Product Bundling in a Vertical Distribution Channel

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Abstract

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Many industries feature a vertical distribution channel structure in which a downstream player (retailer) sells a bundle, composed of products from multiple upstream producers (manufacturers). For example, cable or satellite TV carriers bundle dozens of channels from multiple studios and programming networks. The bundling literature offers deep insights about the economic benefits of bundling, but is limited to a direct manufacturer-buyer setting. Conversely, marketing and supply chain studies on channel management have not considered the possibility of product bundling by the retailer. How do the economic incentives of the manufacturers and retailers in a vertical channel impact the mechanics of bundling? We develop a unique two-stage model to capture the competitive dynamics between the upstream manufacturers and the downstream retailer, which combines the horizontal competition between manufacturers with the strategic choice of selling mechanism (bundle or components) by the retailer. We show that bundling need not emerge as an equilibrium outcome in a vertical channel, even under conditions ripe for its prevalence in a direct manufacturer-buyer market. If the retailer were forced to bundle, the manufacturers exploit this restriction and over-price their components, leading to substantial economic losses especially to the retailer. The retailer’s threat to unbundle the products does not negate these losses, rather it veers the system to a component-selling equilibrium. Despite this failure of a bundling equilibrium, bundling has the Pareto-improving property of raising the surplus of all firms and the collective consumer surplus. Bundling outcomes may, therefore, yet emerge due to alternative industry dynamics. However, these outcomes will be subject to constant pressure due to individual firms’ self-interest and desire to grab a greater share of the gains from bundling, as indeed is frequently witnessed in the form of “carriage disputes” within the TV industry. Other mechanisms must therefore be sought that lead to bundling as a natural competitive outcome.

Keywords: Bundling, Channel, Supply Chain
1 Introduction

[From AP, Oct 16, 2010:] Cablevision, the service provider for 3 million customers in the New York area, and Fox parent News Corp. failed to solve a dispute over rates Saturday, leaving baseball fans who wanted to watch the opener of the National League Championship Series with a blank screen instead of a marquee pitching matchup. The stalemate ... was the latest in a series of programming fee disputes that have led to blackouts of programs such as the Oscars.

Such “carriage fee” disputes occur increasingly often in the TV industry, between Manufacturers (programming networks or studios who own rights to content) and Retailers (service providers such as Comcast, TimeWarner, DirecTV and Dish Network who offer consumers bundles of channels comprising content from multiple manufacturers). This paper studies market structures in which a Retailer adds value primarily by bundling products created by independent Manufacturers. We develop an analytical model which articulates and analyzes the dominant economic forces underlying this structure. We identify the economic frictions that cause frequent tensions between participating firms, demonstrating that pricing disputes such as in the TV industry arise from the intersection of two prominent economic forces: product bundling, which enables both sets of firms a way to extract higher surplus; and channel conflict, which prevents the firms from reaching stable agreements for sharing these additional revenues made possible by product bundling. We also propose future research opportunities aimed at producing more stable outcomes.

Bundling, by a downstream firm, of products made by multiple independent upstream firms, is a common phenomenon. The TV industry is a prototypical example. Similarly, online music and movie retailers aggregate content from a number of third-party firms (see Fig. 1). Cultural organizations and theaters sell annual subscriptions which provide patrons access to a bundle of artists and shows. PC manufacturers such as HP and Dell can be viewed as bundling several components from suppliers who provide disk drives, memory, screens, CPU chips, etc. System integration is another form of bundling which is practiced
widely, for instance, in the defense industry. As the examples indicate, this kind product bundling is particularly prevalent in the information and technology industries, and is a core economic strategy for many firms such as information aggregators. Yet, and despite a massive literature on both product bundling and channel conflict, there is little academic literature on the practice of product bundling across a vertical distribution channel.

Figure 1: Product bundling in entertainment industry. Rhapsody gives consumers access to “all the music you want” (over 10 million songs as of July 2010) for a flat fee; DirecTV bundles over 150 channels into its basic offering; Netflix charges a monthly flat fee to “watch as many movies as you want” (100,000 DVD titles and 12,000 streaming titles as of February 2009).

1.1 Related Work

Stigler (1963) is one of the earliest papers to discuss the demand-smoothing benefits of product bundling (the paper refers to “block booking” of movies), followed by Adams and Yellen (1976)’s seminal work on a firm’s reasons to adopt either *mixed bundling* (sell bundles as well as individual products, e.g., most theaters sell both annual packages and individual shows) or *pure bundling* (sell only the bundle, e.g., Rhapsody). The bundle demand is flatter, hence more predictable, than that for component products, and bundling raise the firm’s profit even though the bundle price is typically heavily discounted relative to the price for purchasing individual products separately. The gains are highest when marginal costs are very low relative to value, and when consumers’ valuations of the individual products are negatively correlated (Schmalensee, 1984). Bakos and Brynjolfsson (1999) demonstrated that bundling is especially useful for information goods, i.e., when marginal costs are low.
relative to valuations. Later research examined bundling under competition (Bakos and Brynjolfsson, 2000), bundling of complements and substitutes (Venkatesh and Kamakura, 2003) customized bundling (Wu et al., 2008), strategic tying of products (Carlton and Waldman, 2002), and an empirical analysis of implicit price bundling of retail products Mulhern and Leone (1991). For a comprehensive, but slightly dated, survey of bundling literature, please see Stremersch and Tellis (2002).

While the literature on bundling has analyzed many facets of bundling, it is entirely focused on a direct producer-buyer distribution structure (e.g., a restaurant that packages a burger, chips and a drink; or a software firm that bundles multiple components). The models treat the firm which bundles products as the one which also produces them, at least insofar as having exogenous marginal costs for the individual products: to wit, the costs influence, rather than are influenced by, the bundling strategy. In contrast, aggregator firms—the ones that create and execute the bundling strategy—operate in a multiple-echelon distribution channel: their costs for the individual products they bundle are in fact the prices set by the manufactures and, hence, are themselves susceptible to the choice and implementation of bundling strategy. These differences naturally raises several questions—is bundling as effective when the products are purchased from manufacturers? how does the interplay between manufacturers and the retailer impact their pricing strategies and the economics of bundling? can the retailer employ bundling as a coordination device?—that are not addressed in existing literature.

Several papers have studied “composite goods” where two or more component products, made by different firms, must work together to create utility. Almost two hundred years ago, Cournot (1929) provided a classic example (“zinc + copper = brass”) and, assuming that the buyer does the composition, established that two independently-acting component makers would overprice their component products relative to the demand function for the composite good. Sarvary and Parker (1997) study the same problem for information goods
under a variety of product and utility characteristics. He and Yin (2010) examine alliance formation between a monopoly provider of one component and several firms that make a complementary component that can be sold separately or jointly with the first. These models do not consider an active bundling decision by an intermediate firm such as a retailer. Bakos and Brynjolfsson (2000) study a vertical channel with both upstream competition for content and downstream competition for consumers but these two processes are modeled separately of each other. The upstream analysis covers potential purchase of a single component product (thereby avoiding the complexity caused by intra-manufacturer price setting in the backdrop of inter-manufacturer-retailer dynamics) by two competing aggregators who already possess a collection of goods; their Proposition 2 shows that the already bigger aggregator wins this competition. Bakos and Brynjolfsson (1999) acknowledge market structures in which an aggregator sells a bundle of products made by independent monopolists, but do not model the problem or examine the underlying competitive dynamics and tradeoffs.

The interplay between multiple firms in a vertical channel—lacking in the bundling literature—is studied in supply chain management and marketing in a vertical distribution channel. Spengler (1950) identified the double marginalization problem wherein margin-seeking activity in a multilateral monopoly causes the respective firms to make choices that hurt their profits and overall welfare. Pasternack (1985) showed that a buyback policy could achieve channel coordination in a one-manufacturer one-retailer setting (bilateral monopoly). McGuire and Staelin (1983) examined the choice of channel structure when multiple manufacturers sell their products through separate retailers, and Choi (1991) examined the impact of manufacturers selling through a common retailer and engaging in price competition, while Corbett and Karmarkar (2001) examined manufactures engaged in quantity competitions. Adida and DeMiguel (2010) is the first to consider both multiple retailers and multiple competing manufacturers. Again, while these papers cover many facets of channel economics, they do not consider the possibility of a bundling strategy. In all the models—even the
ones with multiple manufacturers—the retailer essentially passes the manufacturer’s products through to the consumer, and price competition signifies that consumers can compare multiple products or must choose because of a fixed budget. Still, the insights about double marginalization and ways to restore efficiency are relevant to understanding the mechanics of bundling in a vertical distribution channel.

1.2 Overview of Model and Results

While the practice of across-channel product bundling is widespread, there is, to our knowledge, no work on bundling strategy for a firm that aggregates products from several manufacturers. This article fills this gap and develops theory that can be applied to settings such as the cable industry, music distribution, system integrators, etc. We develop and analyze a model in which a single retailer bundles products from multiple manufacturers. We are able to address a number of novel and challenging issues that arise in this unique multi-echelon distribution channel. How do the economic incentives of the manufacturers and retailers impact the mechanics of bundling? Would a firm find it as attractive to bundle products from several manufacturers as it would its own? What kind of inefficiencies are created due to the combination of the vertical channel structure and the retailer’s bundling strategy? How is the economic loss distributed among various players in the channel? At the root of these questions is a fundamental question: How does bundling at the retail level impact the manufacturer’s pricing decisions in the vertical channel? Answering these questions requires analyzing a complex interplay between the manufacturers and retailer, as well as competitive interplay between the manufacturers.

A model with two independent manufacturers and one retailer who combines the manufacturers’ component products is sufficient to explore the issues identified above. With zero marginal costs and independent consumer demand across the products, pure bundling would clearly dominate if a single manufacturer owned both products and could sell directly
to consumers (i.e., in an integrated system). But when the products flow through a vertical distribution channel, we show that bundling is no longer an equilibrium outcome despite being attractive to all parties. Our model incorporates a two-stage game in which the retailer chooses its bundling strategy and product prices, while the manufacturers simultaneously set their prices knowing the reaction function of the retailer. We note that the traditional setting of bundling by a multi-product manufacturer already poses substantial modeling challenges due to co-dependence between demand curves for the individual products and bundles. Our setting adds two levels of complexity, the first due to the multi-echelon channel structure, and the second due to the price “competition” between multiple manufacturers whose products are being bundled. Due to these substantial complexities, we develop a series of models—rather than one monolithic model—to formally investigate the research questions while deriving the bulk of our results using formal analytical proofs.

§2 examines a retailer which can only sell a bundle of products from independent monopolists (i.e., it cannot or will not offer individual products). This model enables us to isolate the impact of the vertical channel on the economic characteristics of bundling. First, we establish that bundling in a channel structure causes manufacturers to over-price their products, leading to reduction in overall system and firm profits beyond the effects of double marginalization; we demonstrate how to separate these losses. Second, this reduction is primarily borne by the retailer, who sees lower sales and higher input costs. Third, differences in quality (or popularity) of the items in the bundle get aggregated and shared among all manufacturers, rather than accrue to the high-quality firm, and cannot be converted into higher profits.

Next, §3 models a retailer who strategically chooses whether to sell individual products or to combine them into a bundle. Manufacturers must, therefore, factor the retailer’s strategic choice of selling strategy (and prices) when setting wholesale prices. As before, manufacturers tend to over-price their products realizing that high price will restrain price level of the
other manufacturer. But this renders bundling unattractive to the retailer. Conversely, if manufacturers set prices low enough to induce bundling by the retailer, then the retailer sets a relatively high price to earn high margins, leaving the manufacturers to suffer from relatively low sales. Thus, a bundling strategy is inconsistent with unilateral self-interest of firms that operate across a vertical distribution channel. This presents the firms with a Prisoner’s Dilemma: both the manufacturers and the retailer could improve profits by adopting a bundling solution in which manufacturers adopt prices which are lower than their optimal prices in a component selling strategy, and the retailer agrees to adopt a bundling strategy even though it would receive higher profits selling the products independently. Moreover, bundling has the distinctive characteristic that it can improve the surplus of each of the firms as also consumer surplus. Therefore, bundling may yet emerge as an outcome driven by alternative competitive dynamics. Still, such solutions are likely to confront constant tensions and dissatisfaction among participating firms who each jockey to extract a higher share of the gains from bundling. This is indeed the case in the TV industry. This underlines the need to find coordination mechanisms that will make bundling the outcome of the natural competitive dynamics. We discuss these considerations and other research opportunities in §4.

2 Basic Model: Pure Bundling

We begin by analyzing a pure bundling strategy practiced by a retailer $R$ who obtains products from two manufacturers $M_1$ and $M_2$. The only product available in the retail market is the bundle. We choose this restriction in order to isolate the interplay between a downstream bundling strategy and a vertical channel structure in which production is distributed across multiple firms. The restriction may also be a reasonable model in settings where supply-side factors constrict the retailer to offer only the bundle (e.g., if there are large
economies of scope in distribution). As in Choi (1991) we employ a manufacturer-Stackelberg framework (manufacturers employ the retailer’s price reaction function in setting their prices) to solve the retailer-manufacturer pricing decisions. The two products may be asymmetric in cost or quality (i.e., the range of consumer valuations). Without loss of generality, assume that M2’s product contributes more to consumer valuations, with a quality advantage of $\delta$ and an additional marginal cost $c \geq 0$.

Figure 2: Sequence of events when Retailer is pre-committed to a bundle selling strategy.

Figure 2 displays the decision-making game, sequence of events, and essential notation. In the first stage of the game, manufacturers M1 and M2 individually and simultaneously set prices $w_1, w_2$. Let $\omega = w_1 + w_2$, represent the combined per-unit input cost for the manufacturer. In the second stage, the retailer observes manufacturer prices and sets the market price $p = p(w_1, w_2)$ for the bundle. The retail sales $Q$ are then derived from the market demand function $D(p)$. In the first stage, the manufacturers anticipate the retailer’s price function and the corresponding bundle demand, which also becomes the realized sales for each component product. Let $\Pi_R$ denote the retailer’s profit and $\Pi_1, \Pi_2$ the manufacturers’ profit functions. Then the triple $(w_1^*, w_2^*, p^*)$ is an equilibrium solution of the game if and only if
\[ p^*(w_1^*, w_2^*) = \arg \max_p \Pi(p; w_1^*, w_2^*) = \arg \max_p \{(p - (w_1^* + w_2^*)) D(p)\} \]

\[ w_1^* = \arg \max_{w_1} \Pi_1(w_1, w_2^*; p^*) = \arg \max_{w_1} \{w_1 \cdot D(p^*(w_1, w_2^*))\} \] (1)

\[ w_2^* = \arg \max_{w_2} \Pi_2(w_1^*, w_2; p^*) = \arg \max_{w_2} \{w_2 \cdot D(p^*(w_1^*, w_2))\}. \]

### 2.1 Equilibrium Prices

To solve for the equilibrium, we employ a linear formulation of the demand for the bundle, \( D(p) = a - b \cdot p \), where \( a = a(\delta) \) signifies that strength of demand depends on \( M2 \)'s incremental quality (keeping the baseline quality constant). The linear formulation is a reasonable approximation for bundle demand (see the reviewer’s supplement) and enables a closed-form solution.

**Lemma 1** Each manufacturer’s optimal price response is a decreasing function of its competitor’s price. The overall pricing game has a unique equilibrium \((w_1^*, w_2^*, p^*)\), with

<table>
<thead>
<tr>
<th>Unit Cost</th>
<th>Price</th>
<th>Quantity</th>
<th>Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M1</strong></td>
<td>0</td>
<td>( w_1^* = \frac{a(\delta)-bc}{3b} )</td>
<td>( \frac{(a(\delta)-bc)^2}{18b} )</td>
</tr>
<tr>
<td><strong>M2</strong></td>
<td>( c )</td>
<td>( w_2^* = \frac{a(\delta)-bc}{3b} + c )</td>
<td>( \frac{(a(\delta)-bc)^2}{18b} )</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>( w^* = \frac{2a(\delta)+bc}{3b} )</td>
<td>( p^* = \frac{5a(\delta)+bc}{6b} )</td>
<td>( \frac{(a(\delta)-bc)^2}{36b} )</td>
</tr>
<tr>
<td><strong>M1+M2</strong></td>
<td>( c )</td>
<td>( w^* = \frac{2a(\delta)+bc}{3b} )</td>
<td>( \frac{(a(\delta)-bc)^2}{9b} )</td>
</tr>
<tr>
<td><strong>Firms’ Total</strong></td>
<td>( c )</td>
<td></td>
<td>( \frac{5(a(\delta)-bc)^2}{36b} )</td>
</tr>
</tbody>
</table>

The proof for Lemma 1 (all proofs are in the Appendix) emphasizes a key distinction between traditional horizontal competition and the current setting where intra-manufacturer competition is mediated by the vertical distribution channel. The manufacturers’ price-response functions (of the form \( \Omega_j(w_i) = (\cdot) - \frac{w_i}{2} \), see Eq. 11–12) indicate that manufacturer price acts as a public resource: if firm \( M_i \) consumes more (sets a higher price) then firm
Mj is best off consuming less, setting a lower price in order to obtain a desirable price and quantity outcome at the retail level. Intuitively, Mi’s price increase pushes higher the retail price of the bundle and lower overall sales. For instance in the TV industry, if one studio increases the price of its shows (which are part of a cable bundle), a corresponding price increase in prices for subscription packages would reduce demand for all shows. This demand reduction alters Mj’s margin-volume tradeoff in the direction of lower margin, hence Mj’s price reduction is an effort to mitigate the negative effects of increase in Mi’s price. Second, however, each manufacturer has a tendency to overprice because the retailer’s bundle price opaquely combines the input prices set by the manufacturers, reducing the negative impact of high price on demand. These two factors combine to generate a unique equilibrium in which manufacturer prices are higher, and profits lower, than if they could coordinate their prices (this is discussed further in §2.3).

2.2 Differences in Manufacturer Quality and Cost

One notable aspect of the outcomes is that the higher-cost manufacturer sets a higher price, with differential just enough ($w_2 - w_1 = c$) to offset the higher cost, and as a result both manufacturers make the same profit. Intuitively one would expect the higher cost manufacturer to be penalized and earn a lower profit. However, because products get merged into a bundle, both manufacturers sell the same quantity $q$. And since each manufacturer sets price such that marginal revenue equals marginal cost—and M2 has the higher cost—therefore, at any quantity $q$, M2 demands a higher marginal revenue and therefore wants to price higher than M1. However, as the retailer’s input price goes up (and quantity goes down) M1 experiences greater pain, so that M1 has a preference for a lower price level. Due to the combination of these preferences, the two manufacturers share the effect of M2’s higher cost: M2 raises price by $\frac{2c}{3}$ while M1 drops price by $\frac{c}{3}$ (relative to the price levels for symmetric manufacturers, $c = 0$).
A second notable feature is that M2’s quality differential $\delta$ equally benefits both manufacturers, and it also benefits the retailer, rather than just the manufacturer responsible for the high quality. Higher quality does raise overall demand for the bundle, but the consequent rewards get distributed across the firms. This “free riding” effect causes tensions between participating firms. Manufacturers who perceive their product popularity to be above-average will tend to negotiate separately for higher shares of the bundle price. True popularity is not a necessary prerequisite for such demands, just a perception of popularity is sufficient to cause tension in the bundling agreement. For example, cable TV packages include some shows that are considered more popular—and drive a subscriber to purchase the package—while others are hangers-on which command little pricing power by themselves. The “higher quality” shows will tend to demand higher shares, while the “lower quality” shows will resist a second-tier status in revenue-sharing. Because such dissatisfaction occurs in the context of a per-subscriber pricing arrangement between manufacturers and the retailer, a useful research question is whether it can be addressed through per-view pricing; this is possible, for example, when the retailer can meter consumption of individual products in the bundle. For now, the salient point to note is that quality differential $\delta$ can (other than being incorporated into the valuations for the bundle) essentially be eliminated from the analysis. Thus, the two manufacturers can be considered equivalent in quality and differentiated in cost.

2.3 Interplay between Bundling Strategy and Channel Structure

Lemma 1 describes the equilibrium under a competitive structure where the manufacturers and retailer compete in a vertical channel while the manufacturers compete horizontally as well. To facilitate analysis of this solution, we describe the outcomes for two additional structures: (i) an **integrated system** in which the manufacturers’ and retailer’s interests are merged, leaving the final retail price of the bundle as the only decision variable, and (j)
Table 1: Equilibrium Outcome under Different Structures. The first row, \( w^* \) is the equilibrium input cost faced by the bundling firm. The row \( \Pi^*_M \) is the total profit realized by the two manufacturers. The rightmost column is based on the differences in outcome between the structure (j)—in which a single upstream firm sets the wholesale price for the bundle—and the fully competitive structure in which the retailer creates a bundle out of products made by independent upstream firms.

<table>
<thead>
<tr>
<th>( w^* )</th>
<th>( p^* )</th>
<th>( Q^* )</th>
<th>( \Pi^*_M )</th>
<th>( \Pi^*_R )</th>
<th>Firms’ combined profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>( \frac{c}{2} + \frac{6a}{12b} )</td>
<td>( \frac{6(a-bc)}{12} )</td>
<td>( \frac{18(a-bc)^2}{144b} )</td>
<td>( \frac{9(a-bc)^2}{144b} )</td>
<td>( \frac{36(a-bc)^2}{144b} )</td>
</tr>
<tr>
<td>( \frac{c}{2} + \frac{3a}{6b} )</td>
<td>( \frac{3(a-bc)}{12} )</td>
<td>( \frac{3(a-bc)}{12} )</td>
<td>( \frac{16(a-bc)^2}{144b} )</td>
<td>( \frac{4(a-bc)^2}{144b} )</td>
<td>( \frac{27(a-bc)^2}{144b} )</td>
</tr>
<tr>
<td>( \frac{c}{2} + \frac{3a}{6b} )</td>
<td>( \frac{3(a-bc)}{12} )</td>
<td>( \frac{3(a-bc)}{12} )</td>
<td>( \frac{16(a-bc)^2}{144b} )</td>
<td>( \frac{4(a-bc)^2}{144b} )</td>
<td>( \frac{20a^2 - (2bc)^2}{144b} )</td>
</tr>
<tr>
<td>( \frac{c}{2} + \frac{3a}{6b} )</td>
<td>( \frac{3(a-bc)}{12} )</td>
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a bilateral monopoly where the two joined manufacturers set the input price \( w \) while the retailer chooses the price of the bundle in the retail market. Table 1 summarizes the outcomes under these three structures. Comparing the competitive solution against (i) highlights the efficiency loss (and other factors, including over-pricing by manufacturers) caused by the combination of bundling strategy and the vertical channel structure. Of greater interest to us is the incremental difference from solution (j) to the competitive solution; this difference captures the impact of the practice by a downstream of creating a bundle out of products
made by independent upstream firms. This is because the difference between columns (j) and (i) already captures the double marginalization effect for selling a bundle of products 1 and 2. The rightmost column of Table 1 highlights the key findings; we use the special case \( c = 0 \) in order to get a clear numerical impact. First, consider the final row which lists the total profit of all firms, and the previous two rows \( \Pi_M^* \) and \( \Pi_R^* \) which describe the profit earned by the retailer and the manufacturers under the different structures.

**Proposition 1 (Profit reduction)** The practice of an aggregator who bundles products from independent upstream manufacturers reduces overall system profit beyond the reduction caused by double marginalization. This additional reduction in system profit is primarily borne by the retailer, who sees lower sales and higher input costs.

Counterintuitively, Proposition 1 highlights that the retailer would benefit greatly, rather than lose, if the manufacturers could coordinate on setting prices. Consumers would also realize a small benefit (see the \( p^* \) row) if the manufacturers merged. In the absence of such coordination, self-interested manufacturers set higher prices (see the first row, \( w^* \)) which lowers their own profit, in addition to harming the retailer’s interest. The manufacturers’ price competition is akin to quantity competition in a Cournot duopoly, where each firm’s quantity reaction function is decreasing in the other firm’s quantity choice, causing an incentive to overproduce relative to a coordinated solution. In our setting, each manufacturer’s profit penalty from pricing too high is diminished because the price increase is subsumed into a bundle price. Moreover, manufacturers are aware that the retailer cannot pass higher input costs into the retail price, because retail demand for the bundle is highly elastic (owing to the demand smoothing effect which makes the demand curve flatter than for individual products). These forces give the manufacturers an incentive to overprice their products, reminiscent of (Cournot, 1929, Ch. IX)’s theory of composite goods in a direct channel.
Proposition 2 (Manufacturer over-pricing) Retailer bundling of products from independent upstream manufacturers causes manufacturers to over-price their products in equilibrium (relative to the joint manufacturer pricing), raising the retailer’s price and reducing system sales.

Propositions 2 and 1 shed light on the economic effects of bundling products from independent upstream manufacturers. Collectively, the retailer’s commitment to a pure bundling strategy puts it at a disadvantage. Many retailers practice product bundling (i.e., become aggregators) because it has lower menu costs or other supply-side economies of scope. A key implication of Proposition 1 is that a retailer who chooses to practice bundling must create mechanisms and product pricing policies that reverse the profit reduction effect of manufacturers’ overpricing. One such strategy is to leave open the possibility of selling the products individually, conveying to manufacturers that bundle sales would no longer occur if the retailer’s input cost were too high. We investigate this next.

3 Strategic Choice of Bundle or Component Sales

The previous section demonstrated that a retailer’s practice of product bundling incentives manufacturers to over-price, thereby negating some of the economic benefits of bundling. We also noted that the threat of unbundling could partially reverse the above incentive. To what extent is this true, and what conditions lead to a bundling equilibrium? Now we analyze this question by extending our analysis to a retailer who strategically chooses whether to sell the component products or a bundle. To what extent will manufacturer pricing be restrained by the mere possibility of unbundled component sales by the retailer? What is the nature of the equilibrium outcome? Will it result in bundling to the same extent as in a direct channel? How will it impact the surplus of various players relative to the components-only strategy and to other non-equilibrium strategies that involve collaboration among some parties? We
answer these questions in this section.

3.1 Modeling Framework

The market has one retailer, two manufacturers, and consumers who demand up to one unit of each of two products. The sequence of actions chosen by these players is as follows. Each manufacturer sets a per-unit wholesale price \( w_1 \) or \( w_2 \) for her product. The retailer observes manufacturer prices and then determines (a) whether or not to bundle the products and (b) how to price them. Let \( \Theta \) be the set comprising all \( (w_1, w_2) \) for which the retailer prefers to bundle the products. Let \( p \) represent retail prices, with \( p^B \) the bundle price if the retailer chooses this strategy, and \( p_1, p_2 \) the individual product prices if the retailer sells as components. Consumers observe the selling structure and prices and make their purchase decisions, leading to the realized sales level \( Q_1, Q_2 \) for each of the two products (note: if the retailer sells as a bundle, then all firms achieve the same sales, \( Q^B \)). Please see Figure 3 for a visual depiction of the steps. Intuitively, the retailer will prefer a bundling strategy when the combined input cost \( \omega = w_1 + w_2 \) is not too high. The manufacturers can therefore reason by comparing their profits under the bundle and unbundled scenarios, while noting that the product prices chosen by them will impact the retailer’s choice of bundling strategy.

An equilibrium outcome in the overall game is a strategy profile \( (w_1, w_2, BC, p_1, p_2, p^B) \) where \( BC \) is \( B \) or \( C \) representing the retailer’s choice of Bundling or Components, respectively. By convention, \( p_1 \) and \( p_2 \) are null (sufficiently high) when the retailer picks \( B \) while \( p^B \) is null when the retailer picks \( C \). Since the retailer deterministically sets prices and strategy after observing \( w_1 \) and \( w_2 \), any outcome of the game is fully described by the pair of manufacturer prices \( (w_1, w_2) \). Define \( \Pi_i^C(w_1, w_2) \) as manufacturer \( i \)'s profit when the retailer sells Components and picks the best component prices corresponding to \( (w_1, w_2) \); this is not necessarily an equilibrium because manufacturers may deviate from \( (w_1, w_2) \) if the retailer picks Components. Similarly, let \( \Pi_i^B(w_1, w_2) \) denote manufacturer \( i \)'s profit when the retailer
Figure 3: Price and bundling strategy game with two manufacturers and one retailer.

picks a bundling strategy. Let $\Pi^B_R(w_1, w_2)$ be the retailer’s profit if it sells a bundle (priced optimally given $(w_1, w_2)$), and let $\Pi^C_R(w_1, w_2)$ be its best profit from selling components. Then the pair of prices $(w_1, w_2)$ constitutes an equilibrium if and only if at least one of the following two pairs of conditions is satisfied. Note that in formulating these conditions, we expect each manufacturer to pick its best price, taking the competitor’s price as a given, but while anticipating the retailer’s bundling/components strategy (and prices) as a function of the pair of input prices.

**Bundling Equilibrium:** $(w_1, w_2)$ such that the retailer prefers a bundling strategy, and each manufacturer prefers its own price over any other price $w$ (including prices that induce the retailer to sell as components).

\[
\begin{align*}
\Pi^B_R(w_1, w_2) & \geq \Pi^C_R(w_1, w_2) \\
\Pi^B_1(w_1, w_2) & > \max_w \{ \Pi^B_1(w, w), \Pi^C_1(w, w) \} \\
\Pi^B_2(w_1, w_2) & > \max_w \{ \Pi^B_2(w_1, w), \Pi^C_2(w_1, w) \}
\end{align*}
\]

(2)

**Component Equilibrium** $(w_1, w_2)$ such that the retailer prefers a components selling strategy, and each manufacturer prefers its own price over any other price $w$ (including
prices that induce the retailer to bundle).

\[
\left( \Pi^C_R(w_1, w_2) \geq \Pi^B_R(w_1, w_2) \right) \text{ AND } \left\{ \begin{array}{ll}
\Pi^C_1(w_1, w_2) > \max_w \left\{ \Pi^B_{(w, w_2) \in \Theta} \Pi^C_{(w, w_2) \not\in \Theta} \right\} \\
\Pi^C_2(w_1, w_2) > \max_w \left\{ \Pi^B_{(w_1, w) \in \Theta} \Pi^C_{(w_1, w) \not\in \Theta} \right\} \end{array} \right. \quad (3)
\]

Further analysis of this problem requires generating the bundle demand curve from the consumer valuations of the individual products, and solving the two-stage game between manufacturers and retailer, with a strategic choice on bundling embedded in the second stage, and price competition between manufacturers in the first stage. For computational tractability, therefore, we assume that consumer valuations for each of these products are independent and identically distributed. Formally, let \( u_1(v) \) and \( u_2(v) \) be the valuations of an arbitrary consumer \( v \) for products 1 and 2 respectively. Both \( u_1 \) and \( u_2 \) are drawn from a common distribution \( f \) with support \([0, b]\), so that the density function \( g \) that describes consumer valuations for a bundle comprising the two products is the convolution

\[
g(z) = (f * f)(z) = \int_0^z f(z - y) f(y) \, dy \quad \text{for} \quad z \in [0, 2b], \ 0 \text{ elsewhere}.
\]

If the product valuations for the two individual products are uniformly distributed, then

\[
g(z) = \begin{cases} 
z, & \text{if } z \in [0, 1] \\
2 - z, & \text{if } z \in [1, 2] \\
0, & \text{elsewhere.}
\end{cases}
\]

Without loss of generality, let \( b = 1 \). Then, consumer demand for the two individual products
is expressed as \( D_i(p) = 1 - p \), while the demand curve for the bundle is

\[
D_B(p) = \begin{cases} 
\int_p^1 x \, dx + \frac{1}{2} & = 1 - \frac{p^2}{2} \quad \text{if } p \in [0, 1] \\
\int_p^2 (2 - x) \, dx & = \frac{1}{2} (2 - p)^2 \quad \text{if } p \in [1, 2] \\
0, & \text{elsewhere.}
\end{cases}
\]

It is important to note that bundle selling is the optimal strategy under this formulation, both under an integrated system (i) and a bilateral monopoly (j) where the two manufacturers coordinate to present a joint input cost to the retailer. The result is obvious under (i) due to zero marginal costs: firms’ total profit from selling components is 0.5, while bundling increases this 11% to 0.554. More importantly, under setting (j), the retailer’s use of bundling increases its own profit (40% from 0.125 to 0.176) as well as manufacturer profit and consumer surplus. For brevity, we postpone the details of these results to §3.4, where these are presently jointly with the equilibrium outcomes under the competitive structure.

### 3.2 Retailer Strategy

Recall that the set \( \Theta \) is defined as \( \{(w_1, w_2) : \Pi^B_R(w_1, w_2) \geq \Pi^C_R(w_1, w_2)\} \). To compute this set, we develop the retailer’s optimal prices conditional on each of the selling strategies and then compare the profits under the two strategies. This yields

**Proposition 3 (Retailer’s Selling Strategy)** The retailer’s optimal pricing strategy and profits under each \((w_1, w_2)\) are

<table>
<thead>
<tr>
<th>Sell Bundle : ((w_1, w_2) \in \Theta)</th>
<th>Sell Components : ((w_1, w_2) \notin \Theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Price</strong></td>
<td>( p^B = \frac{1}{3} (\omega + \sqrt{6 + \omega^2}) )</td>
</tr>
<tr>
<td><strong>Quantity</strong></td>
<td>( Q^B = \frac{2}{3} - \frac{\omega}{9} (\omega + \sqrt{6 + \omega^2}) )</td>
</tr>
<tr>
<td><strong>Profit</strong></td>
<td>( \Pi^B_R = \frac{1}{27} (\omega^3 + (6 + \omega^2)^{\frac{3}{2}} - 18\omega) )</td>
</tr>
</tbody>
</table>

(4)
and the retailer picks a bundle selling strategy when manufacturer prices are in

\[ \Theta = \left\{ (w_1, w_2) : (\omega \leq \frac{1}{2}) \land \left( \frac{1}{27}(\omega^3 + (6 + \omega^2)^{\frac{3}{2}} - 18\omega) \geq \frac{(1 - w_1)^2 + (1 - w_2)^2}{4} \right) \right\}. \quad (5) \]

Fig. 4 graphically depicts the range of manufacturer prices for which the retailer prefers selling a bundle. This result offers a simple contrast to an integrated system in which manufacturer sells its own two products as well as a bilateral monopoly where a retailer sells two products from the same manufacturer; bundling is always optimal in both cases under i.i.d demand and zero marginal costs (this can be verified from Table 2). Whereas, in the competitive case, the retailer’s choice of bundling requires that the sum of manufacturer prices be below 0.5 (see Proof of Proposition 3); in addition, Fig. 4 suggests each of the prices must be below \( \frac{1}{4} \). We can therefore limit further analysis of the bundling outcome to \( \omega < 0.5 \).

### 3.3 Manufacturer Strategy

As given in Eq. 4, \( Q_i \), the number of units sold by manufacturer \( i \), depends on the retail prices \( (p_1, p_2) \) or \( p^B \) which depend, in turn, on the wholesale prices \( (w_1, w_2) \). Therefore (i) for manufacturer prices \( (w_1, w_2) \in \Theta \), \( Q_i \) is identical for both manufacturers, while (ii) otherwise, \( Q_i = \frac{1-w}{2} \). We plug this into the manufacturer \( i \)'s profit function to compute \( \Pi_i(w_1, w_2) = w_i \cdot Q_i(w_1, w_2) \).

Consider the strategic thinking of Manufacturer M2 on its price. For an M1 price of \( w_1 \), M2 can either pick the best price \( w_2 \) that induces the Retailer to bundle or it can pick the best price corresponding to a components outcome. In the latter case, M2’s optimal price is trivially computed at \( \frac{1}{2} \) and profit \( \frac{1}{8} \).\footnote{We know that at this price, the Retailer will NOT merge the two products into a bundle, hence computing a component-strategy profit for M1 is consistent with the retailer’s move.} Under retailer bundling, M2 sells \( Q^B \) units as given in Eq. 4, hence M2’s profit is \( w_2 \cdot \left( \frac{2}{3} - \frac{w_1+w_2}{9} (w_1 + w_2 + \sqrt{6 + (w_1 + w_2)^2}) \right) \). Within this
branch, M2’s (best) profit is the maximum value of this term over all \( w : (w_1, w) \in \Theta \) (see Eq. 5). Therefore, M2’s best-response to \( w_1 \) is be the maximizer of this function when the (maximum) function value exceeds \( \frac{1}{8} \), and \( \frac{1}{2} \) otherwise.

Now consider the inter-dependency between the manufacturers’ prices. Intuitively, one firm’s price increase is detrimental to the other (when the retailer is bundling the products) because it causes the retailer to increase price, thereby reducing sales for both. Looking at this formally and applying the Envelope Theorem, it is easily seen that M2’s maximum profit (inside the \( \Theta \) region) reduces as \( w_1 \) increases. This implies the existence of a threshold \( \hat{w}_1 \) such that \( w_1 > \hat{w}_1 \) causes M2 to pick a price that leads to a Components outcome (i.e., \( w_2 = \frac{1}{4} \) for all \( w_1 > \hat{w}_1 \)). Now consider how M2’s optimal price response \( \Omega_2(w_1) \) varies with change in \( w_1 \) (for \( w_1 < \hat{w}_1 \)). Applying Topkis’s Theorem (Topkis, 1998) yields the counterintuitive finding that M2’s price reduces as \( w_1 \) increases, parallel to the finding in

Figure 4: Retailer’s choice of pure components vs pure bundle strategy, as a function of manufacturer input prices.
Lemma 1. Combining all of these conclusions yields the following result.

**Proposition 4 (Manufacturer’s Optimal Price Response)** Each manufacturer’s best price response is a decreasing function of the other manufacturer’s price up to some threshold, beyond which optimal price jumps to $\frac{1}{4}$.

![Figure 5: Manufacturers’ pricing strategy. The two curves (dashed and regular) indicate each of the two manufacturer’s optimal price responses to the other’s price. The points labeled $B^?$ and $C^?$ are candidate Bundling and Component equilibria, respectively.](image)

The main idea of Proposition 4 is encapsulated in Fig. 4. The price response functions are discontinuous and non-monotonic. The retailer’s use of bundling causes a co-dependence between the manufacturers’ prices and demand. Consider the problem from M2’s perspective. At a low M1 price, M2 covets a bundling outcome because the retailer can set a relatively low price causing high demand for all. As M1’s price goes up (causing lower demand for the bundle and for each manufacturer) M2’s trade-off between margin and quantity sold is perturbed, and she prefers to lower price in order to maintain the retailer’s bundling strategy. But when M1’s price is too high, M2 no longer covets a bundle outcome and jumps to a high price that is optimal under a components outcome. Thus, Proposition 4 points to an intra-manufacturer dynamics of accommodation and posturing, causing each to price higher in order to capture a greater share of retail revenues. The question is whether this posturing
and high pricing will force prices so high to make bundling unattractive to the retailer. We resolve this issue in the next section.

3.4 Equilibrium Properties

Above, we have derived some general properties about the manufacturers’ pricing strategies. Solving the best-response functions for the two manufacturers yields candidate equilibria (of the B and C type), but these must additionally be checked for consistency with the retailer’s bundle vs. components selling strategy (see Eq. 2–3). For the Components selling regime, the solution \( w^*_1 = w^*_2 = \frac{1}{4}; p^*_1 = p^*_2 = \frac{3}{4} \) —which yields \( \Pi^C_R = \Pi^C_1 = \Pi^C_2 = \frac{1}{8} \)—is an equilibrium, with neither firm benefitting from a unilateral deviation.

For a potential bundling equilibrium, examining the manufacturers’ best-response pricing strategy, we find that the manufacturer prices \( w_1, w_2 \) must be roots of a computationally complex system of 3 equations (of order 4, 4, 2 respectively) in 3 variables \( (w_1, w_2 \text{ and } z = \sqrt{6 + (w_1 + w_2)^2}) \). While this system admits multiple solutions we can assert that there is a unique real feasible solution

\[
    w_1 = w_2 = \sqrt{\frac{5\sqrt{33} - 21}{32}} \approx 0.49.
\]

This solution violates the Retailer’s conditions for choosing a bundling strategy (recall one of the constraints, \( w_1 + w_2 < 0.5 \)), and moreover there is no solution \( (w_1, w_2) \in \Theta \) that satisfies the manufacturers’ optimal pricing rules. This leads to the conclusion that bundling is not an equilibrium outcome in the vertical channel structure (please see the Appendix for all details behind these computations).

**Proposition 5 (Non-existence of Bundling Equilibrium)** The two-stage game has a unique equilibrium solution in which the manufacturers set prices \( w_1 = w_2 = \frac{1}{2} \), and the retailer sells the component products at prices \( p_1 = p_2 = \frac{3}{4} \). Each firm makes a profit of \( \frac{1}{8} \),
the total profit across all three firms is $\frac{3}{8}$, and total consumer surplus is $\frac{1}{16}$, achieving a total system surplus $\frac{7}{16}$ (of a possible 1).

The significance of this result is that it occurs in a setting—with two products, independent and additive consumer valuations, and zero marginal costs—where bundling is ideal under a direct manufacturer-consumer market. In fact, bundling is ideal even in a bilateral monopoly where both products are made by one manufacturer and sold through the retailer (refer the end of §3.1). This property is snuffed out by the competitive interplay between the three different firms (the two independent manufacturers and the retailer). This happens because, on the one hand, a low enough manufacturer price (which induces the retailer to bundle the products) allows the retailer to appropriate the gains by setting a relatively high bundle price, earning a high profit margin but lowering the level of product sales thereby diminishing manufacturer profits. Whereas, on the other hand, a relatively high manufacturer price leaves the retailer with no benefits from bundling.

![Figure 6](image_url)

Figure 6: Payoffs under joint manufacturer vs fully competitive structures, conditional on bundling and component selling strategies. Bundling improves the payoffs when manufacturers can coordinate on pricing. Payoffs are lower in the competitive structure, with component selling being superior to bundling.
Table 2: Outcomes under different channel structures (†: not an equilibrium solution). Columns (i) describe an integrated system; (j) covers manufacturers jointly setting prices in a bilateral monopoly; C is the fully competitive solution. ω is the sum of manufacturer prices, and Π_M is the total manufacturer profit.

Table 2 lists the outcomes under alternate channel structures conditional on each of the two selling strategies. In particular, compare the competitive structure to structure (j) where the manufacturers jointly present the component prices to the retailer. Under structure (j), bundling is the equilibrium outcome, and it raises all firms’ profits even though manufacturers compete for channel profits with the retailer. The bundle price in the (j) equilibrium \((\frac{10}{9})\) is lower than the sum of component prices in the C equilibrium, hence consumer surplus is also higher under bundling. And, of course, all players’ payoffs are even lower in the competitive structure (with its components equilibrium). Figure 6 graphically depicts the relative payoffs of all players under the two channel structures and the bundle vs. components selling strategies. Last, an integrated system obviously produces even better payoffs and also
leads to a bundling outcome.

**Proposition 6 (Pareto-improving Collaboration between Firms)** Collaboration between manufacturers or between the manufacturers and retailer can increase all of their profits and total surplus. Moreover, there exist specific collaboration rules that can increase surplus of each firm as also consumer surplus.

With regard to structure (j) where manufacturers coordinate (raising system profit to 0.375 from 0.305), there is a straightforward solution for revenue-sharing between manufacturers at least when the firms are symmetric: split the total manufacturer profits. Revenue-sharing between asymmetric manufacturers would be a useful topic for further research. Potential gains are greater, however, from coordination between the retailer and manufacturers, which raises industry profits to 0.5. However, revenue-sharing is no longer straightforward and requires further research (as does revenue-sharing between asymmetric manufacturers). Regardless of the specific revenue-sharing rules, Proposition 6 makes the case for some degree of price coordination between firms. This counters the usual idea that such coordination between firms is anti-competitive and leads to higher prices. However, there are other contexts such as *complementary* products (e.g., platforms and applications) for which price coordination is argued to be desirable (Lichtman, 2000). Proposition 6 emphasizes that price coordination between firms would increase consumer (and firm) surplus even for non-complementary products.

4 Conclusion

This paper has examined the possibilities for product bundling in a vertical distribution channel where an aggregating retailer bundles individual products from independent upstream manufacturers. What have we learnt so far? We showed that the vertical distribution channel, combined with intra-manufacturer competition, causes substantial efficiency loss and
It seems like an annual rite: to usher in the new year, cable providers and networks squabble over programming fees.

Cable companies, burdened by the cost of programming, are starting to seriously consider ... letting television subscribers pay for just those channels they want to watch.

Figure 7: A few examples of carriage disputes in the TV industry (Jan-Sep 2010).

distortion. The retailer’s use of bundling causes manufacturers to over-price their products (relative to the joint manufacturer pricing or to a single-product vertical channel), raising the retailer’s price and reducing system sales. These effects go beyond the losses due to double marginalization and are primarily borne by the retailer. Moreover, these losses cannot be reversed by a retailer’s threat of unbundled sales. In the presence of this threat (or opportunity) the manufacturers set prices so high to make bundling unattractive for the retailer. Conversely, if manufacturers were to set prices (so low) to make bundling attractive, then the retailer sets a relatively high price to earn high margins while leaving the manufacturers to suffer from relatively low sales. Thus, the vertical channel structure has the potential to destroy the economic benefits of product bundling.

Our finding that a bundling strategy is inconsistent with unilateral self-interest of firms that operate across a vertical distribution channel does not imply that bundling practices would not be observed in such settings, indeed they are. What our findings suggest is that
such practices would emerge due to other factors, such as alternative industry dynamics. For instance, bundling would be consistent with a setting in which the retailer was dominant and could move first to set a bundle price and make take-it-or-leave-it offers to the manufacturers. It might also emerge if there was substantial competition between manufacturers of each product, or if the firms focused on long-term strategic interests while keeping the incentives of other players into account. In this kind of a repeated game, firms could well agree to a bundle-inducing solution.

But regardless of the driving force behind the realization of bundling in practice, such outcomes would be subject to constant pressure due to individual firms’ self-interest and desire to grab a greater share of the gains from bundling. In the cable TV industry, which is a prime example of bundling in a vertical distribution channel, such tensions have publicly risen to the fore in the form of carriage disputes, with several studios and broadcast networks demanding a greater share of the cable TV operator’s per-subscriber (or “carriage”) fee. As manufacturers (i.e., studios) jockey for higher per-subscriber fees, the retailer (TV carrier) could either raise subscriber fees (reducing demand and hurting all firms) or drop the high-fee manufacturer. Fig. 7 provides a glimpse of a few recent examples of carriage disputes and their consequences. For instance, millions of Cablevision subscribers were unable to watch the Academy Awards this year when the carrier dropped ABC channels over a carriage dispute; in another case, angry subscribers formed a Facebook group to pressure Cablevision to bring FoodNetwork and HGTV back into its bundle; and News Corp. is believed to have asked Time Warner Cable a high $1/month/subscriber for rights to carry the Fox channels (which is an extra $300 million/year, assuming about 25 million subscribers). The Cablevision-Fox dispute in October 2010 was resolved (with what Cablevision described as an “unfair price”) on October 29, only after much controversy, demands from the FCC and political leaders, the impending start of the baseball playoffs on October 31, and finally the resolution of a dispute between Fox and the Dish Network.
One notable aspect of our analysis is that bundling can be surplus-enhancing solution for all parties, the manufacturers, retailer, and consumers. Price coordination between firms is one way to facilitate a bundling solution. But it may also be useful to examine more formal incentive-compatible mechanisms that will make bundling the natural outcome in a competitive game. One possibility is the practice of a mixed bundling strategy by the retailer, wherein certain expensive products are omitted from basic bundles and offered only in a super bundle or, alternately, certain less expensive products are excluded from the bundle and offered for independent purchase. This threat of omission from the bundle can reduce the manufacturers’ incentive to over-price because they can no longer free-ride on the bundle. This may explain the practice of cable TV channels which tend to offer expensive channels such as HBO separately rather than as part of the basic package. Another possibility is the use of alternative pricing formulas by the retailer. The current model, following industry practice, assumes a per-user or per-subscriber price, leading to manufacturer revenue regardless of whether their product is actually consumed. This enables manufacturers to hide poor quality or offer high price. An alternative model is a per-use or per-view fee, in which manufacturer compensation depends on the actual consumption of their contribution to the bundle. The extreme implementation of this approach, of course, would degenerate to a component selling strategy, hence a hybrid of per-user and per-use fee would be a useful topic for future research.

Industry-specific arrangements may also help achieve coordination. In the TV industry, a possible coordination mechanism and a useful topic for research is the role of advertising. Since ad revenue accrues primarily to the manufacturers (content owners) they have an increased incentive to maximize subscribers rather than margins, which makes a bundling equilibrium more desirable. A related factor is the brand image of the manufacturer and its position in the product lifecycle. For instance, certain emerging channels are included in cable TV bundles at zero carriage fee, with the aim to establish the brand as well as
generate advertising revenue. Moreover, industry structure is generally more complex than assumed in our simple model with two single-product manufacturers. Most of the mythical 500 channels on TV are offered by about 20 studios each of whom may have more than a dozen channels, with some bundling occurring at the studio (manufacturer) level. For example, Disney executives negotiate for the inclusion of certain less-popular channels in exchange for the right to carry ESPN, similarly News Corp. charges a bundle price for the collection of FOX channels. Another complicating factor is that some retailers are also manufacturers who provide their own content (e.g., Comcast several networks including E! The Style Network, G4, and the Golf Channel).\footnote{http://www.comcast.com/corporate/about/pressroom/comcastcablenetworks/comcastcablenetworks.html} Incorporating any of these features into our model would be challenging but would highly enrich the analysis. Our model is a useful starting point and provides an analysis framework that is computationally tractable and insightful. We hope that it will spur substantial new work in this exciting area.
A Appendix: Technical Details

Proof of Lemma 1.

Plug \( D(p) = (a - b \cdot p) \) into the firms’ profit functions (Eq. 1). First, solving the second-stage retailer’s optimization function (treating \( w = w_1 + w_2 \) as the retailer’s input cost) yields the retailer’s optimal solution

\[
\begin{align*}
p^* &= \frac{w}{2} + \frac{a}{2b} \\
Q^* &= \frac{a - bw}{2} \\
\Pi_R^* &= \frac{1}{4b}(a - bw)^2
\end{align*}
\]

Next, to compute manufacturer prices, note that these prices consider (a) the retailer’s pricing strategy (Eq. 6), which will influence how many units the manufacturer can sell (Eq. 7), and (b) the competing manufacturer’s pricing strategy, which will affect the retailer’s input cost and hence price and quantity sold. Manufacturer \( i \)'s profit function is

\[
\begin{align*}
\Pi_1 &= w_1Q^* = w_1 \left( \frac{a - bw_1 - bw_2}{2} \right) \\
\Pi_2 &= (w_2 - c)Q^* = (w_2 - c) \left( \frac{a - bw_1 - bw_2}{2} \right)
\end{align*}
\]

which yields the two best response functions

\[
\begin{align*}
\Omega_1(w_2) &= \frac{a}{2b} - \frac{w_2}{2} \\
\Omega_2(w_1) &= \frac{a + bc}{2b} - \frac{w_1}{2}
\end{align*}
\]
Solving these yields the equilibrium solution

\[ w_1^* = \frac{a - bc}{3b} \]  \quad (13)  
\[ w_2^* = \frac{a + 2bc}{3b} \]  \quad (14)  
\[ w^* = \frac{2a + bc}{3b} \]  \quad (15)  
\[ p^* = \frac{5a + bc}{6b} \]  \quad (16)  
\[ Q^* = \frac{a - bc}{6} \]  \quad (17)  

with the last two terms obtained by substituting Eq. 15 into Eq. 6-7. Plugging these optimal solutions into the retailer and manufacturer profit functions yields each player’s equilibrium surplus.

Proof of Proposition 3 (Retailer’s Selling Strategy). First consider the retailer’s pricing under a pure components strategy (with demand curves \( D_i(p) = 1 - p \)). The solution is trivially obtained by solving the first-order condition and plugging back the optimal price.

**Pure Components**

\[ p_i^* = \frac{1}{2} + \frac{w_i}{2}; \quad Q_i^* = \frac{1}{2} - \frac{w_i}{2} \]
\[ \Pi_{CR} = \frac{(1-w_1)^2+(1-w_2)^2}{4} \]  \quad (18)  

Next consider the retailer’s pricing under a pure bundling strategy. Given the bundle demand \( D_B(p) \) and a combined input cost \( \omega = w_1 + w_2 \), the optimal bundle price is obtained by maximizing the profit function \( \Pi_B^R = (p - \omega)D_B(p) \). Consider the profit function for the two price segments \([0, 1] \) and \([1, 2] \).

\( p \leq 1 \): \( \Pi_R^B = (p - \omega)(1 - \frac{p^2}{2}) \) yields \( p^* = \frac{1}{3}(\omega + \sqrt{6 + \omega^2}) \). This is valid \((p \leq 1)\) only for \( \omega \leq \frac{1}{2} \).

\( p \geq 1 \): \( \Pi_R^B = \frac{1}{2}(p - \omega)(2 - p)^2 \) yields \( p^* = \frac{2}{3}(1 + \omega) \) (valid only when \( \omega \geq \frac{1}{2} \)), valid for \( \omega > \frac{1}{2} \).
By definition, $\Pi^*_B = \max\{\Pi^*_B(p \leq 1), \Pi^*_B(p \geq 1)\}$. Intuitively, the firm should prefer the higher price ($p \geq 1$) when the input cost is substantial. Comparing the two profits, the switchover point is $\omega = \frac{1}{2}$. This yields the pricing strategy

$$\text{Pure Bundling}$$

$$\begin{cases} 
\text{if } \omega \leq \frac{1}{2}, & p^B = \frac{1}{3}(\omega + \sqrt{6 + \omega^2}); \\
& Q^B = \frac{2}{3} - \frac{\omega}{9}(\omega + \sqrt{6 + \omega^2}) \\
\Pi^B = \frac{1}{27}(\omega + 1)(6 + \omega^2)^{\frac{3}{2}} - 18\omega \\
\text{if } \omega \geq \frac{1}{2}, & p^B = \frac{2}{3}(1 + \omega) \\
& Q^B = \frac{2}{9}(2 - \omega)^2 \\
& \Pi^B = \frac{2}{27}(2 - \omega)^3.
\end{cases}$$

(19)

Next, we eliminate the second case ($\omega > \frac{1}{2}$) from further consideration because bundling can never dominate components strategy when $\omega > \frac{1}{2}$. This is because, in this case, the retailer’s best bundling profit is no more than $\frac{1}{4}$ (using Eq. 19) whereas it can make at least $\frac{9}{32}$ by selling components (this follows from Eq. 18). Now, comparing Eq. 18 with (the $\omega \geq \frac{1}{2}$ part of) Eq. 19 yields the desired result.

Proof of Proposition 4 (Manufacturer’s Optimal Price Response). The first part of the result (that M2’s optimal profit reduces as $w_1$ increases) is trivially obtained using the Envelope Theorem. For the second part, M2’s profit under retailer bundling is $\Pi_2(w_1, w_2) = w_2 \cdot (\frac{2}{3} - \frac{w_1 + w_2}{9}(w_1 + w_2 + \sqrt{6 + (w_1 + w_2)^2})).$ Now, applying Topkis’s Theorem (combining comparative statics with the implicit function theorem) to Eq. ?? (over $(w_1, w_2) \in \Theta$), the direction of change in $\Omega_2(w_1)$ as $w_1$ changes is the same as the sign of $\frac{\partial^2 \Pi^B_2}{\partial w_1 \partial w_2}$. We want to show that this derivative is negative:

$$\frac{\partial^2 \Pi_2}{\partial w_1 \partial w_2} = -\frac{2}{3} - \frac{\omega}{3 \sqrt{6 + \omega^2}} + \frac{\omega^3}{9(6 + \omega^2)^{\frac{3}{2}}} < 0$$

Algebraic manipulation of the term reduces the requirement to $2(6 + \omega^2)^{\frac{3}{2}} + 18\omega + 2\omega^3 > 0$, which is obviously true.

Proof of Proposition 5 (Non-Existence of Bundling Equilibrium). In a bundling
equilibrium, manufacturer prices $w_1, w_2$ must satisfy the conditions

$$w_1 = \arg \max_{w:(w,w_2) \in \Theta} \left( w \cdot \left( \frac{2}{3} - \frac{w + w_2}{9} (w + w_2 + \sqrt{6 + (w_1 + w)^2}) \right) \right)$$

$$w_2 = \arg \max_{w:(w_1,w) \in \Theta} \left( w \cdot \left( \frac{2}{3} - \frac{w_1 + w}{9} (w_1 + w + \sqrt{6 + (w_1 + w)^2}) \right) \right)$$

Computing derivatives and re-formulating the first-order conditions with the variable substitution $z = \sqrt{6 + (w_1 + w_2)^2}$, we note that the manufacturer prices must be roots of the following simultaneous system of equations

$$z \cdot (6 - (w_1 + w_2)(3w_1 + w_2)) - (12w_1 + 3w_1^3 + 6w_2 + 7w_1w_2 + 5w_1^2w_2^2 - w_2^3) = 0 \ (20)$$

$$z \cdot (6 - (w_1 + w_2)(w_1 + 3w_2)) - (12w_2 + 3w_2^3 + 6w_1 + 7w_2w_1 + 5w_2w_1^2 - w_1^3) = 0 \ (21)$$

$$z - \sqrt{6 + (w_1 + w_2)^2} = 0 \ (22)$$

Plugging these equations into the Mathematica symbolic computing environment yields one real solution $w_1 = w_2 = \sqrt{\frac{5\sqrt{33} - 21}{32}} \approx 0.49$; there are also several imaginary solutions, and there might be additional real solutions. However, employing a Taylor approximation of the $Q^B_H$ term we can confirm that the real solution above is unique (several alternative approximations produce solutions close to this one).

Next it is possible that there is a solution $(w_1, w_2)$ at the boundary of $\Theta$. However, in this case the manufacturers’ best-response price functions have no intersection, hence there does not exist a bundling equilibrium. The only candidate equilibrium for the original problem, therefore, is $C^\alpha$ and it is easy to confirm that neither firm has any benefit from a unilateral deviation from this solution.
References


Figure 8: Linear demand is a reasonable approximation for the bundle, with customer valuations distributed uniformly over a support \([a_0, a]\) with \(a_0\) substantially greater than 0. For products 1 and 2, the dots represent customer valuations sampled from a normal distribution, while the linear regression line is an approximation of a linear demand curve based on these valuations.

B Supplementary Information for Reviewers

B.1 Linear Demand for Bundle

It is known that demand-smoothing makes the bundle demand “flatter” relative to demand for the individual products. While the distribution of customer valuations for the bundle tends towards the normal (causing a sigmoid demand function), a reasonable approximation to this function is a linear demand curve \(a - bp\), where \(a\) is an increasing function of the quality differential \(\delta\). The linear approximation is exhibited in Figure 8 and is, from a price optimization perspective, better suited to the bundle than to individual products (due to the demand smoothing effect). The key implication of demand smoothing is that customer valuations are distributed (uniformly) over an interval \([A_0, A]\) whose left bound \(A_0\) may be substantially higher than 0, therefore any optimization routine must ensure that the optimal price not be below \(A_0\).
B.2 Taylor Approximation of Quantity Sold (Pure Bundling vs. Components)

The Taylor approximation of $\sqrt{1+y}$ is $1 + \frac{1}{2}y - \frac{1}{8}y^2 + \frac{1}{16}y^3 - \ldots$. We plug this into M1’s “quantity sold” term ($\frac{2}{3} - \frac{\omega}{9}(\omega + \sqrt{6+\omega^2})$), transforming $\frac{\omega^2}{6}$ into $y$. As shown in Fig. 9, picking just the first two terms in the expression provides a powerfully close approximation, because the $\omega^2$ in the replacement term leads to high order terms that contribute little to the value of the function (note that the next term would be $-\frac{\omega^4}{288}$, a tiny amount for $\omega < 0.5$).

Figure 9: Taylor Approximation of Quantity Sold