Does Household Finance Matter?
Small Financial Errors with Large Social Costs

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Abstract

Households with familiarity bias tilt their portfolios towards a few risky assets creating two kinds of inefficiencies: underinvestment and underdiversification. To understand their implications for growth and social welfare, we solve in closed form a model of a stochastic, dynamic, general-equilibrium economy with a large number of firms and households who bias their investment toward a few familiar assets. We find that although the static losses from investment mistakes are modest, the dynamic effects on intertemporal consumption, and hence, welfare can be much larger for individual households. We demonstrate that even when we force the biases in portfolios to cancel out across households, the implications of familiarity bias for consumption choices do not cancel—individual behavioral biases can have significant aggregate effects. We also show that the effects of household-level distortions to individual consumption are amplified by aggregation and have a substantial effect on aggregate growth and welfare. Our results imply that financial markets are not a sideshow and that financial education, regulation, and innovation that improves the portfolio choices of households can have a significant positive impact on social welfare.

Keywords: Portfolio choice, underdiversification, ambiguity aversion, familiarity, behavioral finance, growth, social welfare

JEL: G11, E44, E03, G02
1 Introduction and Motivation

One of the fundamental insights of standard portfolio theory (Markowitz (1952, 1959)) is the advice to hold diversified portfolios. However, evidence from natural experiments (Huberman (2001)) and empirical work (Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2014)) shows households invest in portfolios biased toward familiar assets.\footnote{For surveys of the portfolio behavior of households, see Guiso, Haliassos, and Jappelli (2002), Haliassos (2002), Vissing-Jorgensen (2003), Campbell (2006), and Guiso and Sodini (2013).} Familiarity biases may be a result of geographical proximity, employment relationships or perhaps even language, social networks and culture (Grinblatt and Keloharju (2001)). Holding portfolios biased toward a few familiar assets forces households to bear more financial risk than is optimal. An important question studied by macroeconomists such as Lucas (1987, 2003) is the determination of the welfare costs to households of variability in aggregate consumption growth. The analogous question at the microeconomic level of how important variability in household wealth is for welfare has been studied empirically by financial economists such as Calvet, Campbell, and Sodini (2007). They find that the welfare costs for individual households arising from underdiversified portfolios are modest within a static mean-variance framework. We extend the static framework to a dynamic, general equilibrium production economy setting to examine how familiarity biases in individual household portfolios can impact intertemporal consumption choices of households, and hence, social welfare, aggregate growth, and real investment.

In our paper, we address the following questions. Are pathologies such as familiarity bias in financial markets merely a sideshow or do they impact the real economy?\footnote{For a review of the literature on the interaction between financial markets and the real economy, see Bond, Edmans, and Goldstein (2012).} Even if the single-period welfare loss from bearing too much risk in financial markets is small, how large is the impact on dynamic consumption and real investment decisions, in particular over longer horizons? Furthermore, do household-level portfolio errors cancel out, or does aggregation amplify them, imposing significant social costs and distorting growth?

Our paper makes three contributions. First, we show that even if the static losses from familiarity bias are modest, the dynamic effects on intertemporal consumption, and hence, welfare can be much larger. Second, even if biases in portfolios cancel out across households, their implications for consumption choices do not, indicating that there are spillovers from financial markets to the real economy. Third, household-level distortions to individual
consumption stemming from excessive financial risk taking are amplified by aggregation and have a substantial effect on aggregate growth and welfare—financial markets are not a sideshow.

Our model builds on the production economy framework developed in Cox, Ingersoll, and Ross (1985). As in Cox, Ingersoll, and Ross, there are a finite number of firms whose physical capital is subject to exogenous shocks. But, in contrast with Cox, Ingersoll, and Ross, we have heterogeneous households with Epstein and Zin (1989) and Weil (1990) preferences and familiarity bias. Each household is more familiar with a small subset of firms. Familiarity creates a desire to concentrate investments in familiar firms at the expense of holding a portfolio that is well-diversified across all firms. Importantly, we specify the model so that investors are symmetric in their familiarity biases. The symmetry assumption ensures that the familiarity biases cancel out—that is, the portfolio aggregated over all households is unbiased.

We conceptualize the idea of greater familiarity with certain assets presented in Huberman (2001) via ambiguity in the sense of Knight (1921). The lower the level of ambiguity about an asset, the more “familiar” the asset. To allow for differences in familiarity across assets, we extend the modeling approach in Uppal and Wang (2003) along three dimensions: one, we distinguish between risk across states of nature and over time by giving households Epstein-Zin-Weil preferences, as opposed to time-separable preferences; two, we consider a production economy instead of an endowment economy; three, we consider a general-equilibrium rather than a partial-equilibrium framework.

We then compute the optimal portfolio decisions for each household. We show that the optimal portfolio is similar to the standard mean-variance portfolio, but with expected returns adjusted downward for lack of familiarity. The adjustment is downward because investors are averse to ambiguity and is greater for the less familiar assets. The adjustments tilt portfolio weights toward familiar assets and also reduce the overall investment in risky assets. Because of the familiarity-induced tilt, the portfolio is excessively risky relative to the return of the optimally diversified portfolio. Bearing this extra risk has only a small negative effect on the welfare of a household. However, this extra financial risk also changes the intertemporal consumption-saving decision of a household. The resulting consumption decisions of a household are much more volatile than in the absence of familiarity. The welfare loss from extra consumption volatility is substantial. In equilibrium, the excessively
volatile consumption of individual households distorts growth and severely reduces social welfare.

Our results suggest that financial education, financial regulation, and financial innovation designed to reduce investment mistakes by households can have a substantial impact on social welfare and growth. Our work thereby provides an example of how improving the decisions made by households in financial markets can generate positive investment externalities; for a further discussion of such externalities, see the presidential address to the American Finance Association by Titman (2013).

We now describe the related literature. The evidence that households hold poorly diversified portfolios is substantial. See for example, Guiso, Haliassos, and Jappelli (2002), Haliassos (2002), Campbell (2006), and Guiso and Sodini (2013) who highlight this behavior in their surveys of household portfolio characteristics. Polkovnichenko (2005), using data from the Survey of Consumer Finances, finds that for households that invest in individual stocks directly, the median number of stocks held was two from 1983 until 2001, when it increased to three, and that poor diversification is often attributable to investments in employer stock, which is a significant part of equity portfolios. Barber and Odean (2000) and Goetzman and Kumar (2008) report similar findings of underdiversification based on data for individual investors at a U.S. brokerage firm. Calvet, Campbell, and Sodini (2007), based on detailed government records covering the entire Swedish population, also find that thirty-eight percent of Swedish households do not participate in the equity market, and of the ones that do, many are poorly diversified and bear significant idiosyncratic risk. Campbell, Ramadorai, and Ranish (2012) report that for their data on Indian households, “the average number of stocks held across all accounts and time periods is almost 7, but the median account holds only 3.4 stocks on average over its life.” They also estimate that mutual fund holdings are between 8% and 16% of household direct equity holdings over the sample period, and that 65% of Indian households did not own any bonds or mutual funds. Guiso and Sodini (2013) find that even though participation is much higher for wealthy households, there is limited participation in each asset class even among the richest households; for example, 10% of the wealthiest households do not hold equity. It is clear that lack of diversification is not purely a U.S. phenomenon; it is prevalent worldwide.\footnote{Lack of diversification has been documented in Australia (Worthington (2009)), France (Arrondel and Lefebvre (2001)), Germany (Börsch-Supan and Eymann (2002) and Barasinska, Schäfer, and Stephan (2008)), India (Campbell, Ramadorai, and Ranish (2012)), Italy (Guiso and Jappelli (2002)), Netherlands (Alessie and Van Soest (2002)), and the United Kingdom (Banks and Smith (2002)).}
Typically, the few assets that investors do hold are ones with which they are “familiar.” Huberman (2001) introduces the idea that people invest in familiar assets and provides evidence of this in a multitude of contexts; for example, households in the United States prefer to hold the stock of their local telephone company. Grinblatt and Keloharju (2001), based on data on Finnish investors, find that investors are more likely to hold stocks of Finnish firms that are “familiar;” that is, firms that are located close to the investor, that communicate in the investor’s native language, and that have a chief executive of the same cultural background. Massa and Simonov (2006) also find that investors tilt their portfolios away from the market portfolio and toward stocks that are geographically and professionally close to the investor, resulting in a portfolio biased toward familiar stocks.

French and Poterba (1990) and Cooper and Kaplanis (1994) document that investors bias their portfolios toward “home equity” rather than diversifying internationally. Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2014) test the relation between familiarity bias and several household portfolio-choice puzzles. Based on a survey of U.S. households, they find that familiarity bias is related to stock market participation, the fraction of financial assets in stocks, foreign stock ownership, own-company stock ownership, and underdiversification.

The most striking example of investing in familiar assets is the investment in “own-company stock,” that is, stock of the company where the person is employed. Haliassos (2002) reports evidence of limited diversification based on the tendency of households to hold stock in the employer’s firm. Mitchell and Utkus (2004) report that five million Americans have over sixty percent of their retirement savings invested in company stock and that about eleven million participants in 401(k) plans invest more than twenty percent of their retirement savings in their employer’s stock. Benartzi, Thaler, Utkus, and Sunstein (2007) find that only thirty-three percent of the investors who own company stock realize that it is riskier than a diversified fund with many different stocks. Surveys conducted by John Hancock Financial Services during the 1992–2004 period find that even after financial education is provided by funds and plan sponsors, investors continued to rate own-company stock as safer than a domestic stock fund. Remarkably, a survey of 401(k) participants by the Boston Research Group (2002) found that half of the respondents said that their company stock had the same or less risk than a money market fund, even though there was a
high level of awareness amongst the respondents about the experience of Enron’s employees, who lost a substantial part of their retirement funds that were invested in Enron stock.\footnote{At the end of 2000, 62 percent of Enron employees’ 401(k) assets were invested in company stock; between January 2001 and January 2002, the value of Enron stock fell from over $80 per share to less than $0.70 per share.}

The rest of this paper is organized as follows. We describe the main features of our model in Section 2. The choice problem of an investor who exhibits a bias toward familiar assets is solved in Section 3, and the equilibrium implications of aggregating these choices across all households are described in Section 4. In Section 5, we discuss the sensitivity of our results to our modeling assumptions. We conclude in Section 6. Proofs for all results are collected in the appendix.

2 The Model

In this section, we develop a simple model of a stochastic dynamic general equilibrium economy with a finite number of production sectors and a finite number of household types. When defining the preferences of households, we show how to extend Epstein and Zin (1989) and Weil (1990) preferences to allow for familiarity biases, where the level of the bias differs across risky assets.

2.1 Firms

There are $N$ firms indexed by $n \in \{1, \ldots, N\}$. The value of the capital stock in each firm at date $t$ is denoted by $K_{n,t}$ and the output flow by

$$Y_{n,t} = AK_{n,t},$$

for some constant technology level $A > 0$. The level of a firm’s capital stock can be increased by investment at the rate $I_{n,t}$. A firm’s capital stock depreciates at the constant rate $b > 0$ and is subject to exogenous shocks which are proportional to the level of capital. We thus have the following capital accumulation equation for an individual firm:

$$dK_{n,t} = (I_{n,t} - bK_{n,t})dt + \sigma K_{n,t} dZ_{n,t},$$

where $\sigma$, the volatility of the exogenous shock to a firm’s capital stock, is constant. The term $dZ_{n,t}$ is the increment in a standard Brownian motion and is firm-specific; the correlation...
between \( dZ_{n,t} \) and \( dZ_{m,t} \) is denoted by \( \rho \), which is also assumed to be constant over time, and the same for all \( n \neq m \). Firm-specific shocks create heterogeneity across firms. The \( N \times N \) correlation matrix of returns on firms’ capital stocks is given by \( \Omega = [\Omega_{nm}] \), where the elements of the matrix are

\[
\Omega_{nm} = \begin{cases} 
1, & n = m, \\
\rho, & n \neq m.
\end{cases}
\]

Given the fixed level of technology, a firm’s capital stock can only be expected to increase when its investment flow exceeds the reduction in capital through depreciation. Firm-level heterogeneity creates benefits from diversifying investments across firms. We assume the expected rate of return is the same across the \( N \) firms. Thus, diversification benefits manifest themselves solely through a reduction in risk—expected returns do not change with the level of diversification.

A firm’s output flow is divided between its investment flow and dividend flow:

\[
Y_{n,t} = I_{n,t} + D_{n,t}.
\]

We can therefore rewrite the capital accumulation equation as

\[
dK_{n,t} = ((A - b)K_{n,t} - D_{n,t})dt + \sigma K_{n,t} dZ_{n,t}.
\]

By defining \( \alpha = A - b \), we obtain

\[
dK_{n,t} = (\alpha K_{n,t} - D_{n,t})dt + \sigma K_{n,t} dZ_{n,t}.
\]

### 2.2 The Investment Opportunities of Households

There are \( H \) households, indexed by \( h \in \{1, \ldots, H\} \). Households can invest their wealth in two classes of assets. The first is a risk-free asset, which has an interest rate \( i \) that we assume is constant over time—we will show below, in Section 4.2, that this is indeed the case in equilibrium. Let \( B_{h,t} \) denote the stock of wealth invested by household \( h \) in the risk-free asset at date \( t \):

\[
\frac{dB_{h,t}}{B_{h,t}} = i dt.
\]

Additionally, households can invest in \( N \) risky firms. We denote by \( K_{hn,t} \) the stock of household \( h \)’s wealth invested in the \( n \)’th risky firm. Given that household wealth, \( W_{h,t} \),
is held in either the risk-free asset or invested in a risky firm,

\[ W_{h,t} = B_{h,t} + \sum_{n=1}^{N} K_{hn,t}. \]

The proportion of a household’s wealth invested in firm \( n \) is denoted by \( \omega_{hn} \), and so

\[ K_{hn,t} = \omega_{hn} W_{h,t}, \]
\[ B_{h,t} = \left(1 - \sum_{h=1}^{N} \omega_{hn}\right) W_{h,t}. \]

Hence, the dynamic budget constraint for household \( h \) is given by

\[ \frac{dW_{h,t}}{W_{h,t}} = \left(1 - \sum_{n=1}^{N} \omega_{hn}\right) \alpha dt + \sum_{n=1}^{N} \omega_{hn,t} (\alpha dt + \sigma dZ_{n,t}) - \frac{C_{h,t}}{W_{h,t}} dt, \]

where \( C_{h,t} \) is the consumption rate of household \( h \) and \( C_{hn,t} \) is the consumption rate of household \( h \) from the dividend flow of firm \( n \), with \( C_{h,t} = \sum_{n=1}^{N} C_{hn,t} \). The dividends distributed by firm \( n \) are consumed by household \( h \), that is:

\[ C_{hn,t} = D_{hn,t} = \frac{K_{hn,t}}{K_{n,t}} D_{n,t}. \]

2.3 Preferences and Familiarity Biases of Households

In the absence of any familiarity bias, each household maximizes her date-\( t \) utility level, \( U_{h,t} \), defined as in Epstein and Zin (1989) by an intertemporal aggregation of date \( t \) consumption flow, \( C_{h,t} \), and the date-\( t \) certainty-equivalent of date \( t + dt \) utility, i.e.

\[ U_{h,t} = A(C_{h,t}, \mu_t[U_{h,t+dt}]), \]

where \( A(\cdot, \cdot) \) is the time aggregator, defined by

\[ A(x, y) = \left[ (1 - e^{-\delta dt}) x^{1-\frac{1}{\psi}} + e^{-\delta dt} y^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \quad (1) \]

in which \( \delta > 0 \) is the rate of time preference, \( \psi > 0 \) is the elasticity of intertemporal substitution, and \( \mu_t[U_{h,t+dt}] \) is the date-\( t \) certainty equivalent of \( U_{h,t+dt} \).\(^5\)

\(^5\)The only difference with Epstein and Zin (1989) is that we work in continuous time, whereas they work in discrete time. The continuous-time version of the recursive preferences is known as stochastic differential utility (SDU), and is derived formally in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness.
The standard definition of a certainty equivalent amount of a risky quantity is the equivalent risk-free amount in static utility terms, and so the certainty equivalent $\mu_t[U_{h,t+dt}]$ satisfies

$$u_\gamma(\mu_t[U_{h,t+dt}]) = E_t[h(U_{h,t+dt})],$$

where $u_\gamma(\cdot)$ is the static utility index defined by the power utility function

$$u_\gamma(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma > 0, \gamma \neq 1 \\ \ln x, & \gamma = 1, \end{cases}$$

and the conditional expectation $E_t[\cdot]$ is defined relative to a probability measure $P$.\(^6\)

We can exploit our continuous-time formulation to write the certainty equivalent of household utility an instant from now in a more intuitive fashion:

$$\mu_t[U_{h,t+dt}] = E_t[U_{h,t+dt}] - \frac{1}{2} \gamma U_{h,t} E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right].$$

The above expression reveals that the certainty equivalent of utility an instant from now is just the expected value of utility an instant from now adjusted downwards for risk. Naturally, the size of the risk adjustment depends on how risk averse the household is, that is, $\gamma$. The risk adjustment depends also on the volatility of the proportional change in household utility, which is given by $E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right]$. Additionally, the risk adjustment is scaled by the current utility of the household, $U_{h,t}$.\(^7\)

Typically, standard models of portfolio choice assume that households know the true expected return $\alpha$ on the value of each capital stock. Such perfect knowledge would make each household fully familiar with every firm and the probability measure $P$ would then be the true objective probability measure.\(^8\) However, in practice households do not know the true expected returns, so they do not view $P$ as the true objective probability measure—they treat it merely as a common reference measure. The name “reference measure” is chosen to capture the assumption that even though households do not observe true expected returns,

\(^6\)In continuous time the more usual representation for utility is given by $J_{h,t}$, where $J_{h,t} = u_\gamma(U_{h,t})$.

\(^7\)The scaling ensures that if the expected proportional change in household utility and its volatility are kept fixed, doubling current household utility also doubles the certainty equivalent. For a further discussion, see Skiadas (2009, p. 213).

\(^8\)In continuous time when the source of uncertainty is a Brownian motion, one can always determine the true volatility of the return on the capital stock by observing its value for a finite amount of time; therefore, a household can be uncertain only about the expected return.
they do observe the same data and use it to obtain identical point estimates for expected returns.

We assume households are averse to their lack of knowledge about the true expected return and respond by reducing their point estimates. For example, household \( h \) will change the empirically estimated return on capital for firm \( n \) from \( \alpha \) to \( \alpha + \nu_{hn,t} \), thereby reducing the size of the firm’s expected risk premium \( (\nu_{hn,t} \leq 0 \text{ if } \alpha > i \text{ and } \nu_{hn,t} \geq 0 \text{ if } \alpha < i) \). The size of the reduction depends on each household’s familiarity with a particular firm—the reduction is smaller for firms with which the household is more familiar. Differences in familiarity across households lead them to use different estimates of expected returns in their decision making, despite having observed the same data. We can see this explicitly by observing that in the presence of familiarity, the contribution of risky production to a household’s expected return on wealth changes from \( \sum_{n=1}^{N} \omega_{hn,t} \alpha dt \) to \( \sum_{n=1}^{N} \omega_{hn,t}(\alpha + \nu_{hn,t})dt \). The adjustment to the expected return on a household’s wealth stemming from familiarity bias is thus

\[
\sum_{n=1}^{N} \omega_{hn,t} \nu_{hn,t} dt. \quad (4)
\]

Without familiarity bias, the decision of a household on how much to invest in a particular firm depends solely on the certainty equivalent. Therefore, to allow for familiarity bias it is natural to generalize the concept of the certainty equivalent. For date \( t + dt \) utility, we follow Uppal and Wang (2003) and define the familiarity-biased certainty equivalent by

\[
\mu_{h,t}^{U}[U_{h,t}+dt] = \mu_t[U_{h,t}+dt] + U_{h,t} \times \left( \frac{W_{h,t}}{U_{h,t}} \nu_{h,t}^{\top} \omega_{h,t} + \frac{1}{2\gamma} \frac{\nu_{h,t}^{\top} \Gamma_h^{-1} \nu_{h,t}}{\sigma^2} \right) dt, \quad (5)
\]

where \( W_{h,t} = \frac{U_{h,t}}{U_{h,t}} \), \( \omega_{h,t} = (\omega_{h,1,t}, \ldots, \omega_{h,N,t})^{\top} \) is the column vector of portfolio weights, \( \nu_{h,t} = (\nu_{h,1,t}, \ldots, \nu_{h,N,t})^{\top} \), and \( \Gamma_h = [\Gamma_{h,nm}] \) is the \( N \times N \) diagonal matrix defined by

\[
\Gamma_{h,nm} = \begin{cases} 
\frac{1-f_{hn}}{f_{hn}}, & n = m, \\
0, & n \neq m,
\end{cases}
\]

where \( f_{hn} \in [0, 1] \) is a measure of how familiar the household is with firm \( n \). A larger value for \( f_{hn} \) indicates more familiarity, with \( f_{hn} = 1 \) implying perfect familiarity, and \( f_{hn} = 0 \) indicating no familiarity at all.
The first term in (5), the pure certainty equivalent $\mu_t[U_{h,t+dt}]$, does not depend directly on the familiarity-bias adjustments. As before, we introduce the scaling factor $U_h,t$ (see Footnote 7 for the role of the scaling factor). The next term, $\frac{W_{h,t}U_{h,t}}{V_{h,t}}V_{h,t}^T\omega_{h,t}$, is the adjustment to the expected change in household utility. It is the product of the elasticity of household utility with respect to wealth, $\frac{W_{h,t}U_{h,t}}{V_{h,t}}$, and the change in the expected return on household wealth arising from the adjustment made to returns, which is given in (4).

The tendency to make adjustments to expected returns is tempered by a penalty term, $\frac{1}{2\gamma}\nu_h,t\Gamma_h^{-1}\nu_h,t$, which captures two distinct features of household decision making. The first pertains to the idea that when a household has more accurate estimates of expected returns, she will be less willing to adjust them. The accuracy of household expected return estimates is measured by their standard errors, which are proportional to $\sigma$. With smaller standard errors, there is a stiffer penalty for adjusting returns away from their empirical estimates. The second feature pertains to familiarity—when a household is more familiar with a particular firm, she is less willing to adjust its expected return.

3 Portfolio Choice and Welfare of an Individual Household

In this section, we study the optimal consumption and portfolio choice problem of an individual household and its impact on the household’s welfare. We first write down the household’s intertemporal decision problem. We show that the portfolio-choice problem can be interpreted as the problem of a mean-variance investor, where the familiarity bias is captured by adjusting expected returns. Finally, we show how the mean-variance portfolio choice impacts intertemporal consumption choice and the welfare of an individual household.

3.1 The Intertemporal Choice Problem of Individual Households

In the absence of familiarity-bias, an individual household would choose her consumption rate and portfolio policy according to the standard choice problem:

$$\sup_{C_{h,t}} A\left(C_{h,t}, \sup_{\omega_{h,t}} \mu_{h,t}[U_{h,t+dt}] \right).$$

In our continuous-time framework, an infinite number of observations are possible in finite time, so standard errors equal the volatility of proportional changes in the capital stock, $\sigma$, divided by the square root of the length of the observation window.
In the presence of familiarity bias, the time aggregator in (1) is unchanged—all we do is replace the maximization of the certainty-equivalent $\sup_{\omega_{h,t}} \mu_t[U_{h,t+dt}]$, with the combined maximization and minimization of the familiarity-based certainty equivalent, $\sup_{\omega_{h,t}} \inf_{\nu_{h,t}} \mu_{h,t}^\nu[U_{h,t+dt}]$ to obtain

$$\sup_{C_{h,t}} A \left( C_{h,t}, \sup_{\omega_{h,t}} \inf_{\nu_{h,t}} \mu_{h,t}^\nu[U_{h,t+dt}] \right).$$

(7)

A household chooses $\nu_{h,t}$ to minimize its familiarity-biased certainty equivalent—the household adjusts expected returns more for firms with which it is less familiar, which acts to reduce the familiarity-biased certainty equivalent. By comparing (6) and (7), we can see that once a household has chosen the vector $\nu_{h,t}$ to adjust the expected returns of each firm for familiarity bias, it makes consumption and portfolio choices in the standard way.

Given any portfolio choice $\omega_{h,t}$ for a household, finding the adjustments to firm-level expected returns is a simple matter of minimizing the familiarity-biased certainty equivalent in (5). The solution is given by

$$\nu_{hn,t} = -\frac{W_{h,t}U_{h,t}}{U_{h,t}} \left( \frac{1}{f_{hn}} - 1 \right) \sigma^2 \gamma \omega_{hn,t}.$$  

(8)

The above expression shows that if a household is fully familiar with firm $n$, $f_{hn} = 1$, then she makes no adjustment to the firm’s expected return. When she is less than fully familiar, $f_{hn} \in [0, 1)$, one can see that $\nu_{hn,t}$ is negative (positive) when $\omega_{hn,t}$ is positive (negative), reflecting the idea that lack of familiarity leads a household to moderate its portfolio choices, shrinking both long and short positions toward zero.

To solve a household’s consumption-portfolio choice problem under familiarity bias we use Ito’s Lemma to derive the continuous-time limit of (7), which leads to the following Hamilton-Jacobi-Bellman equation:

$$0 = \sup_{C_{h,t}} \left( \delta u_{\psi} \left( \frac{C_{ht}}{U_{ht}} \right) + \sup_{\omega_t} \inf_{\nu_{h,t}} \frac{1}{U_{h,t}} \mu_{h,t}^\nu \left[ dU_{h,t} \right] \right),$$

(9)

where the function

$$u_{\psi}(x) = \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0,$$

10In the language of decision theory, households are averse to ambiguity and so they minimize their familiarity-biased certainty equivalents.
and
\[ \mu_{h,t}^\nu \left[ dU_{h,t} \right] = \mu_{h,t}^\nu \left[ U_{h,t} + dt - U_{h,t} \right] = \mu_{h,t}^\nu \left[ U_{h,t} + dt \right] - U_{h,t}, \]
with \( \mu_{h,t}^\nu \left[ U_{h,t} + dt \right] \) given in (5).

Assuming a constant risk-free rate, homotheticity of preferences combined with constant
returns to scale for production implies that maximized household utility is a constant mul-
tiple of household wealth. Hence, the Hamilton-Jacobi-Bellman equation can be decomposed
into two parts: a single-period mean-variance optimization problem for a household with
familiarity bias and an intertemporal consumption choice problem:
\[
0 = \sup_{C_{h,t}} \left( \delta u_{\psi} \left( \frac{C_{h,t}}{W_{h,t}} \right) - \frac{C_{h,t}}{W_{h,t}} + \sup_{\omega_t} \inf_{\nu_{h,t}} MV(\omega_{h,t}, \nu_{h,t}) \right).
\]
In the above expression, \( MV(\omega_{h,t}, \nu_{h,t}) \) is the objective function of a single-period mean-
variance investor with familiarity bias:
\[
MV(\omega_{h,t}, \nu_{h,t}) = i + \left( \alpha - i \right) 1^T \omega_{h,t} - \frac{1}{2} \gamma \sigma^2 \omega_{h,t}^T \Omega \omega_{h,t} + \nu_{h,t}^T \omega_{h,t} + \frac{1}{2} \nu_{h,t}^T \Gamma_h^{-1} \nu_{h,t},
\]
where \( i + \left( \alpha - i \right) 1^T \omega_{h,t} \) is the expected portfolio return, \(-\frac{1}{2} \gamma \sigma^2 \omega_{h,t}^T \Omega \omega_{h,t}\) is the penalty for portfolio variance, \( \nu_{h,t}^T \omega_{h,t}\) is the adjustment to the portfolio’s expected return arising from
familiarity bias, and \( \frac{1}{2} \nu_{h,t}^T \Gamma_h^{-1} \nu_{h,t} \) is the penalty for adjusting expected returns.\(^{11}\)

In the first part of the mean-variance problem with familiarity bias, the firm-level ex-
pected returns are optimally adjusted downward because of lack of familiarity. Because
household utility is a constant multiple of wealth, (8) simplifies to:
\[
\nu_{h,t} = -\gamma \sigma^2 \Gamma_h \omega_{h,t}.
\]
Substituting the above expression into (10), we see that the household faces the following
mean-variance portfolio problem:
\[
\sup_{\omega_{h,t}} MV(\omega_{h,t}) = \left( i + \left( \alpha 1 + \frac{1}{2} \nu_{h,t} - i 1 \right)^T \omega_{h,t} \right) - \frac{1}{2} \gamma \sigma^2 \omega_{h,t}^T \Omega \omega_{h,t},
\]
where, of course, \( \nu_{h,t} \) is given by (11). When the household is fully familiar with all firms,
then \( \Gamma_h \) is the zero matrix, and from (11) we can see the adjustment to expected returns

\(^{11}\)The familiarity-bias adjustment is obtained from a minimization problem, so the associated penalty is positive, in contrast with the penalty for return variance.
is zero and the portfolio weights are exactly the standard mean-variance portfolio weights. For the case where the household is completely unfamiliar with all firms, then each $\Gamma_{h,nn}$ becomes infinitely large and $\omega_h = 0$: complete unfamiliarity leads the household to avoid any investment in risky firms, in which case we get non-participation in the stock market.

### 3.2 Solution to the Choice Problem of Individual Households

Solving the first-order condition for the optimal portfolio weights from (12), we get:

$$\omega_h = \Omega^{-1} \frac{\alpha 1 + \nu_h - i 1}{\gamma \sigma^2}, \quad (13)$$

where the adjustment to the vector of expected returns, $\alpha 1$, is now given explicitly in terms of exogenous variables by:

$$\nu_h = -(\alpha - i) a_h, \quad (14)$$

with

$$a_h = (I + \Omega \Gamma^{-1})^{-1} 1.$$

Substituting (14) into (13) gives

$$\omega_h = \frac{\alpha - i}{\gamma \sigma^2} \Omega^{-1} (1 - a_h). \quad (15)$$

Hence, the proportion of a household’s wealth invested in risky assets, $1^T \omega_h$, is given by

$$1^T \omega_h = \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2} f_h, \quad (16)$$

where

$$f_h = \frac{1}{N} 1^T \Omega^{-1} (1 - a_h)$$

is the aggregate familiarity of household $h$. The familiarity-biased portfolio of only risky assets, denoted by $x_h = \frac{\omega_h}{1^T \omega_h}$, where the weights in each of the risky assets, $\omega_{hn}$, is normalized by their sum is given by

$$x_h = \frac{(\Omega + \Gamma_h)^{-1} 1}{1^T (\Omega + \Gamma_h)^{-1} 1}.$$

To see the intuition behind the expression for $a_h$, we set $\rho = 0$. Doing so reduces the correlation matrix $\Omega$ to the identity matrix, and so we obtain that the elements of the $a_h$
matrix are:

\[ a_{hn} = 1 - f_{hn}. \]

In this case, the adjustment made to the expected returns of firm \( n \) becomes

\[ \nu_{hn} = -(\alpha - i)(1 - f_{hn}), \quad (17) \]

and the portfolio weight for firm \( n \) is

\[ \omega_{hn} = \frac{\alpha - i}{\gamma \sigma^2} f_{hn}. \quad (18) \]

From (17), we can see that the size of a household’s adjustment to a firm’s return is smaller when the level of familiarity, \( f_{hn} \), is larger; if \( f_{hn} = 1 \), then the adjustment vanishes.

From (18), we see that the standard mean-variance portfolio weight for firm \( n \), \( \frac{\alpha - i}{\gamma \sigma^2} \), is scaled by the level of the household’s familiarity with firm \( n \), \( f_{hn} \). As a household’s level of familiarity with a particular firm decreases, the proportion of her wealth she chooses to invest in that firm also decreases. Also, if a household’s average familiarity, which for \( \rho = 0 \) simplifies to

\[ \bar{f}_h = \frac{1}{N} \sum_{n=1}^{N} f_{hn}, \]

decreases, she invests less of her wealth in risky firms.

The familiarity-biased portfolio of only risky assets, \( x_h \), is the minimum-variance portfolio with a familiarity-biased adjustment.\(^{12}\) Given that all risky assets have the same volatility and correlation, the minimum-variance portfolio with no familiarity bias is given by \( x_{hn} = \frac{1}{N} \). Familiarity bias tilts the portfolio of only risky assets away from \( \frac{1}{N} \), thereby increasing its variance. This also leads the household to reduce the proportion of her overall wealth held in risky assets.

Defining by \( SR_{x_h} \) the Sharpe ratio of the portfolio of only risky assets,

\[ SR_{x_h} = \frac{\alpha - i}{\sigma_{x_h}}, \]

where \( \sigma_{x_h} \) is the volatility of the return on the portfolio of only risky assets, given by

\[ \sigma_{x_h}^2 = \sigma^2 x_h^\top \Omega x_h. \]

\(^{12}\)If \( \rho = 0 \), then \( x_{hn} = \frac{f_{hn}}{\sum_{n=1}^{N} f_{hn}}. \)
We now compute the welfare loss from familiarity bias of a household who maximizes just the single-period mean-variance objective function in (10). With familiarity bias, the optimized objective function is given by

$$\sup_{\omega_{h,t}} \inf_{\nu_{h,t}} MV(\omega_{h,t}, \nu_{h,t}) = i + \frac{1}{2\gamma} \left( \frac{\alpha - i}{\sigma} \right)^2 T_h$$

$$= i + \frac{1}{2\gamma} SR_{x,h}^2.$$  

Without familiarity bias, the optimized single-period objective function is

$$\sup_{\omega_{h,t}} MV(\omega_{h,t}, \nu_{h,t} = 0) = i + \frac{1}{2\gamma} SR_{1/N}^2,$$

where $SR_{1/N}$ is the Sharpe ratio of the equal-weighted portfolio of only risky assets, given by

$$SR_{1/N} = \frac{\alpha - i}{\sigma \sqrt{\frac{1}{N} + (1 - \frac{1}{N}) \rho}}.$$  

The single-period mean-variance utility loss is equivalent to a decrease in the risk-free interest rate of

$$\frac{1}{2\gamma} \left( SR_{1/N}^2 - SR_{x,h}^2 \right) = \frac{1}{2} \left( 1^\top \omega_h \right) \sigma_{x,h} \left( \frac{SR_{1/N}^2 - SR_{x,h}^2}{SR_{x,h}} \right),$$

where $\gamma$ has been substituted out using the result $\omega_h = \left( \frac{1}{\gamma} \frac{SR_{x,h}}{\sigma_{x,h}} \right) x_h$. This measure of utility loss is the same as in Campbell (2006, p. 1574).

Next, we solve for optimal consumption in terms of household utility. From the Hamilton-Jacobi-Bellman equation in (9), the first-order condition with respect to consumption is

$$\delta \left( \frac{C_{h,t}}{U_{h,t}} \right)^{-\frac{1}{\psi}} = \frac{U_{h,t}}{W_{h,t}}.$$  

Substituting the above first-order condition into the Hamilton-Jacobi-Bellman equation allows us to solve for household utility and hence optimal consumption. We find that

$$U_{h,t} = \left( \frac{C_{h,t}/W_{h,t}}{\delta^\psi} \right)^{\frac{1}{1-\psi}} W_{h,t}, \quad (19)$$

where a household’s optimal consumption-to-wealth ratio is given by

$$\frac{C_{h,t}}{W_{h,t}} = \psi \bar{\delta} + (1 - \psi) \left( \left[ i + \left( \alpha 1 + \frac{1}{2} \nu_{h,t} - i1 \right) \right] \omega_{h,t} \right) - \frac{1}{2} \gamma^2 \omega_{h,t}^\top \Omega \omega_{h,t}$$
\[ \psi \delta + (1 - \psi) \left( i + \frac{1}{2\gamma} \left[ \frac{\alpha - i}{\sqrt{N}} \right]^2 \tilde{f}_h \right) \]

(20)

\[ \psi \delta + (1 - \psi) \left( i + \frac{1}{2\gamma} SR_{x_h}^2 \right). \]

(21)

The above expression shows that the households portfolio choice impacts her intertemporal consumption choice. We see from (21) that the optimal consumption-wealth ratio is a weighted average of the impatience parameter \( \delta \) and optimized single-period, mean-variance objective function. When the substitution effect dominates \( (\psi > 1) \), choosing a portfolio subject to familiarity bias makes consumption more attractive relative to saving. In contrast, when the income effect dominates \( (\psi < 1) \), familiarity bias makes consumption less attractive than saving. Familiarity bias changes a household’s intertemporal consumption choice as follows

\[ \frac{C_{h,t}}{W_{h,t}} \bigg|_{\text{no bias}} - \frac{C_{h,t}}{W_{h,t}} \bigg|_{\text{bias}} = (1 - \psi) \left( \frac{1}{2\gamma} (SR_{1/N}^2 - SR_{x_h}^2) \right), \]

assuming a partial equilibrium perspective, where the risk-free interest rate, \( i \), is held fixed.

We now see how the single-period welfare loss caused by familiarity bias translates into a multiperiod loss via the change in intertemporal consumption. If a household is subject to familiarity bias, her lifetime-utility level, \( U_{h,t} \), is given by

\[ U_{h,t} \bigg|_{\text{bias}} = \left[ \frac{\psi \delta + (1 - \psi) \left( i + \frac{1}{2\gamma} SR_{x_h}^2 \right)}{\delta^\psi} \right]^{1/\psi} W_{h,t}, \]

which we obtain by substituting (21) into (19). On the other hand, the lifetime-utility level of a household that does not suffer from the familiarity bias is given by the following expression, where \( SR_{x_h} \) is replaced by \( SR_{1/N} \):

\[ U_{h,t} \bigg|_{\text{no bias}} = \left[ \frac{\psi \delta + (1 - \psi) \left( i + \frac{1}{2\gamma} SR_{1/N}^2 \right)}{\delta^\psi} \right]^{1/\psi} W_{h,t}. \]

We measure the multiperiod welfare loss from familiarity bias as the equivalent percentage reduction in the level of an individual household’s wealth. That is, where a household’s date-\( t \) utility is given in terms of her wealth by \( U_{h,t} = U_h(W_{h,t}) \), we find \( \lambda_{PE} \) such that

\[ U_h((1 - \lambda_{PE})W_{h,t})\bigg|_{\text{no bias}} = U_h(W_{h,t})\bigg|_{\text{bias}}. \]
\[
\lambda_{PE} = 1 - \left[ \frac{\psi \delta + (1 - \psi) \left( i + \frac{1}{2} \gamma \delta h \right) \left( i + \frac{1}{2} \gamma \delta x \right) \left( i + \frac{1}{2} \gamma \delta x_1 \right) \right]^{\frac{1}{\psi}}.
\]  

(22)

We obtain the corresponding welfare measure for a single-period, mean-variance household by making the intertemporal consumption choice perfectly inelastic by letting \( \psi \to 0 \), which gives

\[
\lambda_{MV} = 1 - \frac{i + \frac{1}{2} \gamma \delta x_1}{i + \frac{1}{2} \gamma \delta x}.
\]  

(23)

We can also analyze the impact of a small single-period welfare loss on multiperiod welfare. Taking the partial derivative of \( U_{h,t}/W_{h,t} \) with respect to the single-period mean-variance objective function gives

\[
\frac{\partial}{\partial MV} \left( \frac{U_{h,t}}{W_{h,t}} \right) = \left( \frac{1}{\delta} \frac{U_{h,t}}{W_{h,t}} \right)^\psi.
\]

The first-order change in welfare induced by a small change, \( \Delta MV \), in the single-period, mean-variance objective function is thus given by

\[
\left( \frac{1}{\delta} \frac{U_{h,t}}{W_{h,t}} \right)^\psi \Delta MV,
\]

where \( \left( \frac{1}{\delta} \frac{U_{h,t}}{W_{h,t}} \right)^\psi \) is an amplification factor which is greater than or equal to 1, when \( MV > \delta \). The amplification factor is equal to 1 if and only if \( \psi = 0 \). Furthermore we can show that if \( \psi > 1 \), then a lower bound for the amplification factor is given by \( e^{\frac{MV}{\delta} - 1} \) when \( MV > \delta \). In summary, if a household is more patient (smaller \( \delta \)) or has a greater tolerance for intertemporal consumption fluctuations (larger \( \psi \)), the small single-period loss is amplified.

3.3 Quantitative Assessment of Familiarity Bias for Individual Household Welfare

We now undertake a quantitative analysis of how familiarity bias impacts the welfare of an individual household. In our analysis, we set the mean-variance welfare loss equal to the value identified in Calvet, Campbell, and Sodini (2007). We then examine the magnitude of the multiperiod welfare loss when the investor is free to choose intertemporal consumption.
In the partial equilibrium setting, the household preference parameters for rate of time preference, \( \delta \), relative risk aversion, \( \gamma \), and the elasticity of intertemporal substitution, \( \psi \) are restricted only by the well-known transversality and existence conditions. Therefore, we examine the welfare losses for a range of values for these parameters, with our choices disciplined by the transversality and existence conditions.

We examine the size of the multiperiod welfare loss, as measured by \( \lambda_{PE} \) defined in Equation (22), and compare it with the corresponding single-period welfare loss, \( \lambda_{MV} \), defined in Equation (23). To compute these quantities, the per annum Sharpe ratios, \( SR_{x_h} \) and \( SR_{1/N} \) are set equal to 0.30 and 0.45, respectively, based on the empirical estimates reported in Calvet, Campbell, and Sodini (2007). We set the value of the annualized interest rate, \( i \), equal to 0.56%, based on the annual estimate reported in Beeler and Campbell (2012, their Table 2) for the period 1930-2008. The base-case values that we use for the preference parameters are the following: relative risk aversion of the household is \( \gamma = 2 \); elasticity of intertemporal substitution of the household is \( \psi = 1.5 \); and, subjective rate of time preference is \( \delta = 0.02 \). We also report the welfare loss for a range of values around the base-case values of the preference parameters, \( \delta \), \( \psi \), and \( \gamma \).

In Figure 3, we report on the vertical axis three quantities: \( \lambda_{PE} \), \( \lambda_{MV} \), and the ratio \( \lambda_{PE}/\lambda_{MV} \). On the horizontal axis, we let the Sharpe ratio of the full-diversified benchmark portfolio range from a low of 0.31 to a high of 0.45. From the plot, we see that both the single-period measure of welfare loss, \( \lambda_{MV} \), and the multiperiod measure of welfare, \( \lambda_{PE} \), increase as the Sharpe ratio of the benchmark portfolio increases beyond 0.30, which is the Sharpe ratio of the familiarity-biased portfolio of the household. We also see from the figure that the multiperiod welfare loss \( \lambda_{PE} \), given by the dashed red line, is greater than the single-period welfare loss, \( \lambda_{MV} \), given by the solid blue line. The black-dotted line plots the ratio \( \lambda_{PE}/\lambda_{MV} \) and shows that the multiperiod loss is about 1.7 to 2.0 times the value of the single-period loss.

In Figure 4, we study the same three quantities on the vertical axis, \( \lambda_{PE} \), \( \lambda_{MV} \), and the ratio \( \lambda_{PE}/\lambda_{MV} \), and on the horizontal axis we allow the household’s relative risk aversion to range from 1.5 to 2. We see from the figure that the single-period loss \( \lambda_{MV} \), given by the solid-blue line, is flat; the multiperiod loss increases when risk-aversion is low, and then declines because at high levels of risk aversion, the investor would have found it optimal to invest a larger share of wealth in the risk-free asset, and so the welfare loss from holding a
portfolio that is underdiversified because of familiarity bias declines. The black-dotted line that plots $\lambda_{PE}/\lambda_{MV}$ shows that the multiperiod loss is about 1.4 to 2 times the single-period loss.

Figure 5, shows the welfare loss for different levels of elasticity of intertemporal substitution (EIS), given by $\psi$. We see from the figure that the single-period loss $\lambda_{MV}$, given by the solid-blue line is flat because the single-date measure implicitly considers consumption at only one date, and therefore, excludes the possibility of reallocating consumption intertemporally; the multiperiod loss increases with EIS, because the more willing is the investor to substitute current consumption for future consumption, the greater is the benefit from choosing a fully-diversified portfolio. The black-dotted line that plots $\lambda_{PE}/\lambda_{MV}$ shows that the difference between the multiperiod loss and the single-period loss increases with EIS. The multiperiod loss is about 1.2 times the single-period loss if EIS is 0.5, it is 1.5 times the single-period loss if EIS is 1.0, and it is 2 times the single-period loss if EIS is 1.5.

Figure 6, shows the welfare loss for different subjective rates of time preference, given by $\delta$. We see from the figure that the single-period loss $\lambda_{MV}$, given by the solid-blue line is flat because the single-date measure implicitly considers consumption at only one date, and therefore, the rate of time preference has no effect on this measure; the multiperiod loss decreases with the rate of time preference, because it implies that the investor discounts future consumption more heavily, and hence, the benefits from increasing future consumption are less valuable. The black-dotted line that plots $\lambda_{PE}/\lambda_{MV}$ shows that the difference between the multiperiod loss and the single-period loss decreases with the rate of time preference. The multiperiod loss is almost 2 times the single-period loss if the rate of time preference is 0.02, it is 1.4 times the single-period loss if the rate of time preference is 0.03, and there is almost no difference if the rate of time preference is 0.04.

Finally, we look at the effect of the risk-free interest rate on the welfare loss from familiarity bias. As the risk-free rate increases, both the single-period and multiperiod welfare-loss measures from holding an underdiversified portfolio decline. The black-dotted line that plots $\lambda_{PE}/\lambda_{MV}$ shows that the multiperiod loss is about 1.8 times the single-period loss if the risk-free rate is 0.25%, 1.9 times the single-period loss if the risk-free rate is 0.50%, and 1.5 times the single-period loss if the risk-free rate is 2.00%.

To summarize, we find that the multiperiod welfare loss is greater than the single-period welfare loss. The intuition is that in a multiperiod setting with intermediate consumption
a household has the choice to reallocate consumption intertemporally. Thus, holding a portfolio less affected by familiarity bias not only reduces the risk of the portfolio, but also improves the intertemporal allocation of consumption. Intuitively, the more elastic the consumption of an individual household the greater the impact of a more efficient portfolio on its welfare.

We conclude this section by explaining why the losses in multiperiod welfare from familiarity bias are so large compared to the small change in single-period welfare losses from the perspective of discounting. Calvet, Campbell, and Sodini (2007) report that the single-period welfare loss in terms of a household’s Sharpe ratio of about 0.30 relative to the benchmark portfolios Sharpe ratio of 0.45 is equivalent to a decrease of about 1% in the risk-free interest rate. By a back of the envelope calculation we can see what impact such a fall in the interest rate would have on the value of a household’s intertemporal consumption stream. We start from the simple Gordon growth formula for valuing a household’s consumption stream:\(^{13}\)

\[
\frac{W_{h,t}}{C_{h,t}} = \frac{1}{i - \hat{g}_h},
\]

where \(\hat{g}_h\) is the risk-neutral expected growth rate of household \(h\)’s consumption flow. If the current interest rate is 3% and the risk-neutral expected growth rate of household \(h\)’s consumption flow is 1%, a fall in the interest rate of only 1% doubles household wealth; that is, \(W_{h,t}/C_{h,t}\) increases from 50 to 100.

In the above calculation, we took a partial equilibrium perspective and assumed an exogenous interest rate. In the next section, we show that above insights hold in general equilibrium where the interest rate is endogenous.

4 Social Welfare and Growth

In this section, we study social welfare and growth. In the previous section, we examined how familiarity bias increased the risk of a household’s portfolio and distorted its consumption choice. When we aggregate across households, the distortion in individual consumption choices has implications for aggregate investment, growth, and welfare.

\(^{13}\)We interpret a household’s wealth as the present value of its consumption stream. We thus replace dividend flow in the Gordon growth formula by a household’s consumption stream.
4.1 Aggregate familiarity bias across households

A common criticism of behavioral economics is that, while it shows individual households make errors in their decisions, it has not shown that the errors by individual households lead to aggregate effects. We address this criticism by showing that the aggregate effects of the familiarity bias are substantial in equilibrium, even though the familiarity bias in portfolios cancels out in expectation when aggregated across households.

By canceling out we mean that the expected amount of wealth invested in each firm is the same and independent of households’ familiarity biases; that is, the proportion of expected aggregate wealth invested in each firm is always $1/N$. Expressed formally,

$$E_0 \left[ \sum_{h=1}^{H} \omega_{hn} W_h \right] = \frac{1}{N} E_0 \left[ \sum_{n=1}^{N} \sum_{h=1}^{H} \omega_{hn} W_h \right],$$

where the left-hand side is the expected wealth held by all households in firm $n$ and the right-hand side is the fraction $1/N$ of expected wealth held by all households in all firms.

When $\rho = 0$, the following symmetry condition implies that the familiarity bias cancels out:

$$\sum_{n=1}^{N} f_{hn} = \sum_{h=1}^{H} f_{hn}, \quad \forall \ h \text{ and } n.$$  \hspace{1cm} (25)

Intuitively, the above condition means that the total familiarity of a household across all firms is equal to the total familiarity toward a firm from all households. Furthermore, the total familiarity of a household is the same across all households and the total familiarity toward a firm is the same for all firms. Another way of expressing the symmetry condition is to define a $H \times N$ familiarity matrix,

$$F = [f_{hn}]$$

and observe that the condition in (25) is equivalent to the condition that the sums across rows and columns of $F$ are all the same.$^{14}$ In the appendix, we show a similar condition holds for the case where $\rho \neq 0$.

We illustrate the symmetry condition with two examples. In both examples, we set the number of firms to be equal to the number of households, $N = H$ and assume each

$^{14}$If $H = N$, then the matrix $F$ is a multiple of doubly-stochastic matrix; that is, a square matrix with row and column sums equal to 1.
household is equally familiar with a different subset of firms, and the household is unfamiliar with the remaining firms. The number of firms in the familiar subset is always the same for each household.

In the first example, illustrated in Figure 1, we assume that household 1 is familiar with firm 1, household 2 is familiar with firm 2, and so on, with each household investing only in the firm with which it is familiar; that is, \( f_{1,1} = f_{2,2} = f_{3,3} = \ldots = f_{N,N} = f \), where \( f \in (0, 1] \), while \( f_{hn} = 0 \) for \( h \neq n \). Thus, the familiarity matrix in this case is:

\[
F = \begin{bmatrix}
  f & 0 & \cdots & 0 \\
  0 & f & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & f
\end{bmatrix}.
\]

In the second example, illustrated in Figure 2, we assume that each household is familiar with 2 firms. Let the firms be arranged in a circle, and let each household \( h \) be equally familiar with the two firms nearest to it on either side. Thus, in this case the familiarity matrix is:

\[
F = \begin{bmatrix}
  f & f & 0 & \cdots & \cdots \\
  0 & f & f & 0 & \cdots \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  f & 0 & \cdots & 0 & f
\end{bmatrix}.
\]

4.2 The Equilibrium Risk-free Interest Rate

We now impose market clearing in the risk-free bond market. The risk-free bond is in zero net-supply, so we have

\[
\sum_{h=1}^{H} B_{h,t} = 0.
\]

The amount of wealth held in the bond by household \( h \) is given by

\[
B_{h,t} = (1 - 1^\top \omega_h)W_{h,t},
\]

where \( 1^\top \omega_h \), the proportion of household \( h \)'s wealth invested in risky assets, is given in (16). Summing over households gives

\[
0 = \sum_{h=1}^{H} B_{h,t} = \sum_{h=1}^{H} (1 - 1^\top \omega_h)W_{h,t} = \sum_{h=1}^{H} \left( 1 - \frac{1}{\gamma} \frac{\alpha - i}{\sigma_N^2} f_h \right) W_{h,t}.
\]
As a consequence of the symmetry assumption, each household will have the same average familiarity, \( f_h = \bar{f} \), and hence the same demand for the risk-free asset. Therefore, the bond-market clearing condition simplifies to

\[
0 = \left(1 - \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2} \right) \sum_{h=1}^{H} W_{h,t}.
\]

The equilibrium risk-free interest rate is thus given by the constant

\[
i = \alpha - \frac{\gamma \sigma^2}{\bar{f} N}. \tag{26}
\]

We can see immediately that reducing familiarity (that is, a reduction in \( \bar{f} \)) increases the riskiness of household portfolios, leading to a greater demand for the risk-free asset, and hence a decrease in the risk-free interest rate.

### 4.3 Aggregate Growth and Social Welfare

Substituting the equilibrium interest rate in (26) into the expression for the consumption-wealth ratio in (20) gives the consumption-wealth ratio in general equilibrium, which is common across households:

\[
\frac{C_{h,t}}{W_{h,t}} = c,
\]

where

\[
c = \psi \delta + (1 - \psi) \left( \alpha - \frac{1}{2} \frac{\gamma \sigma^2}{\bar{f} N} \right).
\]

Exploiting the fact that the consumption-wealth ratio is constant across households allows us to obtain the ratio of aggregate consumption, \( C^{\text{agg}}_t = \sum_{h=1}^{H} C_{h,t} \), to aggregate wealth, \( W^{\text{agg}}_t = \sum_{h=1}^{H} W_{h,t} \):

\[
\frac{C^{\text{agg}}_t}{W^{\text{agg}}_t} = c.
\]

In equilibrium, the aggregate level of the capital stock equals the aggregate wealth of households, because the bond is in zero net supply: \( K^{\text{agg}}_t = W^{\text{agg}}_t \), where \( K^{\text{agg}}_t = \sum_{n=1}^{N} K_{h,t} \), is the aggregate level of the capital stock. We therefore obtain the aggregate consumption-capital and consumption-output ratios:

\[
\frac{C^{\text{agg}}_t}{K^{\text{agg}}_t} = c,
\]

\[
\frac{C^{\text{agg}}_t}{Y^{\text{agg}}_t} = c.
\]
and
\[ \frac{C_{t}^{\text{agg}}}{Y_{t}^{\text{agg}}} = \frac{c}{A}, \]
where aggregate output is given by
\[ Y_{t}^{\text{agg}} = \sum_{n=1}^{N} Y_{n,t} = A \sum_{n=1}^{N} K_{n,t}. \]

We now derive the aggregate investment-capital ratio. The aggregate investment flow, \( I_{t}^{\text{agg}} \), is the sum of the investment flows into each firm, \( I_{t}^{\text{agg}} = \sum_{n=1}^{N} I_{n,t} \). The aggregate investment flow must be equal to aggregate output flow less the aggregate consumption flow, i.e.
\[ I_{t}^{\text{agg}} = AK_{t}^{\text{agg}} - C_{t}^{\text{agg}}. \]
It follows that the aggregate investment-capital ratio is given by
\[ \frac{I_{t}^{\text{agg}}}{K_{t}^{\text{agg}}} = A - c = b + \psi(\alpha - \delta) + (\psi - 1) \left( \alpha - \frac{1}{2} \frac{\gamma \sigma^{2}}{f N} \right). \]
A decrease in an individual household’s average familiarity makes its portfolio riskier, that is, \( \frac{1}{f N} \sigma^{2} \) increases. There is then a reduction in the equilibrium expected return on an individual household’s portfolio adjusted for risk and familiarity bias, given by \( \alpha - \frac{1}{2} \frac{\gamma \sigma^{2}}{f N} \). When the substitution effect dominates (\( \psi > 1 \)), the aggregate investment-capital ratio falls because households will consume more of their wealth.

We now determine trend output growth, \( g \), defined by
\[ g = E_{t} \left[ \frac{dY_{t}^{\text{agg}}}{Y_{t}^{\text{agg}}} \right]. \]
Depreciation reduces the growth effect of investment. Firms all have constant returns to scale and differ only because of shocks to their capital stocks. Therefore, the aggregate growth rate of the economy is the aggregate investment-capital ratio less depreciation:
\[ g = \frac{I_{t}^{\text{agg}}}{K_{t}^{\text{agg}}} - b = \alpha - c = \psi(\alpha - \delta) - \frac{1}{2} (\psi - 1) \frac{\gamma \sigma^{2}}{f N}. \]
From the above, we see that a fall in the aggregate investment-capital ratio reduces output growth.

We now study social welfare, that is the aggregate welfare of all households. An individual household’s utility level, \( U_{h,t} \), is given by
\[ U_{h,t} = \kappa_{h} W_{h,t}, \]
where $\kappa_h$ is given by

$$\kappa_h = \left[ \frac{\psi \delta + (1 - \psi) \left( i + \frac{1}{2} \gamma \sigma^2 \right)}{\delta \psi} \right]^{\frac{1}{1-\psi}} = \left[ \frac{\psi \delta + (1 - \psi) \left( i + \frac{1}{2} \left( \frac{\alpha - i \sigma}{\sqrt{N}} \right)^2 \bar{J}_h \right)}{\delta \psi} \right]^{\frac{1}{1-\psi}}.$$

Our symmetry condition implies that average familiarity, $\bar{J}_h$, is equal across households. Hence, each household has the same utility-wealth ratio $\kappa_h = \kappa$. Substituting in the expression for the market-clearing interest rate, we obtain

$$\kappa = \left[ \frac{\psi \delta + (1 - \psi) \left( \alpha - \frac{i}{2} \sigma^2 \right)}{\delta \psi} \right]^{\frac{1}{1-\psi}}.$$

Social welfare is given by $U_t^{social}$, where

$$U_t^{social} = \sum_{h=1}^{H} U_{h,t} = \kappa \sum_{h=1}^{H} W_{h,t} = \kappa K_{t}^{agg}.$$

In the last equality, we have used the fact that aggregate household wealth $\sum_{h=1}^{H} W_{h,t}$ must equal the level of the aggregate capital stock $K_t^{agg} = \sum_{n=1}^{N} K_{n,t}$, because the bond is in zero net supply.

From the above expression, we can see that for a given level of the aggregate capital stock, familiarity biases at the household level decrease social welfare. The intuition is that familiarity biases induce individual households to invest less in risky assets and to hold underdiversified portfolios. Higher portfolio risk distorts households’ intertemporal consumption decisions. Consequently, aggregate investment is also distorted, creating a loss in social welfare.

### 4.4 The Externality from Reducing Familiarity Bias

Education in finance theory is not widespread. For instance, the vast majority of high school students receive no education in portfolio choice. Even at the university level, only a minority of students study economics or finance and only a small proportion of the population undertake graduate study with a finance element. We know that households benefit from their own individual financial education if it allows them to choose better diversified portfolios as a consequence of conquering their familiarity biases. But how significant would
be the gains of widespread financial education? In particular would any macro-externalities make such a policy particularly worthwhile in terms of the economic welfare of households?

To answer this question, we need to understand that the welfare gains take place via two different channels. One is a micro-level volatility channel, whereby a household’s welfare is increased purely from choosing a more well-balanced set of investments—the return on a household’s financial wealth then becomes less risky, which reduces her consumption-growth volatility. The second is a macro-level externality, which raises the welfare of all household’s. From where does this macro externality arise? Its source lies in the decline of risk in every household’s portfolio. When the income effect dominates the substitution effect ($\psi > 1$), household’s prefer to consume less today and invest more for the future in risky firms. Therefore, aggregate investment increases, raising the trend growth rate of the economy, and increasing welfare. When the substitution effect dominates ($\psi < 1$), household’s prefer to consume less today and invest less in risky production, thereby reducing trend growth, but still increasing welfare.

We now show analytically how to disentangle the micro-level volatility channel from the macro-level externality channels. In equilibrium, the level of social welfare can be written as

$$U_t = (\delta^\psi P^{agg}_t)^{1 - \psi} K^{agg}_t,$$

where $P^{agg}_t$ is the price-dividend ratio of the aggregate capital stock, or equivalently, the aggregate wealth-consumption ratio:

$$P^{agg}_t = \frac{K^{agg}_t}{C^{agg}_t} = \frac{W^{agg}_t}{C^{agg}_t}.$$

Importantly, we choose to write the aggregate price-dividend ratio in terms of the endogenous expected growth rate of aggregate output, $g$, and the volatility of household portfolios, $\sigma_p$, i.e.

$$P^{agg}_t = \frac{1}{\delta + \left(\frac{1}{\psi} - 1\right) \left(g - \frac{1}{2} \gamma \sigma_p^2\right)};$$

where

$$\sigma_p^2 = \frac{1}{f N} \sigma^2.$$

The micro-level welfare gain stems from a reduction in household portfolio risk, brought about by improved financial education. The macro-level externality manifests itself via a
change in expected aggregate consumption growth, $g$. We can separate the micro-level volatility and macro-level externality effects on social welfare by observing that

\[
\frac{d \ln \left( \frac{U_{agg}}{K_{agg}} \right)}{d \ln (\sigma^2_P)} = \frac{\partial \ln \left( \frac{U_{agg}}{K_{agg}} \right)}{\partial \ln (\sigma^2_P)} + \frac{\partial \ln \left( \frac{U_{agg}}{K_{agg}} \right)}{\partial \ln g} \cdot \frac{\partial \ln g}{\partial \ln (\sigma^2_P)},
\]

where the first term on the right-hand side captures the micro-level volatility effect and the second term gives the effect of the macro-level externality. Computing the relevant derivatives gives

\[
\frac{d \ln \left( \frac{U_{agg}}{K_{agg}} \right)}{d \ln (\sigma^2_P)} = -\frac{1}{2} \gamma P^2 \sigma^2 + 1 - \frac{1}{\psi},
\]

where the first term captures the micro-level volatility effect and the second term gives the effect of the macro-level externality. We can see that a decline in the risk of household portfolios always increases social welfare. The relative importance of the micro-volatility and macro-growth channels is determined by the elasticity of intertemporal substitution, $\psi$. When the elasticity of intertemporal substitution is higher, a reduction in risk at the micro-level has a greater impact at the macro-level, because households are more willing to adjust their consumption intertemporally.

4.5 Quantitative Assessment of Familiarity Bias for Social Welfare & Growth

We conclude this section by assessing quantitatively the extent to which familiarity bias reduces social welfare and distorts aggregate growth in general equilibrium. In going to general equilibrium, we discipline our analysis along two dimensions. First, we specifically require that familiarity bias in portfolios cancels when aggregating portfolio demands across households. Second, the interest rate is endogenous and will decrease when households exhibit familiarity bias, because of increased demand for precautionary savings.

Just as we did in our partial-equilibrium analysis, we set the mean-variance welfare loss equal to the value identified in Calvet, Campbell, and Sodini (2007). We then examine the magnitude of the multiperiod welfare loss in general equilibrium. We examine the size of the multiperiod welfare loss in general equilibrium, as measured by $\lambda_{GE}$ defined in
Equation (22) below,

\[
\lambda_{GE} = 1 - \left[ \frac{\psi \delta + (1 - \psi) \left( i_{bias} + \frac{1}{2\gamma} SR_{xh}^2 \right)}{\psi \delta + (1 - \psi) \left( i_{no \; bias} + \frac{1}{2\gamma} SR_{1/N}^2 \right)} \right]^{\frac{1}{1-\psi}},
\]

and compare it with the corresponding partial-equilibrium welfare loss, \(\lambda_{PE}\), defined in Equation (22).

To compute the quantity above, we treat \(i_{no \; bias}\) as an endogenous variable that changes with our choice of parameter values. The per annum Sharpe ratios, \(SR_{xh}\) and \(SR_{1/N}\) are set equal to 0.30 and 0.45, respectively, based on the empirical estimates reported in Calvet, Campbell, and Sodini (2007). We set the value of the annualized interest rate, \(i_{bias}\), equal to 0.56%, based on the annual estimate reported in Beeler and Campbell (2012, their Table 2) for the period 1930-2008. The base-case values that we use for the preference parameters are the following: relative risk aversion of the household is \(\gamma = 5\); elasticity of intertemporal substitution of the household is \(\psi = 1.5\); and, subjective rate of time preference is \(\delta = 0.02\).

In Figure 8, we report on the vertical axis two quantities: \(\lambda_{PE}\) and \(\lambda_{GE}\). On the horizontal axis, we let the Sharpe ratio of the unbiased (benchmark) portfolio range from a low of 0.30 to a high of 0.45. From the plot, we see that both the partial-equilibrium measure of welfare loss, \(\lambda_{PE}\), and the general-equilibrium measure of welfare loss, \(\lambda_{GE}\), increase as the Sharpe ratio of the benchmark portfolio increases beyond 0.30, which is the Sharpe ratio of the familiarity-biased portfolio of the household. We also see from the figure that the general-equilibrium welfare loss \(\lambda_{GE}\), given by the solid blue line, is greater than the partial-equilibrium welfare loss, \(\lambda_{PE}\), given by the dashed red line.

5 Sensitivity of Results to Modeling Assumptions

One may have at least three concerns about the large welfare gains identified above from households switching to diversified portfolios. One, on the financial side, it is assumed that households will switch from holding only a few familiar risky asset to all the risky assets; in practice, it is unlikely that households will switch to the perfectly diversified portfolio. Two, besides holding personal portfolios biased toward familiar assets, it is possible that some households have wealth invested in professionally-managed mutual funds, which are well diversified; in this case, the gains from switching their personally-managed portfolio to
one that is better diversified may be smaller than in the case where households are assumed to invest in only a few familiar assets. Three, on the real side, it is assumed that firms can adjust instantly and at no cost their investment policies within the firm, and also reallocate capital optimally across firms. Below, we examine how sensitive our results are to these assumptions.

To address the first concern, observe that in our quantitative assessment of the implications of familiarity bias of households, rather than specifying explicitly the level of familiarity of individual households, a quantity that would be difficult to estimate, we have instead used the Sharpe ratios estimated by Calvet, Campbell, and Sodini (2007) for the portfolios actually held by households and for the benchmark portfolio. This has been possible because we have found expressions for the welfare loss from familiarity bias in partial and general equilibrium (Equations (22) and (27), respectively) that are expressed in terms of the Sharpe ratio of a typical household’s portfolio and the Sharpe ratio of the benchmark portfolio that would be held by a household if it did not suffer from a familiarity bias.

And, in Figures 3 and 8, we have plotted the welfare losses in partial and general equilibrium for the case where, instead of achieving the Sharpe ratio of the benchmark portfolio (which is estimated to be 0.45), the investor achieves a Sharpe ratio that is less than that of the benchmark portfolio. So, for example, if the household was successful in raising the Sharpe ratio of its portfolio to only 0.375 from 0.30 (which is only half the potential improvement in Sharpe ratio), Figure 3 shows that even in this case the welfare gain would be substantial—about 50% of the household’s wealth.

The second concern, that households may invest not just in individual stocks but also in professionally managed mutual funds, is also mitigated because the estimates that we using from Calvet, Campbell, and Sodini (2007) for our quantitative assessment of the model are based on data that includes household investments in mutual funds. This is the benefit of relying on estimates from Calvet, Campbell, and Sodini (2007) that, in contrast to studies based on survey data or data from a particular brokerage firm, are based on comprehensive data that covers individual financial assets and are reported by Swedish financial institutions and confirmed by taxpayers who face penalties for errors in reporting.15

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15And on the theory side, if one wished, it would be straightforward to extend the theoretical model so that part of household wealth was invested in mutual funds, with only the remaining money that is being managed directly by households subject to the familiarity bias.
Finally, to study the impact of assuming that investment levels can be adjusted instantaneously, one can use the approach in Obstfeld (1994, p. 1325) where it is assumed that the annual welfare gain converges toward the long-run annual gain at an instantaneous rate of $x$ percent, which is about 2.2% per annum based on the work of Barro, Mankiw, and Sala-i-Martin (1992). Therefore, the actual capitalized welfare gain, $\lambda_{\text{actual}}$ is related to the reported welfare gain $\lambda$ as follows:

$$\lambda_{\text{actual}} = \int_{0}^{\infty} i \lambda (1 - e^{-xt}) e^{-it} dt = \frac{\lambda x}{i + x}.$$  

If the interest rate is 0.56% per annum, then $\frac{x}{i + x} = 79\%$. This implies that the actual welfare gains are about 79% of the welfare gains reported in the tables above, indicating that they are still quite large.

6 Conclusion

Our results indicate that even if the static losses from familiarity bias are modest, the dynamic effects on intertemporal consumption, and hence, welfare can be much larger for individual households. These household-level distortions to individual consumption stemming from excessive financial risk taking are amplified by aggregation and have a substantial effect on aggregate growth and welfare. These aggregate effects are present even when biases in portfolios cancel out across households. Thus, underinvestment and underdiversification by individual households distort aggregate growth and reduce social welfare.

The costs arising from underinvestment and underdiversification by households can be alleviated in at least three ways. First, financial education can play an important role by informing households about the benefits of investing in financial assets and holding diversified portfolios. For instance, Bayer, Bernheim, and Scholz (2008) find that both participation in and contributions to voluntary savings plans are significantly higher when employers offer frequent seminars about the benefits of planning for retirement, while the impact of other written materials, such as newsletters and plan descriptions, is negligible. Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2014) also find that, while general education has only a small effect in reducing familiarity bias, an increase in financial competence does reduce this bias.
Second, financial regulation could be designed to encourage participation. For example, if mutual funds were required to simplify investment procedures it would lower the barrier to entry. For example, Iyengar, Huberman, and Jiang (2004) find a negative correlation between the number of investment options offered in a plan and the participation rate in that plan. Financial regulation could be introduced also to prohibit companies from providing funds in the form of own-company stock to match the pension contributions of employees. Similarly, financial regulation could prohibit the use of own-company stock in 401(k) plans.

Finally, one could encourage financial innovation in order to design products that make it easier for households to invest in diversified portfolios. Similar to the policy advocated by Benartzi and Thaler (2004), where people commit in advance to allocating a proportion of their future salary increases toward retirement savings, one could design sensible default options that encourage households to invest in portfolios that are diversified not just across equities but also across asset classes. For example, households could be offered a small number of portfolios to choose from, with the portfolios having different levels of risk, but all of them being well diversified. Simplifying the design of investment plans, offering easy-to-understand default options, and introducing schemes to rebalance the portfolio automatically are relatively cheap ways to ensure that households invest in portfolios that are well diversified.

The analysis in our paper indicates that the answer to the question posed in the title is a resounding “yes”. Household finance matters a great deal because small improvements in the financial decisions of households have the potential to generate large economic gains for society: a small step for a household can be a giant leap for society.
A Appendix

In this Appendix, we provide all proofs not given in the main text.

A.1 The Certainty Equivalent: Derivation of (3)

The definition of the certainty equivalent in (2) implies that

$$\mu_t[U_{h,t+dt}] = E_t \left[ U_{h,t+dt}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}.$$

Therefore

$$\mu_t[U_{h,t+dt}] = E_t \left[ U_{h,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} = E_t \left[ U_{h,t}^{1-\gamma} + d(U_{h,t}) \right]^{\frac{1}{1-\gamma}}.$$

Applying Ito’s Lemma, we obtain

$$d(U_{h,t}^{1-\gamma}) = (1 - \gamma) U_{h,t}^{-\gamma} dU_{h,t} - \frac{1}{2} (1 - \gamma) U_{h,t}^{-\gamma-1} (dU_{h,t})^2$$

Therefore

$$\mu_t[U_{h,t+dt}] = E_t \left[ U_{h,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} = U_{h,t} \left( 1 + (1 - \gamma) \left[ E_t \left[ \frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2} \gamma E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right] \right)^{\frac{1}{1-\gamma}}$$

$$= U_{h,t} \left( 1 + (1 - \gamma) \left[ E_t \left[ \frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2} \gamma E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right] \right)^{\frac{1}{1-\gamma}}.$$

Hence,

$$\mu_t[U_{h,t+dt}] = U_{h,t} \left( 1 + E_t \left[ \frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2} \gamma E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right) + o(dt).$$

Therefore, in the continuous time limit, we obtain

$$\frac{\mu_t[dU_{h,t}]}{dt} = \frac{\mu_t[U_{h,t+dt}] - U_{h,t}}{dt} = U_{h,t} \left( E_t \left[ \frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2} \gamma E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right] \right).$$
A.2 The Familiarity-Biased Certainty Equivalent

We now adjust the certainty equivalent for familiarity bias to obtain the familiarity-biased certainty equivalent. A household treats $\mathbb{P}$ as a reference measure. If she is less familiar with a particular firm, she adjusts its expected return in capital, which is equivalent to changing the reference measure to a new measure, denoted by $\mathbb{Q}^\nu$.

To change measure, it is easier to work with an orthogonal set of Brownian motions. However, the increments of the Brownian motions $Z_{n,t}, n \in \{1, \ldots, N\}$ are not mutually orthogonal. We therefore define a vector Brownian motion, $\mathbf{Z}$ (under $\mathbb{P}$):

$$\mathbf{Z} = (Z_1, \ldots, Z_N)^\top,$$

where $Z_n, n \in \{1, \ldots, N\}$ is a set of mutually orthogonal standard Brownian motions under $\mathbb{P}$ such that

$$M^T \mathbf{Z} = (Z_1, \ldots, Z_N)^\top,$$

where $M$ is an $N \times N$ real matrix such that

$$M^TM = \Omega.$$

We can therefore rewrite the stochastic differential equations for the evolution of firms’ capital stocks as

$$\left( \frac{dK_1 + D_1dt}{K_1}, \ldots, \frac{dK_N + D_Ndt}{K_N} \right)^\top = \alpha dt + \sigma M^\top d\mathbf{Z}.$$

Intuitively, $\mathbf{Z}$ represents a set of mutually orthogonal factors underlying the returns on firms’ capital and we shall refer to this representation as the factor basis.

We now define the new measure $\mathbb{Q}^\nu$. We start from the exponential martingale (under the reference measure $\mathbb{P}$)

$$d\xi_{h,t} = \frac{1}{\sigma} \nu_{h,t}^\top d\mathbf{Z}_t,$$

where the $N \times 1$ vector $\mathbf{\nu}_{h,t}$ is the factor basis representation of the $N \times 1$ vector $\mathbf{\nu}_{h,t}$, i.e.

$$\mathbf{\nu}_{h,t} = (M^\top)^{-1} \mathbf{\nu}_{h,t}.$$

The new measure $\mathbb{Q}^\nu$ is defined by

$$\mathbb{Q}^\nu(A) = \mathbb{E}[1_A \xi_{h,T}],$$

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where $E$ is the expectation under $P$. Applying Girsanov’s Theorem, we see that under the new measure $Q^\nu_h$, the evolution of firm $n$’s capital stock is given by

$$dK_{n,t} = \left[ (\alpha + \nu_{hn,t})K_{n,t} - D_{n,t} \right] dt + \sigma K_{n,t} dZ^\nu_{n,t},$$

where $Z^\nu_{n,t}$ is a standard Brownian motion under $Q^\nu_h$, such that

$$dZ^\nu_{n,t} dZ^\nu_{m,t} = \begin{cases} \rho dt, & n = m, \\ \rho dt, & n \neq m. \end{cases}$$

We now define a penalty function for using the measure $Q^\nu_h$ instead of $P$. Since the factor basis is orthogonal, we can follow Uppal & Wang (2003) and define the penalty function (with respect to the factor basis) as

$$L_{h,t} = \frac{1}{2\gamma} \frac{1}{\sigma} \nu^\top_{h,t} \Gamma^{-1} \frac{1}{\sigma} \nu_{h,t},$$

where $\Gamma$ is the familiarity matrix with respect to the factor basis, i.e.

$$\Gamma = (M^\top)^{-1} \Gamma M^{-1}. $$

The intuition behind the definition of the penalty function is that it measures the familiarity-weighted distance between the reference measure and the measure $Q^\nu_h$, where the distance between them is the conditional Kullback-Leibler divergence between $P$ and $Q^\nu_h$. The penalty function is also a familiarity-weighted measure of the information lost by using $Q^\nu_h$ instead of $P$.

Using the original non-orthogonal basis for shocks to firms’ capital stocks, we obtain

$$L_{h,t} = \frac{1}{2\gamma} \frac{1}{\sigma^2} \nu^\top_{h,t} \nu_{h,t}. $$

Our initial, but somewhat formal definition for the date-$t$ familiarity-biased certainty equivalent of date-$t + dt$ household utility is given by

$$\mu^\nu_{h,t}[U_{h,t+dt}] = \hat{\mu}^\nu_{h,t}[U_{h,t+dt}] + U_{h,t} L_{h,t} dt, \quad (28)$$

where $\hat{\mu}^\nu_{h,t}[U_{h,t+dt}]$ is defined by

$$u_\gamma (\hat{\mu}^\nu_{h,t}[U_{h,t+dt}]) = E_t^Q [u_\gamma (U_{h,t+dt})].$$
We can see that $\hat{\mu}_h[t][U_{h,t+dt}]$ is like a certainty equivalent, but with the expectation taken under $Q^{\nu}_h$ in order to adjust for familiarity bias. From Section A.1, we know that

$$\mu^{Q^{\nu}_h}_h[t][U_{h,t}+dt] = U_{h,t} \left( 1 + E^{Q^{\nu}_h}_t \left[ \frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2} \gamma E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] \right) + o(dt).$$

We therefore obtain

$$\mu^{\nu}_h[t][U_{h,t+dt}] = U_{h,t} \left( 1 + E^{Q^{\nu}_h}_t \left[ \frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2} \gamma E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] + \frac{1}{2\gamma} \nu_{h,t}^{\top} \Gamma^{-1}_h \nu_{h,t} dt \right) + o(dt),$$

Applying Ito’s Lemma, we see that under $Q^{\nu}_h$

$$dU_{h,t} = W_{h,t} \frac{\partial U_{h,t}}{\partial W_{h,t}} dW_{h,t} + \frac{1}{2} W_{h,t}^2 \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2} \left( \frac{dW_{h,t}}{W_{h,t}} \right)^2,$$

where

$$dW_{h,t} = \left( 1 - \sum_{n=1}^N \omega_{h,n,t} \right) dt + \sum_{n=1}^N \omega_{h,n,t} \left( (\alpha + \nu_{h,n,t}) dt + \sigma dZ_{n,t}^{Q^{\nu}_h} \right) - C_{h,t} W_{h,t} dt.$$

Hence,

$$E^{Q^{\nu}_h}_t [dU_{h,t}] = W_{h,t} \frac{\partial U_{h,t}}{\partial W_{h,t}} dW_{h,t} + \frac{1}{2} W_{h,t}^2 \frac{\partial^2 U_{h,t}}{\partial W_{h,t}^2} \left( \frac{dW_{h,t}}{W_{h,t}} \right)^2$$

$$= E_t [dU_{h,t}] - \frac{1}{2} \gamma U_{h,t} E_t \left[ \left( \frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] + W_{h,t} \frac{\partial U_{h,t}}{\partial W_{h,t}} \omega_{h,t}^{\top} \nu_{h,t} dt$$

$$= \mu[t][dU_{h,t}] + W_{h,t} \frac{\partial U_{h,t}}{\partial W_{h,t}} \omega_{h,t}^{\top} \nu_{h,t} dt$$

and so we obtain (5). Given its relatively simple form, we take (5) as a definition as opposed to (28).

### A.3 Derivation of the Hamilton-Jacobi-Bellman Equation

Writing out (7) explicitly gives

$$U_{h,t}^{1-\frac{1}{p}} = (1 - e^{-\delta dt}) C_{h,t}^{1-\frac{1}{p}} + e^{-\delta dt} \left( \mu^{\nu}_h[t][U_{h,t+dt}] \right)^{1-\frac{1}{p}},$$

where for ease of notation sup and inf have been suppressed. Now

$$\left( \mu^{\nu}_h[t][U_{h,t+dt}] \right)^{1-\frac{1}{p}} = (U_{h,t} + \mu^{\nu}_h[t][dU_{h,t}])^{1-\frac{1}{p}}$$
\[
U_{h,t}^{1 - \frac{1}{\psi}} \left( 1 + \mu_{h,t} \left[ \frac{dU_{h,t}}{U_{h,t}} \right] \right)^{1 - \frac{1}{\psi}} = U_{h,t}^{1 - \frac{1}{\psi}} \left( 1 + \left( 1 - \frac{1}{\psi} \right) \mu_{h,t} \left[ \frac{dU_{h,t}}{U_{h,t}} \right] \right) + o(dt).
\]

Hence
\[
U_{h,t}^{1 - \frac{1}{\psi}} = \delta C_{h,t}^{1 - \frac{1}{\psi}} dt + U_{h,t}^{1 - \frac{1}{\psi}} \left( 1 + \left( 1 - \frac{1}{\psi} \right) \mu_{h,t} \left[ \frac{dU_{h,t}}{U_{h,t}} \right] \right) - \delta U_{h,t}^{1 - \frac{1}{\psi}} dt + o(dt),
\]
from which we obtain (9).

A.4 Mean-Variance Portfolio Choice with Familiarity Bias

Assuming a constant risk-free rate, homotheticity of preferences combined with constant returns to scale for production implies that we have \( U_{h,t} = \kappa_h W_{h,t} \), for some constant \( \kappa_h \). Equation (10) is then a direct consequence of (5) and (9).

A.5 Gordon Growth Formula

Household \( h \)'s optimal consumption-wealth ratio given in (21) can be rewritten as
\[
\frac{C_{h,t}}{W_{h,t}} = i + \gamma \sigma^2_{p,h} - g_h,
\]
where
\[
\sigma^2_{p,h} = \frac{1}{f_{h,N}} \sigma^2
\]
is the variance of the optimal portfolio held by household \( h \) in equilibrium and \( g_h \) is household \( h \)'s expected consumption growth, given by
\[
g_h dt = E_t \left[ \frac{dC_{h,t}}{C_{h,t}} \right] = E_t \left[ \frac{dW_{h,t}}{W_{h,t}} \right].
\]
We therefore obtain household \( h \)'s wealth-consumption ratio
\[
\frac{W_{h,t}}{C_{h,t}} = \frac{1}{i + \gamma \sigma^2_{p,h} - g_h}.
\]
Note that the risk-neutral growth rate of household \( h \)'s consumption is given by
\[
\hat{g}_h = g_h - \gamma \sigma^2_{p,h}.
\]
and so

\[
\frac{W_{h,t}}{C_{h,t}} = \frac{1}{i - \bar{g}_h}.
\]

A.6 Symmetry Condition

When \( \rho = 0 \), we have

\[
\omega_{hn} = \frac{1}{\gamma} \frac{\alpha - i}{\sigma^2} f_{hn}.
\]

Substituting the above expression into (24) and simplifying gives (25).

For the general case when \( \rho \neq 0 \), substituting (15) into (24) and simplifying implies that aggregate familiarity \( \bar{f}_h \) is independent of \( h \), i.e.

\[
\forall h, \bar{f}_h = \bar{f}.
\]
Figure 1: First example of symmetry condition

In this figure, we illustrate the first example of the symmetry condition for familiarity of households with certain firms. We set the number of firms to be equal to the number of households, $N = H$, and assume that household 1 is familiar with firm 1, household 2 is familiar with firm 2, and so on, with each household investing only in the firm with which it is familiar; that is, $f_{1,1} = f_{2,2} = f_{3,3} = \ldots = f_{N,N} = f$, where $f \in (0,1]$, while $f_{hn} = 0$ for $h \neq n$. 
Figure 2: Second example of symmetry condition

In this figure, we illustrate the second example of the symmetry condition for familiarity of households with certain firms. We set the number of firms to be equal to the number of households, $N = H$, and assume that each household is familiar with two firms. Let the firms be arranged in a circle, and let each household $h$ be equally familiar with the two firms nearest to it on either side.
In this figure, we plot three quantities while letting the Sharpe ratio of the unbiased benchmark portfolio in partial equilibrium range from 0.31 to 0.45. The three quantities we plot are: one, the measure of multiperiod welfare loss, $\lambda_{PE}$, which is defined in Equation (22); two, the measure of single-period welfare loss, $\lambda_{MV}$, which is defined in Equation (23); and, three, the ratio $\lambda_{PE}/\lambda_{MV}$. The values that we use for the preference parameters are the following: relative risk aversion of the household is $\gamma = 2$; elasticity of intertemporal substitution of the household is $\psi = 1.5$; and, subjective rate of time preference is $\delta = 0.02$. The per annum interest rate is assumed to be 0.56% and the Sharpe ratio of the familiarity-biased portfolio is 0.30. The risk-free interest rate is assumed to be 0.56%.
Figure 4: Welfare loss for different levels of RRA in partial equilibrium

In this figure, we plot three quantities while letting the relative risk aversion (RRA) of the investor, $\gamma$, range from 1.5 to 3. The three quantities we plot, all from the partial-equilibrium economy, are: one, the measure of multiperiod welfare loss, $\lambda_{PE}$, which is defined in Equation (22); two, the measure of single-period welfare loss, $\lambda_{MV}$, which is defined in Equation (23); and, three, the ratio $\lambda_{PE}/\lambda_{MV}$. The values that we use for the preference parameters are the following: elasticity of intertemporal substitution of the household is $\psi = 1.5$; and, subjective rate of time preference is $\delta = 0.02$. The per annum interest rate is assumed to be 0.56%, the Sharpe ratio of the familiarity-biased portfolio is 0.30, and the Sharpe ratio of the unbiased benchmark portfolio is 0.45. The risk-free interest rate is assumed to be 0.56%.

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![Graph](image-url)
Figure 5: Welfare loss for different levels of EIS in partial equilibrium

In this figure, we plot three quantities while the elasticity of intertemporal substitution (EIS), denoted by $\psi$, ranges from 0.5 to 1.6. The three quantities we plot, all from the partial-equilibrium economy, are: one, the measure of multiperiod welfare loss, $\lambda_{PE}$, which is defined in Equation (22); two, the measure of single-period welfare loss, $\lambda_{MV}$, which is defined in Equation (23); and, three, the ratio $\lambda_{PE}/\lambda_{MV}$. The values that we use for the preference parameters are the following: relative risk aversion of the household is $\gamma = 2$; and, subjective rate of time preference is $\delta = 0.02$. The per annum interest rate is assumed to be 0.56%, the Sharpe ratio of the familiarity-biased portfolio is 0.30, and the Sharpe ratio of the unbiased benchmark portfolio is 0.45. The risk-free interest rate is assumed to be 0.56%.

![Graph showing welfare loss for different levels of EIS in partial equilibrium]
Figure 6: Welfare loss for different time preference rates in partial equilibrium

In this figure, we plot three quantities while letting the rate of time preference of the investor, $\delta$, range from 0.02 to 0.04. The three quantities we plot, all from the partial-equilibrium economy, are: one, the measure of multiperiod welfare loss, $\lambda_{PE}$, which is defined in Equation (22); two, the measure of single-period welfare loss, $\lambda_{MV}$, which is defined in Equation (23); and, three, the ratio $\lambda_{PE}/\lambda_{MV}$. The values that we use for the preference parameters are the following: relative risk aversion of the household is $\gamma = 2$; and elasticity of intertemporal substitution of the household is $\psi = 1.5$. The per annum interest rate is assumed to be 0.56%, the Sharpe ratio of the familiarity-biased portfolio is 0.30, and the Sharpe ratio of the unbiased benchmark portfolio is 0.45. The risk-free interest rate is assumed to be 0.56%.
Figure 7: Welfare loss for different risk-free rates in partial equilibrium
In this figure, we plot three quantities while letting the risk-free interest rate, $i$, ranges from 0.25% to 2%. The three quantities we plot, all from the partial-equilibrium economy, are: one, the measure of multiperiod welfare loss, $\lambda_{PE}$, which is defined in Equation (22); two, the measure of single-period welfare loss, $\lambda_{MV}$, which is defined in Equation (23); and, three, the ratio $\lambda_{PE}/\lambda_{MV}$. The values that we use for the preference parameters are the following: relative risk aversion of the household is $\gamma = 2$; elasticity of intertemporal substitution of the household is $\psi = 1.5$; and, subjective rate of time preference is $\delta = 0.02$. The per annum interest rate is assumed to be 0.56%, the Sharpe ratio of the familiarity-biased portfolio is 0.30, and the Sharpe ratio of the unbiased benchmark portfolio is 0.45.
Figure 8: Multiperiod welfare loss in partial and general equilibrium

In this figure, we plot two quantities while letting the Sharpe ratio of the unbiased benchmark portfolio range from 0.30 to 0.45. The two quantities we plot are: one, the measure of multiperiod welfare loss in partial equilibrium, $\lambda_{PE}$, which is defined in Equation (22) and two, the measure of the multiperiod welfare loss in general equilibrium, $\lambda_{GE}$, which is defined in Equation (27). The values that we use for the preference parameters are the following: relative risk aversion of the household is $\gamma = 5$; elasticity of intertemporal substitution of the household is $\psi = 1.5$; and, subjective rate of time preference is $\delta = 0.02$. The per annum interest rate is assumed to be 0.56% and the Sharpe ratio of the familiarity-biased portfolio is 0.30. The risk-free interest rate in the economy with familiarity bias is assumed to be 0.56%. We also assume that the investor can invest in a total of $N = 100$ assets, each of which has a volatility of 16%, a correlation of 0.25%, and an expected rate of return of 5%. The endogenous risk-free rate in equilibrium in the economy with no familiarity bias corresponding to the above parameter values is 1.7%.

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**PE multiperiod**

**GE multiperiod**

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References


