one-year target prices. Those target prices for other horizons are excluded. If the horizon indicator is missing (2,842 observations), the target price forecast is included since it is likely a one-year forecast, judging from observations with non-missing horizon indicator.

Due to the stock price fluctuation, the same target price issued at different time can have different implications (e.g., a target price of $40 issued when the stock price is $35 differs from another forecast of the same target price issued when the stock price is $45). Correspondingly, Bloomberg provides the date of the analyst report and the closing stock price on the same day. We construct the scaled target price forecast for stock \( s \) on day \( \tau \) by analyst \( i \) as

\[
F_{s,\tau,i} = \frac{\text{Target price}_{s,\tau,i}}{\text{Close price}_{s,\tau}}.
\]

Such scaling by market price is consistent with Dokko and Edelstein (1989) and Brav and Lehavy (2003). It represents the rate of return implied by the target price. The resulting data contain
monthly scaled target prices for the ten companies. If an analyst issues multiple forecasts for the same company in a month, only the last forecast before month end is used. Analysts do not issue forecasts in all months. When an analyst does not issue forecast in a month, his most recent forecast in the past is assumed to be in effect for that month.4

For each stock at the end of 2006, we sort analysts into two groups based on their scaled target price. Those analysts whose forecasts are below (equal to or above) the median are classified as pessimists (optimists). There are a total of 20 groups: one optimist group and one pessimist group for each stock. If an analyst covers multiple stocks, it is possible that he is in the optimist group for one stock yet in the pessimist group for another stock. The classification remains fixed afterwards to see how the optimists’ and pessimists’ beliefs evolve during the financial crisis. Let $F_{s,\tau,g}$ denote the average of the scaled price targets from analysts in group $g$ (optimist group or pessimist group) for stock $s$ in month $\tau$. The following regression examines the belief dynamics during 2007-08.

$$F_{s,\tau,g} = \sum_{t=Dec\ 2006}^{Sep\ 2008} \beta_t \times PESSIMIST_{s,g} \times MONTHDUMMY_t + \sum_{t=Dec\ 2006}^{Sep\ 2008} \alpha_t \times MONTHDUMMY_t + \varepsilon_{s,\tau,g}$$  \hspace{1cm} (1)

where the dummy variable $PESSIMIST_{s,g}$ equals 1 (0) for the pessimist (optimist) group of stock $s$. The month dummy $MONTHDUMMY_t$ equals 1 if the forecast month equals $t$ and 0 otherwise. $t$ ranges from December 2006 to September 2008.5 The coefficients $\beta_t$ are the objects of interest, which measure the valuation difference between pessimists and optimists.

The results are shown in Table 1. At the end of 2006, normalizing the stock price to 1, the optimists across the ten stocks on average have a target price of 1.313, which was 31.3% higher than the stock price. The pessimists on average value the stock lower by 0.184. I.e., pessimists have a target price of 1.129. Both the optimists’ and the pessimists’ target prices are higher than the stock price, consistent with earlier findings in Brav and Lehavy (2003). Interestingly, the valuation of pessimists remained lower than optimists during 2007-08. The valuation discount is statistically

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4The 25%, 50%, and 75% quantiles of days between successive forecasts are 9, 21, and 49 calendar days respectively. The results are similar if forecasts older than 1 or 3 months are excluded. These results are omitted for brevity and are available from the author.

5Forecasts of Bear Stearns are included up to February 2008 and forecasts of Lehman Brothers are included up to August 2008, one month before their restructuring events. We have also run the regression (1) excluding Bear Stearns and Lehman Brothers. The results remain similar and are suppressed for brevity.
significant during the crisis before June 2008. After June 2008, the estimates become noisier though the point estimates indicate even larger valuation discounts.

Figure 1 shows a time-series plot of the $\beta_t$ estimates, the S&P 500 index, and an equal-weighted index of the ten financial stocks in 2007-08. In early 2007, before the financial crisis becomes headline, the valuation difference between optimists and pessimists shrinks. This is consistent with the prediction from Bayesian learning based on common information. However, the beliefs diverge after the crisis starts. Such divergence of belief is discussed in Section 4.

When beliefs cross (i.e., an initial pessimist becomes more optimistic than an initial optimist), the initial pessimist (who is the new optimist) may provide liquidity to the initial optimist. That the pessimists’ valuations remain lower than those of the optimists during 2007-08 have direct implications on the difficulty to obtain liquidity from the pessimists. Note that the lack of belief crossing during 2007-08 holds on average. Among individual analysts, some initial pessimists become more optimistic than some of the initial optimists. Liquidity provision in such scenario is discussed in Section 5.

3 A model of bottom fishing

3.1 Evidence of bottom fishing

This section provides a stylized model of investor bottom fishing. Bottom fishing loosely refers to the quest for “that fabled bottoming-out point that suggests a return to more bullish days” (Gaffen (2008)). To confirm investors’ interest in bottom fishing during bear markets, Figure 2 shows the monthly number of Wall Street Journal (WSJ) articles containing the word “bottom.” The data are from Factiva. The following is an example of the usage of “bottom”: “The best-case scenario is that banks will bottom out in the middle of 2009 if the economy starts to rebound” (Sidel (2008)). The assumption is that the media discussions reflect investors’ interest. Hence more mentions of “bottom” likely indicate increased contemplation about bottom fishing. Figure 2 compares the monthly WSJ article counts with two stock indices: S&P 500 and NASDAQ. The plot of article

\footnote{As a robustness check, the analysis is repeated on data one year earlier. I.e., optimists/pessimists are classified at the end of 2005 and held fixed during 2006. In this case, valuation convergence is observed throughout 2006. Unlike the belief dynamics during the crisis in 2007-08, there is no valuation divergence in 2006. Therefore, the belief dynamics in 2007-08 are unlikely driven by seasonality (e.g., Hong and Yu (2008)).}
counts, if placed upside down, closely resembles the plot of stock indices. The correlation coefficient between the monthly article count and the stock index level is $-0.62$ for the NASDAQ index and $-0.46$ for the S&P 500 index. During the bull market at the end of 1990s, the number of articles containing “bottom” drops to 20 per month (i.e., around 1 article per day) during early 2000. Following the dot-com bust, the number of articles increases dramatically and peaks at around 180 per month during 2002. After the stock market touches bottom in 2002 and subsequently rebounds, the number of articles drops to around 90 per month. When the stock market slides near the end of 2007, the number of articles starts to climb again. For the year 2008, an average of 114 articles per month contain the word “bottom”, compared to an average of 97 articles per month in the year 2007. The difference is statistically significant (White (1980) $t$-stat 2.61).

Determining when a downturn bottoms has also received a lot of attention from the academia. For example, the National Bureau of Economic Research (NBER) determines the business cycle peaks and troughs, which are very influential even ex-post.

The Wall Street Journal (Aug 4, 2008) conducted a fund manager interview where 5 managers (asset under management $17.8B, $12.6B, $2.3B, $1.3B, $1B) were selected who were in the top 25% of their Morningstar categories for the past 3 years through June 30, 2008 but in the bottom 25% for the first half of 2008. Many cited the same missteps: They misjudged the energy boom’s staying power, and they underestimated the depth of woes in the financial sector. Looking forward, there is little agreement on what’s ahead. This interview shows that, although fund managers consider the success of bottom fishing an important determinant of investment performance, there is little agreement regarding such forecasts.

3.2 Model setup

This section provides a continuous-time model on bottom fishing and the associated belief dynamics.

Assumption 1 (Securities). There are two securities: a risk-free asset (cash) and a risky asset (for concreteness, the asset is referred to as a mortgage-backed security (MBS) in this paper). The risk-free term structure is flat at rate $r$. The MBS has a total of $K$ shares outstanding. The MBS cannot be shorted. Each share of MBS pays dividend $d_t dt$ between time $t$ and $t+dt$, where $dt$ denotes
an infinitesimal period. There are two possible states: normal and bear market. The dividend

\[ d_t = \begin{cases} 
D & \text{in normal market} \\
D - \delta & \text{in bear market} 
\end{cases} \]

where \( 0 < \delta \leq D \) captures dividend cut during economic downturns.

The transition probability matrix from the state at time \( t \) to the state at time \( t + dt \) is

\[
\begin{array}{c|cc}
& \text{Normal } t + dt & \text{Bear } t + dt \\
\hline
\text{Normal } t & 1 & 0 \\
\text{Bear } t & \lambda dt & 1 - \lambda dt \\
\end{array}
\]

where \( \lambda \) is the parameter that governs the recovery intensity.\(^7\) Higher \( \lambda \) implies that the bear market is likely to end sooner (i.e., the bottom is near). Given \( \lambda \), the bear market is expected to last \( 1/\lambda \).

To simplify the illustration, the state is assumed to be observable (from the dividends) and the normal state is absorbing (i.e., the economy stays in the normal state forever once the existing bear market ends, which is an approximation to the infrequent occurrences of severe crises).

Investors, however, do not observe the recovery intensity. Motivated by the findings in Section 2, investors are assumed to have differences-of-opinion over the recovery intensity and update their beliefs via Bayesian learning.\(^8\)

Assumption 2 (Investors). Each investor’s prior of \( \lambda \) comes from the class of Gamma distributions, denoted by Gamma \( (a,b) \) where the parameters \( a > 1 \) and \( b \) may differ across investors. Investors update beliefs via Bayesian learning. The beliefs are common knowledge and investors agree to disagree (Aumann (1976)). Investors are risk neutral.

A Gamma distribution has two parameters.\(^9\) \( a \) and \( b \) are, respectively, the shape and scale parameters. To simplify notation during Bayesian updating, Gamma distribution in this paper is
expressed in terms of the inverse scale. I.e., $\text{Gamma}(a, b)$ in this paper is equivalent to the typical definition of Gamma distribution with parameters $a$ and $1/b$.

In this framework, differences of opinion come from differences in investors’ priors. Such differences in prior can come from nonstationarity, i.e., perhaps the crisis since 2007 is different from the great depression, hence investors cannot learn from past crises. Alternatively, the rare occurrences of severe downturns imply that learning from such experiences may not completely eliminate the differences in priors.

The Gamma priors allow closed-form characterization of Bayesian belief updating.

**Lemma 1** (Bayesian updating). *For an investor whose prior of recovery intensity $\lambda$ is $\text{Gamma}(a, b)$, the posterior after $\Delta$ periods of bear market is $\text{Gamma}(a, b + \Delta)$.*

**Lemma 2** (Properties of Gamma distribution). *The mean and variance of $\text{Gamma}(a, b)$ distribution is $E(\lambda) = a/b$, $\text{Var}(\lambda) = a/b^2$. When $a > 1$, the expected length of downturn is finite and given by $E[1/\lambda] = b/(a - 1)$.*

Lemmas 1 and 2 imply that, as the bear market lengthens, the posterior becomes more precise and expects longer bear market. Gamma distributions are commonly used to model wait time (wait time to the market bottom in this paper) and allow a rich set of possibilities. For example, they include the exponential and $\chi^2$ distributions as special cases.\(^{10}\) Figure 3 shows several examples of the Gamma probability density function. For an investor with prior $\text{Gamma}(20, 19)$, the belief peaks at $\lambda = 1$ which expects the bear market to last one year. The investor believes that $\lambda$ is very likely to be between 0.5 and 2 (i.e., the bear market is expected to last somewhere between 6 months and 2 years). This prior is more precise than the $\text{Gamma}(2, 1)$ prior, which also expects a one-year bear market. If the prior is $\text{Gamma}(2, 1)$, the posterior becomes $\text{Gamma}(2, 3)$ after observing two periods of bear market. The Bayesian learning has two effects. First, lower recovery intensity is considered more likely. Second, the posterior becomes more precise after observing more observations.

\(^{10}\) $\text{Gamma}(1, \lambda)$ gives the exponential distribution with parameter $\lambda$. $\text{Gamma}(v/2, 1/2)$ is the $\chi^2$ distribution with degree of freedom $v$. 
During the bear market, the net present value (NPV) of the MBS given $\lambda$ is

$$NPV (\lambda) = \int_0^\infty e^{-rt}Ddt - \int_0^\infty \left( \int_0^t e^{-rs} \delta ds \right) \lambda e^{-\lambda t} dt$$

$$= \frac{D}{r} - \frac{\delta}{r + \lambda}$$

which is the NPV of dividends during the normal state minus the expected losses during the bear market.

For an investor with belief $Gamma(a, b)$ regarding the recovery intensity, the fundamental value of the MBS is obtained by integrating (3) over the belief,

$$V(a, b) = E_{Gamma(a, b)} [NPV(\lambda)]$$

$$= D \left[ \frac{1}{r} - \frac{\delta}{D} \cdot b a e^{rb} r a - 1 \Gamma (1 - a, rb) \right]$$

$$\approx \frac{D}{r} - \delta \cdot \frac{b}{a - 1} \quad \text{when } r \text{ is small.}$$

(4) gives an exact formula and an approximation for $V(a, b)$. The exact formula involves the incomplete gamma function $\Gamma(\cdot, \cdot)$, which is well defined if the second argument is positive and can be evaluated numerically to arbitrary precision.\(^\text{11}\) Therefore, the exact valuation formula is essentially in closed form. The last step in (4) follows because, when $r$ is small, the expected loss during the bear market is determined by the expected length of bear market ($b/(a - 1)$ by Lemma 2). The propositions in this paper are proved using the exact as opposed to the approximate formula for $V(a, b)$ unless “$r$ is small” is explicitly stated. All the figures in this paper use the exact formula.

4 Case 1: Lack of belief crossing

The valuation of MBS differs across investors with different beliefs. When the bear market persists, investors’ beliefs change due to Bayesian updating. This raises two possibilities: (1) beliefs do not cross (i.e., initial optimists remain more optimistic than initial pessimists); (2) beliefs cross (i.e.,

\(^{11}\)Specifically, $\Gamma(x, y) \equiv \int_y^\infty t^{x-1} e^{-t} dt$. The software Mathematica can compute the incomplete gamma function to arbitrary precision using its command Gamma[\cdot,\cdot]. See Lebedev and Silverman (1972) for additional details on the incomplete gamma function.
initial optimists become less optimistic than initial pessimists). This section focuses on the first case and Section 5 focuses on the second case. In the first case, only the belief dynamics are involved. Therefore, an equilibrium is not explicitly constructed in this section to maintain generality of the results. However, this section implicitly has in mind that the initial pessimists holding cash are potential liquidity providers to the initial optimists holding the MBS. The lack of belief crossing impedes liquidity provision by the pessimists.

Let us consider two investors: an optimist and a pessimist. In this section, let \( \Gamma(a_O, b_O) \) and \( \Gamma(a_P, b_P) \) denote, respectively, the prior of the optimist and the pessimist. Proposition 1 shows that, as the bear market persists, the pessimist can become even more pessimistic relative to the optimist.

**Proposition 1.** The investors’ belief dynamics have the following properties:

1. **(Initial disagreement)** When \( \frac{b_O}{a_O - 1} < \frac{b_P}{a_P - 1} \) and \( r \) is sufficiently small, \( V(a_O, b_O) > V(a_P, b_P) \).
2. **(Belief updating)** The posteriors, after \( \Delta \) periods of bear market, become \( \Gamma(a_O, b_O + \Delta) \) and \( \Gamma(a_P, b_P + \Delta) \) for the optimist and the pessimist, respectively.
3. **(Initial divergence in valuation)** For given \( \Delta \), where \( a_O > a_P > 1 \) and \( r \) is sufficiently small,

\[
V(a_P, b_P + \Delta) - V(a_P, b_P) < V(a_O, b_O + \Delta) - V(a_O, b_O) < 0.
\]

4. **(Eventual convergence in valuation)** \( \lim_{\Delta \to \infty} V(a_O, b_O + \Delta) = \lim_{\Delta \to \infty} V(a_P, b_P + \Delta) = (D - \delta) / r \).

(5) is the main result of this section. It predicts that the valuation of the pessimist drops faster than the optimist’s valuation if \( a_O > a_P \). Because the parameter \( a \) relates to both the prior mean and the variance of the recovery intensity, the prediction is not too surprising without controlling the belief precision. For example, if the pessimist starts out with a less informative prior, the pessimist’s posterior valuation drops faster with additional observations of bear market. In an extreme case, if the optimist’s prior is infinitely precise (i.e., the optimist knows the recovery intensity), the optimist’s valuation does not change while the pessimist’s valuation drops when the
bear market persists. Interestingly, (5) holds even after controlling the prior precision, as shown in the corollary below.

**Corollary 1.** Proposition 1 applies to the case where $a_P > 1$ and

\[
E_O(\lambda) = \frac{a_O}{b_O} > \frac{a_P}{b_P} = E_P(\lambda) \\
Var_O(\lambda) = \frac{a_O}{b_O^2} = \frac{a_P}{b_P^2} = Var_P(\lambda).
\]

I.e., when investors have the same prior precision.

It is known that Bayesian learning based on common information leads to posteriors that converge. However, Proposition 1 and Corollary 1 show divergent valuation. What explains the difference? To see this, (3) shows that, given recovery intensity, the valuation difference between the optimist and the pessimist is

\[
\left(\frac{1}{r + \lambda_P} - \frac{1}{r + \lambda_O}\right) \delta \approx \left(\frac{1}{\lambda_P} - \frac{1}{\lambda_O}\right) \delta
\]

when $r$ is small. $\lambda_O$ and $\lambda_P$ denote, respectively, recovery intensity in the eyes of the optimist and the pessimist. When the bear market persists, both investors update their belief of recovery intensity towards zero. This convergence in posterior belief is predicted by Bayesian learning. However, since $\lambda$ is in the denominator in (6), the convergence to zero of $\lambda$ can imply divergence in valuation. Because this paper studies liquidity provision during bear market. Proposition 1 and Corollary 1 are conditioning on being in the bear market. Therefore, the beliefs do not exhibit the “eternal switching” in Morris (1996). The divergence in valuation is unique to prolonged bear markets, where the posterior belief converges to $\lambda = 0$. Such low recovery intensity can capture for example the concern of whether the recent downturn since 2007 will last as long as the one in Japan after the 1980s.

Table 2 shows an example where the priors of the optimist and the pessimist are Gamma(2, 1) and Gamma(9/8, 3/4). From Lemma 2, both investors have the same prior precision. The optimist’s (pessimist’s) prior mean of recovery intensity is $E_O(\lambda) = 2$ ($E_P(\lambda) = 3/2$). The difference is 1/2. The optimist (pessimist) expects the bear market to last 1 period (6 periods). The difference is 5 periods. After an additional period of bear market is observed, the posterior of the optimist
(pessimist) is \( \text{Gamma} (2, 2) \) \( (\text{Gamma} (9/8, 7/4)) \). The posterior mean of recovery intensity is now \( E_O (\lambda) = 1 \) \( (E_P (\lambda) = 9/14) \). The difference, 5/14, is smaller than the difference in prior means. However, the optimist (pessimist) now expects the bear market to last 2 (14) periods. The difference in expected bear market length increases to 12 periods, which translates to increased valuation difference between the optimist and the pessimist.

(5) holds under the condition \( a_O > a_P \). This is because the expected bear market length is \( b/(a - 1) \) if the belief is \( \text{Gamma} (a, b) \). After \( \Delta \) periods of downturn, the belief updates to \( \text{Gamma} (a, b + \Delta) \) which expects the bear market to last \( (b + \Delta)/(a - 1) \). The increase in expected bear market length (hence decrease in valuation) is greater if \( a \) is smaller, which reflects the intuition in (6).

When the bear market persists, Proposition 1 shows that eventually the valuations converge. However, the initial valuation divergence can be significant as illustrated by Figure 4. In Figure 4, the optimist’s and the pessimist’s priors are set to \( \text{Gamma} (2, 1) \) and \( \text{Gamma} (9/8, 3/4) \). The priors are the same as those in Table 2. \( r = 0.5\% \) per month. The dividend is \( D = 1 \) and \( D - \delta = 1/10 \) in normal and bear market, respectively. Initially, the optimist (pessimist) expects the bear market to last 1 month (6 months). The valuation discount is less than 1% (optimist valuation: $199.12, pessimist valuation: $197.52). After \( \Delta \)-periods of downturn, the valuation discount of pessimist relative to optimist is \( V (9/8, 3/4 + \Delta) / V (2, 1 + \Delta) - 1 \). Interestingly, as the bear market persists, the valuation discount deepens. After two years in the bear market, the discount increases to over 10% (optimist valuation: $182.67, pessimist valuation: $164.10). Eventually, the discount converges back to zero. However, the convergence does not occur until after a long time. In this example, the valuation discount keeps increasing for about 50 years and peaks at over 25%. That the convergence in valuation occurs so late (after 50 years) makes it less relevant for practical purposes. Therefore, the divergence in valuation can be a significant impediment to liquidity provision by the initial pessimists.

5 Case 2: Option value of cash and delayed liquidity provision

This section studies the case of belief crossing (i.e., initial optimists become less optimistic than initial pessimists). Belief crossing may coexist with lack of belief crossing. For those pessimists
who remain more pessimistic than the initial optimists as in Section 4, it will be shown later that they do not step in to provide liquidity. Therefore, this section assumes away lack of belief crossing without loss of generality. When beliefs cross, a pessimist may step in to purchase the MBS from the initial optimists holding the asset. This can be viewed as liquidity provision by the pessimists. Such liquidity provision depends not only on the pessimist’s own belief dynamics, but also on other aspects of the economy such as the equilibrium prices. Therefore, this section makes some additional assumptions to construct an equilibrium explicitly.

**Assumption 3 (Optimists).** There is a continuum of investors who collectively hold the $K$ units of MBS outstanding at the beginning of the bear market. They are referred to as optimists. Each optimist is identical and has prior $\text{Gamma}(a, b)$ at the beginning of the bear market. $a > 1$.

The assumption of homogeneous optimists simplifies the illustration and allows the focus on liquidity provision by pessimists. Extension to heterogeneous optimists is discussed in Section 6.3.

**Assumption 4 (Pessimists).** There is a continuum of investors indexed by $i \in [0, 1]$ who hold cash at the beginning of the bear market. They are referred to as pessimists. Each pessimist has sufficient capital to buy $M$ shares of MBS. $M > K$, i.e., pessimists collectively can absorb all the MBS outstanding.

The pessimists are assumed to have heterogeneous beliefs. The following assumption ensures that the pessimists have lower MBS valuation than the optimists at the beginning of the bear market and that beliefs cross during the bear market.

**Assumption 5 (Pessimists’ beliefs).** The prior of a pessimist $i \in [0, 1]$ at the beginning of the bear market is $\text{Gamma}(a_L, b_L + ig)$ where $g > 0$ captures belief dispersion among pessimists. Further,

$$a_L > a, \quad \frac{b_L}{a_L - 1} > \frac{b}{a - 1}. \quad (7)$$

The condition $a_L > a$ ensures belief crossing between optimists and pessimists because the condition of (5) is violated. The second condition ensures that the pessimists have lower MBS valuation than optimists at the beginning of the bear market (recall that, when $r$ is small, the MBS valuation in (4) is determined by $b/(a - 1)$ which is the expected length of the bear market if the belief is $\text{Gamma}(a, b)$).
Interestingly, the pessimists can be interpreted as experienced investors who are initially pessimistic, as shown in the following proposition.

**Proposition 2.** At the beginning of the bear market,

\[ \text{Var}_i(\lambda) < \text{Var}(\lambda) \quad \text{for any } i \in [0, 1] \]

where \( \text{Var}_i \) and \( \text{Var} \) denote the variance implied by priors of the pessimist \( i \) and the optimist, respectively, regarding the recovery intensity \( \lambda \).

Intuitively, the sharper priors of the pessimists induce slower belief updating by the pessimists than the optimists. This leads to the possibility of belief crossing. In an extreme case, if a pessimist’s belief is infinitely precise (i.e., the pessimist knows the recovery intensity), the belief does not change with new observations. Therefore, when the optimists become sufficiently pessimistic, belief crossing occurs. Such interpretation of experienced investors can be used to analyze liquidity provision by sophisticated investors.

(7) also implies that a more pessimistic pessimist stays more pessimistic during the bear market, as shown in the following proposition.

**Proposition 3.** For any two pessimists \( i < j \), the expected length of the bear market is smaller for \( i \) than \( j \) at any point in time during the bear market.

Therefore, belief crossing occurs between the optimists and pessimists, but not among the pessimists in this model. A more pessimistic pessimist stays more pessimistic throughout the bear market. Such layering of the potential liquidity providers not only simplifies the analysis, but also can capture the various investors such as hedge funds, private equity funds, sovereign wealth funds, pension funds, mutual funds, individual investors, etc. These investors, for various reasons, may step in under different scenarios to provide liquidity. The modeling choice of heterogeneous belief can capture actual disagreements among these investors, but it may also serve as a parsimonious way to capture other differences such as differences in incentives, required rates of return, etc.

In this model, the optimists start out holding the MBS. When the bear market progresses, the optimists gradually revise downward their valuations. Eventually, belief crossing occurs when the optimists’ valuation drops below that of the pessimist \( i = 0 \). If the bear market persists,
the optimists’ valuation will cross below that of additional pessimists. To simplify terminology, this paper uses pessimists to refer to the initial pessimists even after their valuations subsequently become more optimistic than the initial optimists. After beliefs cross, the pessimists value the MBS higher from a buy-and-hold perspective than the optimists. Will the pessimists step in to provide liquidity? If so, when will they step in? This paper shows next that the answer to the first question is, perhaps naturally, yes. However, the answer to the second question forms another impediment to liquidity provision.

Before the optimists sell out of the MBS, the optimists remain the marginal investor whose valuation determines the MBS price. For a pessimist, this induces a dichotomy between buy-and-hold and short-term returns. A pessimist’s buy-and-hold return is determined only by the pessimist’s belief. However, the pessimist’s short-term expected return is also affected by the market price fluctuation that is determined by the optimists’ belief. Such dichotomy, documented in the following proposition, affects the buying decisions of the pessimists.

**Proposition 4.** Let $\Gamma(\alpha, \beta)$ and $\Gamma(A, B)$ denote, respectively, the belief of optimists (marginal investor) and a pessimist at some point in time during the bear market. The pessimist’s instantaneous expected return from investing in MBS is above the cost of capital $r$ if

$$\frac{A}{B} \geq \frac{\alpha}{\beta}.$$  \hspace{1cm} (8)

Further, when $r$ is sufficiently small, the MBS price is below the pessimist’s buy-and-hold valuation if

$$\frac{A}{B} \geq \frac{\alpha}{\beta} - \left(\frac{1}{\beta} - \frac{1}{B}\right).$$

Note that $B > \beta$ because of (7). Therefore, for the pessimist, there is a parameter region where,

$$\frac{\alpha}{\beta} - \left(\frac{1}{\beta} - \frac{1}{B}\right) \leq \frac{A}{B} < \frac{\alpha}{\beta}. \hspace{1cm} (9)$$

In the situation described by (9), the market price is already below the pessimist’s buy-and-hold valuation. However, the pessimist prefers to delay buying because the short-term expected return is too low. This is because, if the bear market persists, the optimists’ valuation (hence the market
price) drops faster than the pessimist’s valuation. This presents an opportunity to buy at more distressed price that the pessimist prefers to wait for. For the pessimist, this is equivalent to an option value of cash: cash is more overvalued (using the MBS as numeraire) when the bear market worsens.

A pessimist does not wait forever. When condition (8) is met, the pessimist exercises the option and buys the MBS. In this case, the MBS is sufficiently distressed that further waiting risks losing the profit when the recovery takes place. Alternatively, competition from other pessimists may prevent waiting until the ideal entry condition (8). Such effect from competition on option exercise is consistent with Grenadier (2002), Lambrecht and Perraudin (2003), and Kondor (2007). However, different from these studies, the differences-of-opinion that generate the option value of cash also hinder the competition from more pessimistic pessimists. The effect of belief dispersion on competition is formalized in the following proposition.

**Proposition 5.** For a pessimist $i$, let $j(i)$ refer to the pessimist whose buy-and-hold valuation equals the MBS price at the time when $i$’s instantaneous expected return of holding MBS equals $r$. Pessimists $(i, j(i))$ can potentially interfere with $i$’s waiting. When $r$ is sufficiently small,

$$j(i) = i + \frac{bL - b}{a - 1} \frac{i}{g} + \frac{i}{a - 1}.$$

Pessimists $(i, j(i))$ collectively have capital $(j(i) - i) M$. For the most optimistic pessimist $i = 0$, the total amount of competition is

$$j(0) M = \frac{bL - b}{a - 1} \frac{M}{g}.$$  \hspace{1cm} (10)

Irrespective of the total capital $M$, the pessimist $i = 0$ can afford to wait if the belief dispersion $g$ is sufficiently big that $\frac{bL - b}{a - 1} \frac{M}{g} < K$. On the contrary, when belief dispersion disappears ($g \downarrow 0$), pessimists buy immediately after market price equals buy-and-hold valuation.

Belief dispersion affects competition. $M/g$ in (10) is the available capital per unit of belief dispersion. Even if the liquidity providers’ capital is plenty, more belief dispersion implies that the capital is spread thinner. This reduces competition per unit of belief dispersion and exacerbates delay in liquidity provision. Therefore, belief dispersion amplifies the “cash-in-the-market” pricing in Allen and Gale (1994). Even if the available cash is much more than the troubled asset, significant
delays in liquidity provision can occur if there is belief dispersion, which is shown to be the case by Section 2 for the financial crisis since 2007.

Combining Proposition 4 and 5 gives the timing of liquidity provision by pessimists. Note that Proposition 5 places only a lower bound on the delay in liquidity provision because it assumes the competitors step in at their buy-and-hold valuations. Proposition 6 below shows the equilibrium timing of liquidity provision.

**Proposition 6** (Delayed liquidity provision). *When \( r \) is sufficiently small, the time \( t(i) \) when investor \( i \in [0, K/M] \) steps in to provide liquidity is*

\[
t(i) = n(i) + \min{(t_1^*(i) , t_2^*(i))}
\]

where

\[
n(i) = \frac{(a-1)bL - b(aL-1)}{aL-a} + \frac{a-1}{aL-a}ig
\]

\[
t_1^*(i) = \frac{bL-b}{aL-a} + \frac{i}{aL-a}g
\]

\[
t_2^*(i) = g \frac{a-1}{aL-a} \left( \frac{K}{M} - i \right).
\]

\( n(i) \) is the time when the market price reaches \( i \)'s buy-and-hold valuation. \( t_1^* \) is the wait time until the short-term expected return becomes attractive. \( t_2^* \) is the time before competition absorbs all the MBS. \( n, t_1^*, t_2^* \), and hence \( t \) are increasing in \( g \) (longer delay when belief is more dispersed).

Figure 5 visualizes the equilibrium. At the start of the bear market, the most optimistic pessimist \((i = 0)\) values the MBS at $196.11, just below the optimists’ valuation of $196.13. The market price equals the optimists’ valuation until liquidation by optimists is complete. The market price drops below the buy-and-hold valuation of pessimist \(i = 0\) almost immediately into the bear market. However, due to the option value of waiting, pessimist 0 does not buy MBS until 3.24 months into the downturn at which point the market price is $188.05 and pessimist 0’s buy-and-hold valuation is $194.78. This corresponds to a delay of about 3 months in liquidity provision by pessimist 0. Pessimist 0, being the most optimistic liquidity providers, is less constrained by competition from pessimists and can wait until the optimal entry time (\( t_1^* \) in Proposition 6).
Similarly, the MBS price drops to the buy-and-hold valuation of pessimist $i = 0.1$ at 10.94 months into the bear market (MBS price is $175.19$). However, the pessimist does not step in until month 25.45 when all remaining pessimists swarm in together. This represents a delay of over a year. At this time, the buy-and-hold valuation of pessimist 0.1 is $171.16$, which is far above the MBS price of $159.06$. More pessimistic pessimists can afford to delay longer because they perceive a smaller chance of recovery hence a bigger chance of buying at more distressed prices when the bear market continues. Interestingly, without the threat of competition, pessimist 0.1 would have liked to wait even longer. Due to competition, those pessimists $i \geq 0.084$ buy at the same time (25.45 months into the bear market) and, by doing so, absorb all the MBS from the optimists. Among these pessimists, the last liquidity provider $i = 0.2$ buys immediately when the MBS price reaches the buy-and-hold valuation. However, except the last liquidity provider, none of the other pessimists step in immediately when the MBS price reaches the buy-and-hold valuations.

The delay in liquidity provision can be large. The second plot in Figure 5 shows the cumulative fraction of MBS sold by optimists to pessimists over time. The plot shows two cases: the equilibrium in Proposition 6 and the case assuming pessimists do not wait and step in as soon as the MBS price reaches the buy-and-hold valuations. In the equilibrium in Proposition 6, The first sale does not even occur until 3.24 months into the downturn. Without waiting, the pessimists would have already acquired 18% of the MBS by this time. One year into the bear market, only 17% of the MBS are bought by the pessimists compared to 54% without waiting. At the two-year point, more than 60% of MBS is still left stuck with the optimists while, without waiting, the pessimists would have absorbed over 95% of MBS. To put these numbers into perspective, banks have raised about $300$ billion equity one year into the recent liquidity crisis (Calomiris (2008)). Let us assume that this is the equilibrium outcome in Proposition 6 and use the parameters in Figure 5. Without the option value of cash, the banks would have been able to raise a total of $950$ ($300 \times 54/17$) billion equity instead of $300$ billion after one year in the downturn. The extra $650$ billion is comparable in size to the $700$ billion government rescue package signed in October 2008 (of which $250$ billion was made available to purchase equity in financial institutions).

Knowing the optimal entry time $t(i)$ of each pessimist from Proposition 6, the option value of holding cash can be explicitly calculated. Let $\pi(i,t)$ denote the expected (economic) profit per unit of MBS by pessimist $i$ at time $t$. At the actual entry time $t(i)$, the profit is the difference between
the buy-and-hold valuation and the MBS price,

$$\pi (i, t (i)) = V (a_L, b_L + ig + t (i)) - V (a, b + t (i)). \quad (11)$$

The profit is strictly positive for all but the last liquidity providers. Prior to entry, the expected profit needs to be discounted for the time value of money and for the probability of recovery. I.e., for $t < t^*(i)$,

$$\pi (i, t) = e^{-(r(t^*(i) - t))} P \text{ (no recovery before } t (i)) \pi (i, t (i))$$

where the probability is taken under pessimist $i$’s posterior at time $t$. The following proposition provides a closed-form expression of the expected profit.

**Proposition 7.** For pessimist $i \in [0, K/M]$, when $r$ is sufficiently small, the expected profit from waiting is

$$\pi (i, t) = e^{-(r(t^*(i) - t))} \left( \frac{b_L + ig + t}{b_L + ig + t (i)} \right)^{a_L} \pi (i, t (i)) \quad (12)$$

per share of MBS at time $t \leq t^*(i)$. $t^*(i)$ is the optimal entry time by pessimist $i$ in Proposition 6. $\pi (i, t (i))$ is the profit upon entry in (11).

The first plot in Figure 5 shows the dynamics of the reservation value for pessimist $i = 0$. The reservation value equals the buy-and-hold value minus the option value in (12). The pessimist buys when the MBS price reaches the reservation value instead of the buy-and-hold value. Upon exercising the option at about 3 months into the bear market, this pessimist receives a profit of about $6.7, which is the difference between the buy-and-hold value and the MBS price.

Figure 6 plots the expected profit per share from optimally exercising the option to delay liquidity provision. For a given pessimist, the expected profit increases over time due to the discount of both time value of money and the probability of recovery. At a given time in the bear market, the expected profit is hump shaped across liquidity providers. By waiting, each pessimist hopes to buy at more distressed prices when the bear market persists. Any delay, however, risks losing the existing profit if the economy recovers. Because more pessimistic pessimists perceive a smaller probability of recovery, they choose to wait longer and buy at more distressed prices. This tends to increase the expected profit. However, for very pessimistic investors, the threat of competition from other pessimists dominates. This tends to decrease the expected profit. The last
liquidity provider $i = 0.2$ cannot afford to delay at all and has an expected profit of zero.

5.1 Buyer of last resort

A recent development is that the government acts as a “buyer of last resort” in addition to being a lender of last resort. This section studies the effect of the buyer of last resort on the other liquidity providers. Specifically, assuming the government purchases $k < K$ units of MBS, Proposition 8 shows that such government purchase reduces the delay in liquidity provision.

**Proposition 8.** If $r$ is sufficiently small, when the government purchases $k < K$ units of MBS, pessimist $i \in [0, (K - k)/M]$ buys at time

$$t(i, k) = n(i) + \min (t_1^*(i), t_2^*(i, k))$$

where

$$t_2^*(i, k) = g \frac{a - 1}{aL - a} \left( \frac{K - k}{M} - i \right).$$

$n(i)$ and $t_1^*(i)$ are defined as in Proposition 6. Further,

$$t(i, k) \leq t(i)$$

where $t(i)$ is the time of liquidity provision in Proposition 6 and the inequality is strict for some $i$.

Government purchase increases competition for the MBS. This forces some pessimists to exercise their options early. As an example, Figure 7 compares the speed of liquidity provision with and without government intervention. The government is assumed to purchase a quarter of the total supply of MBS. When the government intervenes, the last pessimist required to absorb all the MBS drops from $i = 0.2$ to 0.15. The increased competition forces pessimists $i \in (0.055, 0.15]$ to enter early at 17.89 months into the bear market. Some of these pessimists would have entered as late as 25.45 months, more than half a year later, without the buyer of last resort. However, the effect of government intervention is limited. First, pessimists $i \in (0.055, 0.15]$ still delay buying after the MBS price reaches the buy-and-hold values. Further, those pessimists $i \in [0, 0.055]$ are not affected at all. This is consistent with Proposition 5 in that the most optimistic pessimists can always
afford to wait as long as there is sufficient belief dispersion. Unless the government purchases a bulk of the MBS, it is difficult to eliminate the delay by the most optimistic pessimists. Further, distortionary taxes used to finance the intervention should be weighed against the benefit from improved liquidity provision. The financing of government intervention, along with other issues such as distortions to incentives, is beyond the scope of this paper.

6 Discussions and extensions

6.1 Speculative bubble

Section 5 shows that the liquidity providers do not buy immediately after the MBS price drops to the buy-and-hold values. Rather, the reservation value equals the buy-and-hold value minus the option value to hold cash. The pessimists only buy when the MBS price drops to the reservation values. Such option value of cash, interestingly, relates to the speculative bubble typically associated with boom times. For example, Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003) show a speculative bubble when a speculator bids up the price of an asset in anticipation of selling it to a “greater fool” in the future. The option value to wait in the current paper can be viewed as a bubble in cash when the liquidity providers hope to sell cash higher (using the MBS as numeraire) in the future when the bear market worsens. Due to heterogeneous beliefs, competition does not eliminate the bubble, which is similar to Abreu and Brunnermeier (2003) though the current paper does not depart from common knowledge.

The key ingredient shared by the current paper and Scheinkman and Xiong (2003) is belief crossing: the initial optimists become more pessimistic than the initial pessimists. In this paper, belief crossing is the result of Bayesian learning, when the pessimists are experienced. I.e., they have a sharper prior than the optimists which implies the pessimists reduce valuation slower than the optimists. This can be used to model sophisticated investors trying to do bottom fishing during the bear market.

Similar to Scheinkman and Xiong (2003), short-sales constraint is necessary to generate the option value. In the current model, this means short-sales constraint in cash. Since private investors cannot print money, this short-sales constraint is equivalent to a constraint on leverage. I.e., investors cannot borrow money to buy the MBS unlimitedly, which is not unreasonable during
liquidity crunches. The belief dispersion also raises the financing cost if the optimists try to borrow from the pessimists. Such constraint is captured parsimoniously in the model by assuming a pessimist can buy at most $M$ shares of the MBS.

Different from Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003), the option value is not directly manifested in the MBS price. Section 5 shows that, before the MBS is entirely absorbed by the pessimists, the optimists remain the marginal investor. The MBS price reflects the optimists’ valuation and is not affected by the option value perceived by the pessimists. Instead, the option value manifests as delay in liquidity provision. This is due to the short-sales constraint of the MBS assumed in Section 5, which can be realistic for some of the distressed assets in the financial crisis since 2007. Such delay in liquidity provision due to the option value of cash is a new addition to the speculative bubble literature.

6.2 Short-selling MBS

The short-sales constraint of the MBS is not essential for the option value of cash. Rather, it determines how the option value manifests itself. As discussed in Section 6.1, the short-sales constraint of MBS implies a delay in liquidity provision. When short sales are allowed, the option value manifests directly as lower price of MBS (i.e., higher price of cash when the MBS is numeraire), which is consistent with the speculative bubble literature. An earlier version of the paper finds that allowing shorting of the MBS results in an equilibrium resembling predatory trading in Brunnermeier and Pedersen (2005). Specifically, some pessimists short the MBS at a price below the buy-and-hold values believed by these pessimists. This equilibrium is observationally similar to that in Brunnermeier and Pedersen (2005) except that the predators are not strategic. These results are suppressed and are available from the author.

6.3 Seller heterogeneity

Section 5 assumes homogeneity among the optimists to focus on the option value to hold cash for pessimists and the resulting delay in liquidity provision. When the optimists are heterogeneous, there can also be an option value to continue holding the MBS for optimists. This is due to the same reason of belief crossing. Consider an optimist and a pessimist whose have the same buy-and-hold valuation. If the optimist holds on to the MBS and the bear market persists, the result next
period is “heads I win” (when the bear market ends) or “tails you lose” (when the bear market persists, the optimist can sell to the pessimist because the pessimist’s valuation exceeds that of the optimist). This option value of holding MBS is absent when the optimists are homogeneous due to competition. The option value raises the optimists’ reservation values to sell, consistent with the studies on speculative bubbles in, for example, Scheinkman and Xiong (2003). The equilibrium can be solved similarly to the one in Section 5. An optimist’s reservation value to sell equals the buy-and-hold value plus the option value to hold MBS, which reflects the optimist’s unwillingness to sell. A pessimist’s reservation value to buy equals the buy-and-hold value minus the option value to hold cash, which reflects the optimist’s inability to sell. Trade occurs when the reservation values cross instead of when the buy-and-hold values cross.

6.4 Alternative belief distributions

Sections 4 and 5 use Gamma priors to obtain closed-form solutions. The tractability is due to Gamma distribution being the conjugate prior of exponential distributions (the length of the bear market is exponentially distributed in these two sections), which allows Bayesian updating in closed form. However, the intuition in Section 4 that investors’ beliefs of recovery probability converge towards zero in prolonged bear markets and the intuition in Section 5 that belief crossing generates an option value likely apply more generally. Specifically, in an earlier version of the paper, similar results are obtained from a discrete-time transition matrix (instead of the continuous-time transition matrix (2)) and priors from the class of Beta distributions (the conjugate prior of Bernoulli transition probability in discrete time). These results are omitted for brevity.

6.5 Multiple inferences

Sections 4 and 5 involve inference regarding the recovery intensity. Investors may need to infer additional unobservable variables. For example, the current state (whether in the bear market or not) may be unobservable, or there may be more than two states. Alternatively, the recovery intensity may be time varying and investors need to infer not only the level but also the rate of change of the recovery intensity, etc. If investors who are optimistic in recovery intensity are also optimistic in the other unobservable variables, the additional inferences exacerbate belief dispersion and its effect. On the contrary, if investors who are optimistic in recovery intensity are pessimistic
in the other unobservable variables, the optimism and pessimism cancel out and belief dispersion is smaller. In general, more layers of inference allow more room for the mechanism in this paper.

6.6 Alternative preferences

Sections 4 and 5 simplify the analysis by assuming risk neutrality, which can be viewed as pricing under the risk-neutral measure that can also reflect heterogeneity in risk aversion. Specifically, risk aversion or Knightian uncertainty (e.g., Epstein and Wang (1994)) can affect the liquidity providers’ buy-and-hold values. However, the option value of cash in Section 5 remains as long as beliefs (adjusted for risk/uncertainty aversion) cross.

7 Conclusion

This paper provides a dynamic model of heterogeneous beliefs to illustrate two reasons that can delay liquidity provision. First, the beliefs of pessimists and optimists may diverge during severe bear markets. In this case, liquidity provision by pessimists is ruled out due to their increased pessimism. Further, in the case when the pessimists do become sufficiently optimistic, this paper shows that there is an option value of holding cash, which can result in significant delays in liquidity provision. Such option value of cash, interestingly, is the flip side of the speculative bubble in boom times.

Government, by acting as the buyer of last resort, can reduce the option value of cash and improve the provision of market liquidity. However, the effect of the government is limited. How to eliminate the delay in liquidity provision due to the option value of cash remains a puzzle. Left unresolved, it implies that the right question regarding the next liquidity crunch is not “if” but “when.”

References


Bolton, Patrick, Tano Santos, and José Scheinkman, 2008, Inside and outside liquidity, working paper.


Greenspan, Alan, 2008, Interview with ABC’s “This week”, September 14.


He, Zhiguo, and Wei Xiong, 2008, Delegated asset management and market segmentation, working paper.


Oehmke, Martin, 2008, Gradual arbitrage, working paper.


Appendix Proofs

**Proof of Lemma 1**: This lemma follows from the Bayes rule by noticing the probability of staying in the downturn is $e^{-\lambda t}$ over a period of length $t$. The expected values are from integration over the Gamma distribution function.

**Proof of Proposition 1**: When $r \downarrow 0$, (4) implies

$$V(a_O, b_O) - V(a_P, b_P) = \delta \left( \frac{b_P}{a_P - 1} - \frac{b_O}{a_O - 1} \right).$$

(13)

Therefore, $V(a_O, b_O) > V(a_P, b_P)$ if investor $O$ expects a shorter downturn (optimist).

The belief updating follows from Lemma 1. After $\Delta$ periods of continuous downturn, the investor with prior $\text{Gamma}(a, b)$ updates his belief to $\text{Gamma}(a, b + \Delta)$, and the valuation change is

$$V(a, b + \Delta) - V(a, b) = \delta \left( \frac{b}{a - 1} - \frac{b + \Delta}{a - 1} \right) = -\delta \cdot \frac{\Delta}{a - 1}$$

when $r$ is sufficiently small. Both investors revise down their valuations after observing a prolonged downturn, but investor $O$ revises less if $a_O > a_P > 1$.

The eventual convergence in valuation occurs because, as the downturn persists, asymptotically both types of agents believe they will never get out of the downturn, hence both value the asset at $(D - \delta)/r$. Mathematically, it is because the limit of $b^a e^{rb} a^{-1} \Gamma (1 - a, rb)$ in (4) is $1/r$ when $b \uparrow \infty$.

**Proof of Corollary 1**: The assumption that $\frac{a_O}{b_O} > \frac{a_P}{b_P}$ yet $\frac{a_O}{b_O} = \frac{a_P}{b_P}$ implies $b_O > b_P$ which, together with $\frac{a_O}{b_O} > \frac{a_P}{b_P}$, further implies $a_O > a_P$ and $\frac{a_O - 1}{b_O} > \frac{a_P - 1}{b_P}$.

**Proof of Proposition 2**: (7) implies $a_{L} b_{L} < \frac{a}{b}$ and $\frac{a_{L}}{b_{L}} < \frac{a}{b^2}$. Therefore,

$$\text{Var}_{i}(\lambda) = \frac{a_{L}}{(b_{L} + ig)^2} < \frac{a}{b^2} = \text{Var}(\lambda).$$

**Proof of Proposition 3**: A pessimist $i$’s posterior is $\text{Gamma}(a_L, b_L + ig + \Delta)$ after $\Delta$ periods
of bear market. The expected length of bear market is \((b_L + ig + \Delta) / (a_L - 1)\), which is increasing in \(i\). 

**Proof of Proposition 4:** The liquidity provider’s expected instantaneous payoff

\[
(D - \delta) \, dt + \int \left[ (\lambda dt) \frac{D}{r} + (1 - \lambda dt) V(\alpha, \beta + dt) \right] f_{\text{Gamma}(A,B)}(\lambda) \, d\lambda
\]

\[
= (D - \delta) \, dt + \frac{D}{r} \left( \frac{A}{B} \right) \, dt + \left( 1 - \frac{A}{B} \, dt \right) V(\alpha, \beta + dt)
\]

(14)

where \(f\) is the posterior probability density of recovery intensity of the outsider. When the recovery occurs, price moves to \(D/r\). Otherwise, the price reflects the original MBS investors’ valuation.

The original MBS investors (marginal investors) calculate expected instantaneous payoff to be

\[
(D - \delta) \, dt + \frac{D}{r} \left( \frac{A}{B} \right) \, dt + \left( 1 - \frac{A}{B} \, dt \right) V(\alpha, \beta + dt)
\]

(15)

where, due to time consistency of Bayesian learning, the marginal investor’s expected return is \(r\) (intuitively this is because an optimist’s buy-and-hold and short-term returns are both determined by the optimist’s belief hence both expected returns are consistent with each other). Therefore, the liquidity provider’s expected instantaneous return is higher than \(r\) if and only if \(A/B \geq \alpha/\beta\).

When \(r\) is sufficiently small, (4) implies that the original MBS investors’ valuation is below that of the outsider if \(\frac{\beta}{\alpha - 1} \geq \frac{B}{A - 1}\). 

**Proof of Proposition 5:** After \(n\)-periods into the downturn, the outsider \(i\)’s posterior is \(\text{Gamma}(a_L, b_L + ig + n)\) and the MBS investor’s posterior is \(\text{Gamma}(a, b + n)\). By proposition 4, outsider \(i\) buys for the long term if

\[
\frac{b + n}{a - 1} = \frac{b_L + ig + n}{a_L - 1}.
\]

(16)

If he waits \(t\) more periods until the instantaneous return is attractive, the entry time satisfies

\[
\frac{a_L}{b_L + ig + n + t} = \frac{a}{b + n + t}.
\]

(17)

Let \(j\) be the outsider whose buy-and-hold valuation at time \(n + t\) equals market price, similar to
\[
\frac{b + n + t}{a - 1} = \frac{b_L + jg + n + t}{a_L - 1}.
\]

Eliminating \(n\) and \(t\) from the above three equations yields
\[
j - i = \frac{b_L - b}{a - 1} \frac{1}{g} + \frac{i}{a - 1}.
\]

**Proof of Proposition 6:** The outsider \(i = K/M\) who buys the last unit of MBS will buy as soon as his buy-and-hold valuation is reached. For investor \(i < K/M\), his buy-and-hold entry time \((n)\) and desired wait time \((t^*_1)\) until the instantaneous expected return equals \(r\) can be solved from (16) and (17). However, all the MBS may have been liquidated by the time \(t^*_1\). Therefore, investor \(i\) must step in before the last liquidity provider \(K/M\) does. The time until \(K/M\) steps in (which is \(t^*_2\)) can be calculated from (16) and (18) (specifically, solve for \(t\) by setting \(j = K/M\) in the two equations).

**Proof of Proposition 7:** Assuming the posterior regarding recovery intensity \(\lambda\) is \(\text{Gamma}(A,B)\) at time \(t\), the probability of no recovery before \(t(i)\) is
\[
\int e^{-\lambda(t(i) - t)} f_{\text{Gamma}(A,B)}(\lambda) d\lambda = \left(\frac{B}{B + t(i) - t}\right)^A
\]
where, given intensity \(\lambda\), \(e^{-\lambda(t(i) - t)}\) is the probability of no recovery between \(t\) and \(t(i)\). \(f_{\text{Gamma}(A,B)}(\cdot)\) denotes the probability density function of \(\text{Gamma}(A,B)\) distribution. The proposition follows because the posterior of pessimist \(i\) at time \(t\) is \(\text{Gamma}(a_L, b_L + ig + t)\).

**Proof of Proposition 8:** This follows directly from Proposition 6 by replacing the total net MBS supply to \(K - k\). To see \(t(i,k) < t(i)\), consider those pessimists indexed by \((K - k)/M - \varepsilon\) for small \(\varepsilon > 0\).
Table 1: Belief dynamics for ten major financial stocks during 2007-08

At the end of 2006, for each stock, analysts are classified into two groups. Those analysts whose target price forecasts (divided by the close prices on the days of analyst reports) are below (or above) median are classified as pessimists (or optimists). The classification remains fixed afterwards. For each stock $s$ and group $g$ (optimists or pessimists) in month $\tau$, let $\mathcal{F}_{s,\tau,g}$ denote the equal-weighted average target prices (divided by the close prices on the days of analyst reports) across analysts within that group. $\text{PESSIMIST}_{s,g}$ is a dummy that equals 1 for the pessimist group of a stock and 0 for the optimist group. $\text{MONTHDUMMY}_t$ is a month dummy that equals 1 if the forecast month $\tau$ equals $t$ and 0 otherwise. The target price forecast, forecast date, close stock price are obtained from Bloomberg. The sample consists of the five largest commercial banks and the five largest investment banks (by equity market capitalization) at the end of 2006. It includes Bank of America, Citigroup, JPMorgan Chase, Wachovia, Wells Fargo, Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley. The resulting panel consists of monthly observations of average target prices for each of the 20 analyst groups (one optimist group and one pessimist group for each of the ten stocks). The observations for Bear Stearns are up to February 2008 and the observations for Lehman Brothers are up to August 2008. This table shows the result of the following regression where the average target prices are regressed on the month dummies and the interactions between the pessimist group dummy and the month dummies. The t-statistics are adjusted for heteroskedasticity (White (1980)).

\[
\mathcal{F}_{s,\tau,g} = \sum_{t=Dec2006}^{Sep2008} \beta_t \times \text{PESSIMIST}_{s,g} \times \text{MONTHDUMMY}_t + \sum_{t=Dec2006}^{Sep2008} \alpha_t \times \text{MONTHDUMMY}_t + \varepsilon_{s,\tau,g}.
\]

<table>
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<th>$\alpha_t$</th>
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Table 2: Convergence in posterior yet divergence in valuation

Column (1) shows the prior of the optimist ($\text{Gamma}(2,1)$) and the prior of the pessimist ($\text{Gamma}(9/8,3/4)$), along with the prior mean ($E(\lambda)$) and expected length of the bear market $E[1/\lambda]$. The prior mean and the expected bear market length are calculated using Lemma 2. After one period of bear market, the optimist’s and the pessimist’s posteriors become, respectively, $\text{Gamma}(2,2)$ and $\text{Gamma}(9/8,7/4)$ according to Lemma 1. Column (2) calculates the posterior mean and the posterior expected length of the bear market.

<table>
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<tr>
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<td>1/2</td>
<td>5</td>
<td></td>
<td>5/14</td>
<td>12</td>
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The first plot shows the performance of S&P 500 index and an equal-weighted index of ten financial stocks in 2007-2008 (both indices normalized to 1 at the end of 2006). The ten stocks include Bank of America, Citigroup, JPMorgan Chase, Wachovia, Wells Fargo, Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley. The second plot shows $\beta_t$ in Table 1 which is the time series of the valuation discount of pessimistic analysts relative to optimistic analysts. Optimists and pessimists are classified at the end of 2006 and held fixed during 2007-2008. The third plot shows the time series of optimistic analysts’ valuation ($\alpha_t$ in Table 1) and pessimistic analysts’ valuation ($\alpha_t + \beta_t$). The analyst valuation is defined as the analyst target price forecast divided by the stock close price on the day of analyst report.
Figure 2: Usage of the word “bottom” by Wall Street Journal articles

The first plot shows the monthly number of Wall Street Journal articles containing the word “bottom.” The second plot shows the time series of two stock indices: S&P 500 and NASDAQ. Both indices are normalized to 1 at the end of 1998. The sample period is from the end of 1998 to the end of 2008.
This figure shows the probability density functions of three Gamma distributions: $\text{Gamma}(20, 19)$, $\text{Gamma}(2, 1)$, and $\text{Gamma}(2, 3)$. $\text{Gamma}(2, 3)$ is the Bayesian posterior after observing two periods of bear market of an investor with prior $\text{Gamma}(2, 1)$. 
This figure shows the valuations of the pessimist and the optimist when the bear market persists. The optimist’s prior of recovery intensity is $Gamma (2, 1)$ and the pessimist’s prior is $Gamma (9/8, 3/4)$. $r = 0.5\%$ monthly. The dividend is $D = 1$ and $D - \delta = 1/10$ in the normal and bear market, respectively. The first plot shows the valuations of the pessimist and the optimist during the first two years of the bear market. The second plot shows the valuation discount of the pessimist relative to the optimist up to 200 years into the bear market.
This figure illustrates the delay in liquidity provision. The optimists’ prior is $\text{Gamma}(26/25, 1)$. The pessimists’ priors are $\text{Gamma}(3, 9 + g_i)$ for $i \in [0, 1]$. $g = 500$. Each pessimist can buy $M = 5$ shares of MBS. The total supply of MBS is normalized to $K = 1$. $r = 0.5\%$ monthly. The dividend is $D = 1$ and $D - \delta = 1/10$ in normal and bear markets, respectively. The first plot shows the MBS price, along with the buy-and-hold valuations of pessimists $i = 0, 0.1$, and 0.2. Also shown is the reservation value of pessimist $i = 0$, which is the buy-and-hold value minus the option value of waiting in (12). The pessimists absorb all the MBS after pessimist $i = 0.2$ buys. At time 0, the most optimistic pessimist $i = 0$ values the MBS at $196.11$, just below the optimists’ valuation of $196.13$. Also shown are the equilibrium entry times for $i = 0$, and for $i \in [0.084, 0.2]$ who buy at the same time. The second plot compares the equilibrium cumulative fraction of MBS liquidated to the hypothetical cumulative liquidation when pessimists do not wait and buy as soon as the MBS price drops to the buy-and-hold values.
This figure plots the expected profit per share (12) from optimally exercising the option to delay liquidity provision for pessimists $i \in [0, 0.2]$ at different points in time before exercise. The parameters are the same as those in Figure 5.
This figure shows the entry time of pessimists $i \in [0, 0.2]$ with and without government acting as buyer of last resort. When the government intervenes, it is assumed to absorb a quarter of the total MBS outstanding. The other parameters are the same as those in Figure 5. The last liquidity provider required to absorb all the MBS is, respectively, $i = 0.15$ (with government intervention) and $i = 0.2$ (without government intervention). Also shown is the hypothetical entry time if each pessimist buys as soon as the MBS price reaches the buy-and-hold value.