Multiproduct Competition, Informative Advertising and Cognitive Limitations

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October 2011

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Abstract

In several product categories, such as cereals, ice cream, and detergents, competing firms advertise a variety of products designed to meet the needs of their diverse consumers. Although consumers are exposed to thousands of such product advertisements in a day, they pay attention to only a very small fraction of these advertisements because of their cognitive limitations. Using a novel model of multiproduct competition and informative advertising in a horizontally differentiated market, we examine how consumers’ limited attention span affects a firm’s product line length. Our analysis shows that the number of products offered by a monopolist will not exceed consumers’ attention span, if congesting consumers’ attention span could adversely affect the effective reach of its product advertising. On the contrary, we will observe product proliferation and advertising clutter in a duopoly. Behavioral research suggests that adding many products under a brand umbrella weakens the linkages between the brand and its associated products leading to brand dilution. As such, only if the prospect of brand dilution is weak, a monopolist increases its product line length. In a duopoly, however, brand dilution softens competition and can increase the number of products offered by competing firms. In choosing between product and brand advertising, unlike a monopolist, the duopolists prefer product advertising; but increase the emphasis placed on brand advertising when consumers’ attention span decreases.

Keywords: multiproduct competition, Informative Advertising, Product Line, Behavioral Economics, Game Theory.
1. INTRODUCTION

In many product categories, firms offer a variety of products to meet the diverse needs of consumers. For example, Kellogg’s offers 23 different varieties of cereals while General Mills markets over 10 varieties of cereals. Similarly, Häagen-Dazs produces 29 flavors of ice cream and Ben & Jerry’s sells 39 varieties of its original ice cream. We also see evidence of multiproduct competition in product categories such as toothpaste (Crest versus Colgate) and laundry detergent (Proctor & Gamble detergents versus Unilever detergents). An interesting aspect of these horizontally differentiated markets is that the competition among products is often nonlocal, as consumers can purchase any of the competing products. Moreover, though firms offer a wide variety of products, some consumers’ preferred varieties may still be unavailable. In some instances, even though a consumer’s preferred products are available, she may not be informed about them. Typically, these multiproduct firms use product and/or brand advertising to inform consumers about their product lines. A critical marketing challenge facing a multiproduct firm is: How can it advertise multiple products without cannibalizing its own product line but tapping new market segments and gaining sales from its competitor?

Consumers, however, are exposed to thousands of advertisements in a day and that they manage this information overload by randomly paying attention to only a small fraction of the advertisements (Lanham 2006, Petrecca 2006). New technologies are also making it easier for consumers to zap advertisements (Siddartha and Chattopadhyay 1998). Thus, contrary to a standard assumption in models of informative advertising, consumers are unlikely to process all the advertisements of a firm. This raises the question, how does consumers’ limited attention span affect a firm’s product line length? On one hand, the possibility of congesting consumers’ limited attention span may reduce a firm’s incentive to offer multiple products. On the other hand, a firm may offer more products with the intent of breaking through the clutter and better serving its diverse customers. To gain insight into this question, we build on Chen and Riordan (2007) and Amaldoss and He (2010), and propose a model of multiproduct competition and informative advertising, where consumers pay attention to only a few product advertisements. Our analysis shows that, when consumers’ attention span is limited, the number of products offered by a monopolist will not exceed the cognitive limitation. To appreciate the intuition for this result, note that adding a new product to a firm’s product line reduces the probability of consumers paying attention to
the advertisements of any of its existing products because of consumers’ limited attention span. As a monopolist internalizes the entire negative externality, it reduces its incentive to offer too many products. In the presence of competition, however, the number of products offered exceeds the attention span of consumers and hence we observe advertising clutter. We obtain this result because when a duopolist adds a product to its productline, it reduces the probability of consumers paying attention to each of its products as well as the competing firm’s products. In a duopoly, as an individual firm does not fully internalize the negative externality induced by adding another product to its portfolio, firms offer too many products. Sometimes, instead of using product advertisements to separately promote each of its products, a firm advertises all its products under a common brand umbrella. We next examine such a form of informative brand advertising.

The literature on brand advertising and memory points to the fact that if too many products are advertised under a brand umbrella, the linkages between the brand and its numerous products become weaker (Raaijmakers and Shiffrin 1981, Keller 1991 and 1998). Consequently, a firm needs to carefully weigh the advantage of introducing a new product to better serve the needs of its diverse consumers against the prospect of brand dilution. This raises the question, how does the potential for brand dilution affect a firm’s productline length? As brand dilution reduces the effectiveness of advertising, one might expect a firm’s productline length to increase with the reach of brand advertising only if the elasticity of brand dilution is low. This intuition, although valid for a monopoly, is not applicable to a duopoly. The optimal number of products offered by duopolists increases with the reach of brand advertising, if the elasticity of brand dilution is above a threshold. We obtain this result because, although brand dilution has a negative direct effect by reducing the effectiveness of advertising, it exerts a positive strategic (indirect) effect by attenuating the cannibalization within a firm’s own productline as well as by softening price competition from the competing firm’s products.

While the preceding discussion focused on a firm using either product advertising or brand advertising to inform consumers about its products, in reality firms have the option to engage in product advertising as well as brand advertising. In such a scenario, how much emphasis should a firm place on product advertising vis-à-vis brand advertising? Brand advertising helps the monopolist to better handle consumers’ limited attention span by chunking information into a composite brand advertisement. Thus, despite the prospect
of dilution, brand advertising can help the monopolist to offer more products and better address the needs of its diverse consumers. Our analysis shows that, unlike a monopolist, a duopolist uses both product and brand advertising. Recall that a duopolist does not fully internalize the negative externality induced by its additional product, and hence a duopolist is motivated to offer more products and use product advertising. However, if a duopolist only engages in product advertising, it will aggravate the information overload; brand advertising helps to alleviate this problem and better cater to the needs of its diverse customers. Further analysis clarifies that a duopolist’s optimal level of brand advertising increases when consumers’ attention span becomes more limiting.

Related Literature. Although competition among multiproduct firms is commonly observed in many product categories, theoretical analysis of the phenomenon is limited. The early work on multiproduct competition aggregates a set of consumers with different tastes into a representative consumer who evinces a preference for variety (Spence 1976 and Dixit and Stiglitz 1977). Using this demand formulation, it examines competition among single-product firms. Later work turned attention to competition among multiproduct firms. For example, using an aggregate demand formulation Brander and Eaton (1984) examine whether a multiproduct firm should offer more substitutable products (segmented structure) or more differentiated products (interlaced structure). They find that a segmented structure is more advantageous. To facilitate this analysis, they make the simplifying assumption that the product line length is fixed. In contrast to Brander and Eaton (1984), we investigate how cognitive limitations affect a firm’s product line length.

Our work builds on the theoretical literature on multiproduct competition in a horizontally differentiated market. Traditionally, researchers either used Hotelling’s linear city model or Salop’s circle model to study product competition in a horizontally differentiated market (e.g., Villas-Boas 2004). The linear city model can accommodate either two products from a monopolist or a single product from each duopolist. Although the circle model allows competing firms to offer many more products, competition is local in that a small change in a firm’s price only affects its neighboring firms and not the other firms in the market. The spokes model of Chen and Riordan (2007) overcomes this shortcoming. As such, one can use the spokes model to capture the notion of cannibalization within a firm’s own product line as well as allow competition between the product lines of the competing firms. In a working paper, Caminal and Granero (2010) use the spokes framework to analyze a multiproduct
oligopoly. They show that there will be insufficient variety in the oligopoly compared to the social optimum. Unlike Caminal and Granero (2010), we investigate how cognitive limitations affect a firm’s productline length.

In an interesting piece of work, Kuksov and Villas-Boas (2010) offer an explanation for why might consumers prefer smaller choice sets over large choice sets. They show that if too many or too few alternatives are offered, consumers might not search because of cost considerations. Thus, they focus on search costs, not on limited attention span or brand dilution. Furthermore, they do not analyze how inter-firm competition interacts with cognitive limitations to influence productline length.

Our paper is related to the literature on informative advertising. Using a circle model, Soberman (2004) shows that price decreases with informative advertising when product differentiation is modest but increases with advertising when product differentiation is high. By incorporating informative advertising into the spokes model, Amaldoss and He (2010) examine how prices of single-product firms are affected by the reach of advertising and improvements in advertising technology. Unlike our work, this body of literature studies the behavior of single-product firms.

Our work is also related to the emerging literature on how consumers’ cognitive limitations affect the strategic behavior of firms. For instance, focusing on the strategic interaction between senders in the presence of information overload, Van Zandt (2004) examines mechanisms for allocating the attention of receivers and shows that such mechanisms increase the cost of sending messages, thereby shifting the task of screening messages from the receivers to the senders. In contrast to our work, this paper does not investigate a firm’s productline strategy, or how information overload and brand dilution affect informative advertising.

**Contribution.** In the tradition of behavioral economics literature we incorporate some well-established behavioral findings into a normative model, and examine its strategic implications (see Rabin 2002 and Ho, Lim and Camerer 2006 for overviews). Specifically, we incorporate the notion of limited attention span and brand dilution in a model of informative advertising. Next, we propose a stylized model of multiproduct competition where consumers are distributed on a spokes network and competition is non-localized in that each product is in direct competition with every other product in the market. Furthermore, our analysis clarifies how the effect of limited attention span and brand dilution on a firm’s productline changes with the underlying market structure. We also show how the relative
emphasis placed on product advertising and brand advertising varies with market structure. To the best of our knowledge, these issues have not been investigated in any of the prior literature on multiproduct competition.

The rest of the paper is organized as follows. Section 2 introduces a model of product advertising and multiproduct competition, and analyzes its implications for a monopoly and duopoly. In Section 3, we study the effect of brand advertising on a firm’s productline length. In Section 4, we analyze how firms make the trade-off between product advertising and brand advertising. Finally, Section 5 concludes by summarizing the findings and outlining some directions for further research.

2. A MODEL OF PRODUCT ADVERTISING

In this section, we propose a parsimonious model of multiproduct competition and informative advertising, where a firm separately advertises each of its products. Using this model of product advertising, we investigate how consumers’ attention span and competition affect a firm’s productline length. Later we consider informative advertising where a firm advertises all its products under a brand umbrella (see Section 3). While such brand advertising informs consumers about all the products within a brand umbrella, it may dilute the recall of any given individual product. Using a model of brand advertising, we explore the effect of brand dilution on a firm’s productline length. Finally, we examine the trade-off a firm makes between product advertising and brand advertising (see Section 4). Next we introduce the spokes model and then incorporate informative advertising and multiproduct competition into the model to facilitate our theoretical analysis.

Spokes Model. Consider a product market where consumers seek $N$ different varieties (e.g., flavors). We model this market as a spokes network on a plane. Consumers preferring a variety are represented by a spoke of length $\frac{1}{2}$. Each consumer on a spoke is identified by the distance, $x \in [0, \frac{1}{2}]$, at which she is located from the origin of the spoke. Note that at the origin of a spoke $x = 0$, whereas at the center of the spokes network $x = \frac{1}{2}$. We assume that consumers are distributed uniformly on the spokes network, and the total mass of consumers on the $N$ spokes is $\frac{N}{2}$. To help appreciate the model structure, we illustrate in Figure 1 a multiproduct monopoly where consumers seek nine varieties ($N = 9$) and the monopolist offers five products ($n = 5$).

As in Chen and Riordan (2007), we assume that each consumer considers at most two
products. In addition to considering the product located on the spoke in which she lies (local product), each consumer considers one of the products located in any of the other spokes (nonlocal product). We assume that the nonlocal product preferred by a consumer is exogenously fixed à priori. Furthermore, all nonlocal products are equally likely to be the second preferred product. The assumption that consumers consider at most two products helps to obtain a pure-strategy equilibrium and also reflects the notion that consumers often have a small consideration set (Nedungadi 1990, Hauser and Wernerfelt 1990). Each consumer purchases at most one unit of the product. The base value of all the product varieties is the same and it is denoted by $v$. If the consumer located at $x$ on spoke $l$ is aware of her local product $l$ and chooses to purchase the product, she will derive the following indirect utility:

$$U(x, p_l) = v - tx - p_l,$$

where $t$ is her sensitivity to product characteristics, and $p_l$ is the price of the product. But if the consumer wants to purchase any other product of which she is informed, namely variety $m$ on spoke $m$ such that $m \neq l$, then the indirect utility from this nonlocal product will be:

$$U(x, p_m) = v - t(1 - x) - p_m.$$


Note that the consumer located on spoke \( l \) is \( \frac{1}{2} - x \) units of distance away from the center of the spokes network, and product \( m \) is \( \frac{1}{2} \) unit of distance further away. Thus the total distance between the consumer and nonlocal product \( m \) is \( \frac{1}{2} - x + \frac{1}{2} = 1 - x \). The consumer will purchase local product \( l \) if \( U(x, p_l) > U(x, p_m) \). Hence the marginal consumer, who is indifferent between the two products, is located at a distance \( \frac{1}{2} + \frac{p_m - p_l}{2x} \) from product \( l \). The demand for local product \( l \) is \( \min\{\frac{1}{2} + \frac{p_m - p_l}{2x}, 1\} \), implying that \( \frac{|p_m - p_l|}{x} \leq 1 \). Next, we incorporate informative advertising and limited attention span in the spokes model.

**Informative Advertising.** On average, a consumer is exposed to 3,000 to 5,000 advertisements a day, and this information load can exceed consumers’ limited attention span (Petrecca 2006). Consumers handle this information overload by paying attention to only a small set of the advertisements (Lanham 2006, see also Miller 1956, Simon 1957, Neely 1977). There are even devices to help consumers zap advertisements (Siddarth and Chattopadhyay 1998). Hence, counter to the standard assumption in models of advertising, consumers may not attend to all the advertisements of a firm even if they had an opportunity to see them (e.g., Butters 1977, Grossman and Shapiro 1984, Robert and Stahl 1993).

Van Zandt (2004) proposed a formulation of limited attention span, wherein consumers attend to only a limited number of advertisements in a window of time (see also Anderson and de Palma 2006). If the number of products advertised is less than consumers’ attention span, consumers pay attention to all the advertisements. When the number of advertisements exceeds their attention span, it is no longer guaranteed that consumers pay attention to any given advertisement of a firm. In such instances, consumers randomly attend to only a small set of advertisements from the firm. More generally, as consumers’ attention span increases the effective reach of a product’s advertisement increases among consumers in the market. However, when the number of products advertised increases, the effective reach of a given product’s advertisement reduces. This aggregate formulation of limited attention span abstracts away from some of the finer details of how consumers process advertising message (Bettman 1979). For example, this framework precludes screening of advertisements by consumers based on partial processing of advertising messages. Yet the formulation is helpful in analyzing how limited attention span affects aggregate demand, and in turn the strategic behavior of firms.

Following Van Zandt (2004), we assume that consumers’ attention span is limited to \( \tau > 2 \) advertisements in a window of time. Consumers process all the information in the \( \tau \)
advertisements that they attend to. Furthermore, the firm advertises each product such that, on average, consumers have a chance to see each product advertisement with probability $\omega$, where $0 \leq \omega \leq 1$. If $n > \tau$, the firm recognizes that the probability of consumers being informed about a specific product advertisement is in actuality lower than $\omega$ because of consumers’ limited attention span. Denote the effective reach of a firm’s advertising by $\Omega = \lambda(n, \tau) \omega$. The function $\lambda(n, \tau)$ reduces a given level of product advertising $\omega$ to its effective reach. Consistent with Van Zandt, we assume that $\lambda(n, \tau) = 1$ when $n \leq \tau$. But if when $n > \tau$, we have $0 < \lambda(n, \tau) < 1$, $\frac{\partial \lambda(\cdot)}{\partial \tau} > 0$ and $\frac{\partial \lambda(\cdot)}{\partial n} < 0$, implying that the effective advertising reach increases with consumers’ attention span $\tau$ but decreases with productline length $n$. Denote the elasticity of the $\lambda(n, \tau)$ with respect to productline length by $\eta_\lambda \equiv \frac{\partial \lambda(n, \tau)/\lambda(\cdot)}{\partial n/n}$. To facilitate further analysis, we make the conservative assumption that $\eta_\lambda = -\kappa$, where $\kappa \geq 1$ suggesting that a one percent increase in a firm’s productline length will reduce the effective reach of a given product’s advertisement by at least one percent.

As in prior literature on informative advertising, consumers do not engage in any costly information search. This assumption does not imply that consumers are unaware of the underlying market structure; it just suggests that consumers rely on advertising to learn about prices and product availability. Using this formulation of product advertising, we advance to study how limited attention span affects a firm’s aggregate demand and thereby its product-line decision.

**Multiproduct Competition.** Here we analyze a multiproduct monopoly and a multiproduct duopoly to understand how the nature of competition tempers a firm’s product-line decision.

**Multiproduct Monopoly.** Consider a monopolist who offers $n$ different product varieties, where $2 < \tau < n \leq N$. Recall that Figure 1 illustrates a multiproduct monopoly where consumers prefer $N = 9$ different varieties but the monopolist offers only $n = 5$ products. As a consumer’s consideration set includes only two varieties, the number of possible combinations of varieties that can be preferred by consumers is:

$$\frac{N(N-1)}{2}.$$  \hfill (3)

The possible combinations of product varieties that the monopolist can satisfy is:

$$\frac{n(n-1)}{2}.$$  \hfill (4)

Hence, the fraction of consumers who will have access to the two products in their consider-
ation set and are informed about both of them is given by:

\[
\frac{n(n-1)}{N(N-1)} \Omega^2.
\] (5)

In addition to the above segment of consumers, the demand for the monopolist’s products comes from two other segments. In the case of some consumers, though the monopolist supplies both their preferred products, they are informed about only one of the two products. This group of consumers constitute the second market segment and its size is given by:

\[
2\Omega (1 - \Omega) \frac{n(n-1)}{N(N-1)}.
\] (6)

The third segment of consumers includes people who have only one preferred product available and are informed of the product. The size of this segment is:

\[
2\Omega \frac{n(N-n)}{N(N-1)}.
\] (7)

Let \( A(n, \omega) \) denote the total cost of achieving an effective reach of \( \Omega \) for each of the monopolist’s products. For ease of exposition, we assume that the advertising cost function is linear in advertising reach and products offered. That is, \( A(n, \omega) = sn\omega, A(n,0) = 0, A_n = s\omega > 0, A_{nn} = 0, A_\omega = sn > 0, A_{n\omega} = 0, \) and \( A_{\omega\omega} = s \). A linear cost function helps us to show how demand side factors, such as limited attention span, affect the equilibrium outcome. In the Appendix, we have also considered a convex cost function to illustrate the robustness of our results. We further assume that the fixed cost of introducing a new product is \( F > 0 \) and that the constant marginal cost is \( c = 0 \). As the total fixed cost of introducing \( n \) products is \( nF \) and the total mass of consumers is \( \frac{N}{2} \), the fixed cost per consumer is \( \frac{2nF}{N} \).

Upon aggregating the demand from the three segments of consumers given in equations (5), (6) and (7), we can compute the monopolist’s profits. The monopolist’s profits are given by:

\[
\pi_M = \left[ \frac{n(n-1)}{N(N-1)} \Omega^2 + 2\Omega (1 - \Omega) \frac{n(n-1)}{N(N-1)} + 2\Omega \frac{n(N-n)}{N(N-1)} \right] p_M - \frac{2nF}{N} - A(n, \omega).
\] (8)

where \( p_M \) is the monopolist’s price.

We assume that \( F > F_0 \equiv \frac{1}{2} \left[ (v - \frac{1}{2}) \omega - A(n, \omega) \right] \), so that the spokes model is not reduced to \( N \) independent spokes. Thus our formulation allows for cannibalization within a firm’s product line. As in Chen and Riordan (2007), we also assume that the base valuation of products is high enough that consumers can gain a surplus on purchasing any product,
implying that \( v > p_t + t \). This again permits cannibalization within a firm’s productline. In our model, the monopolist first decides on the number of products to offer, and then sets their prices. This decision structure reflects the notion that it is easier for a firm to change the prices of its products and that a firm treats other marketing mix decisions as fixed while setting prices. To compute the equilibrium of the game, we first solve the pricing subgame and then determine the equilibrium number of products.

On examining the pure-strategy equilibrium of the game, we find that the number of products offered by a monopolist will not exceed consumers’ attention span \( \tau \) if \( \kappa \geq 1 \). We prove this claim in the Appendix. To see the intuition behind this finding about product advertising, first note that a monopolist’s price remains \( v - t \) irrespective of the number of products it supplies.\(^1\) So our result is a consequence of the effect of consumers’ attention span on the demand for the monopolist’s products. Now consider the case where the monopolist offers \( \tau + 1 \) products. As the additional product exceeds consumers’ attention span, the probability of consumers being informed about each of the monopolist’s products reduces. This externality dampens the demand for all its products. As the entire negative externality needs to be internalized by the monopolist, it is not motivated to supply more than \( \tau \) products. Furthermore, offering more products increases the marginal cost of advertising as \( A_n > 0 \).

More specifically, we find that \( \frac{\partial \pi_M}{\partial n} < 0 \) if \( \kappa \geq 1 \) and \( n = \tau \). This implies that if \( \lambda(n, \tau) \) is reasonably elastic with respect to productline length, it is not profitable for the monopolist to offer more than \( \tau \) products. So the monopolist only supplies \( 0 < n \leq \tau \) products. Of course, if the fixed cost of introducing a product is sufficiently high, then the monopolist will not supply any product. Next we advance to compare the monopoly result against the implications of consumers’ limited attention span for a duopolist’s productline.

**Multiproduct Duopoly.** To understand how competition can affect equilibrium behavior, consider a duopoly where firms \( i \) and \( j \) supply \( k_i \) and \( k_j \) products, respectively. Figure 2 shows a multiproduct duopoly where consumers prefer \( N = 8 \) different varieties although each duopolist offers \( k_i = k_j = 3 \) products. As in the previous model, each firm simultaneously chooses the number of products to offer in the first stage of the game. In the second stage,

\(^1\)It is not profitable for the monopolist to offer all the \( N \) varieties at the price \( v - \frac{t}{2} \), as the resulting revenue does not cover the fixed cost of introducing the \( N \) products and advertising them. Specifically, because \( F > F_0 \), we have \( \pi_M = \omega(v - \frac{t}{2}) - 2F - A(n, \omega) < 0 \) (see equation 8).
Fig. 2. A multiproduct duopoly where consumers seek eight varieties \((N = 8)\) and each duopolist offers three products \((k_i = k_j = 3)\)

Notes: Firm \(i\) occupies the spokes in solid lines, while firm \(j\) occupies the spokes in dotted lines. The two spokes in solid gray lines are not served.

Each firm sets a price for its product, namely \(p_i\) and \(p_j\).\(^2\)

To facilitate exposition, we denote the effective reach of firm \(i\)'s advertising by \(\Omega_i \equiv \lambda(k_i, k_j, \tau) \omega_i\), where \(\lambda(k_i, k_j, \tau)\) scales down the advertising level \(\omega_i\) to its effective reach. Consistent with Van Zandt, we assume that \(\lambda(k_i, k_j, \tau) = 1\) when \(k_i + k_j \leq \tau\), but \(0 < \lambda(k_i, k_j, \tau) < 1\) when \(k_i + k_j > \tau\). As the effective advertising reach of a firm’s product advertising reduces with its product line length but increases with consumers’ attention, we have \(\frac{\partial \lambda(\cdot)}{\partial \tau} > 0\) and \(\frac{\partial \lambda(\cdot)}{\partial k_i} < 0\) if \(k_i + k_j > \tau\).

Focusing attention on firm \(i\), we note that the demand for its products can emanate from five different segments of consumers. One segment comprises of consumers whose two preferred products are supplied by firm \(i\) and members of this segment are also informed about the products. The demand from this segment for firm \(i\)'s products is given by:

\[
\Omega_i^2 \frac{k_i (k_i - 1)}{N (N - 1)}.
\]

The second group of consumers also have both their preferred products supplied by firm \(i\),

\(^2\)In this game, as the firms are symmetric, \(k_i = k_j = \frac{N}{2}\) in equilibrium. Further, as all the products are symmetric, \(p_i = p_j\) in equilibrium.
but they are aware of only one of them. The demand from this segment for firm $i$’s product is:

$$2\Omega_i (1 - \Omega_i) \frac{k_i (k_i - 1)}{N (N - 1)}. \quad (10)$$

The third group of consumers prefer one product from each of the two competing firms. Although both the products are available, they are aware of only the product supplied by firm $i$, and hence the demand from this group is given by:

$$2\Omega_i (1 - \Omega_j) \frac{k_i k_j}{N (N - 1)}. \quad (11)$$

In the case of the fourth group of consumers, only one of their two preferred products is available and it is supplied by firm $i$. The demand from this group is:

$$\frac{2\Omega_i k_i}{N} \left( 1 - \frac{k_i + k_j - 1}{N - 1} \right). \quad (12)$$

The fifth group of consumers prefers one product from each of the two firms and they are aware of both the products. The demand for firm $i$’s products from this segment is:

$$\frac{2\Omega_i \Omega_j}{N (N - 1)} \frac{k_i k_j}{2t} \left( 1 + \frac{p_j - p_i}{2t} \right). \quad (13)$$

On aggregating the demand from all the five groups of consumers, and computing the profits of firm $i$, we have:

$$\pi_i = \frac{2k_i F}{N} - A(k_i, \omega_i),$$

where $F > 0$ is the fixed cost of introducing a product and $A(k_i, \omega_i)$ is the total cost of achieving an effective advertising reach of $\Omega_i$ for each of the $k_i$ products. We again assume that the advertising cost function is linear in advertising reach and number of products offered. That is, $A(k_i, \omega_i) = sk_i \omega_i$, $A_{k_i} = s \omega_i > 0$, $A_{k_i k_i} = 0$, $A_{\omega_i} = s k_i > 0$, $A_{\omega_i \omega_i} = 0$, and $A_{k_i \omega_i} = s$. As noted earlier, we use a linear cost function so that we can understand how limited attention span affects a firm’s productline length. In the Appendix, we show that our results are not due to the linear cost function, as a convex cost function also yields similar results. Like before, we assume that consumers can gain surplus on buying any product of their choice and that $F > F_0$. Thus, in our model there is scope for competition between
the two firms as well as within a firm’s own productline. Below we discuss our key findings and relegate the proofs to the Appendix.

On analyzing the pure-strategy equilibrium, we find that the number of products offered by the two duopolists will exceed consumer’s attention span, $\tau$, if the fixed cost per customer is below the threshold $\frac{4(N-2)}{3(N-1)}$. This result, which is very different from the monopoly finding, is established in the Appendix. Note that the threshold for fixed cost per customer will be more than $t$, if the market can accommodate at least two products from each firm. To keep things in perspective, further note that in our model a consumer incurs a transportation cost of at most $t$. Thus, the threshold will be easily satisfied if it is profitable for the multiproduct duopolists to compete in the market.

Now to appreciate the key force behind the duopoly result, consider the case where firm $i$ increases its product offering such that the total number of products in the market exceeds $\tau$. By doing so, firm $i$ may be able to offer the preferred products of some more consumers. However, when the number of advertisements exceeds consumers’ attention span, the probability that consumers are aware of the products of firm $i$ reduces. Furthermore, when firm $i$ congests consumer’s attention span, it has a negative impact on the awareness level of firm $j$’s products as well. Thus, as an individual firm does not fully internalize the negative externality induced by its decision to offer an additional product, we may observe product proliferation and advertising clutter in a duopoly.

In sum, on studying how limited attention affects the productline length of a monopolist and a duopolist, we note that in a duopoly market the number of products could exceed consumer’s attention span and in turn induce advertising clutter. On the other hand, in a monopoly the number of products available will not exceed consumers’ attention. For the monopoly result to prevail, we only need $\kappa \geq 1$, implying a one percent increase in productline length congests consumers’ attention span and scales down the effective reach by at least one percent. In essence, the force behind our findings is that the monopolist fully internalizes the cost of congesting consumers’ attention, whereas a duopolist does not do so. Consequently, the duopolist is motivated to offer more products and induce advertising clutter. As we show in the proof, these results hold for a general advertising cost function whether it is linear, convex, or even concave. The following proposition summarizes our findings on product advertising:

**Proposition 1** Under product advertising, the number of products offered by a monopolist
will not exceed consumers’ attention span, but the total number of products available in duopoly can exceed consumers’ attention span and in turn cause advertising clutter.

Discussion. Our analysis of product advertising shows how limited attention span comes to restrict the product line length of a monopolist. We established this result using a general formulation of limited attention span, namely $\lambda(n, \tau)$, and imposing a conservative condition on its elasticity. This may make one wonder whether our results can be obtained using a micromodel of limited attention span. To save space, here we briefly summarize the results of a micromodel of limited attention span, relegating details of the model and its analysis to the Appendix. Although the micromodel is rich in finer details, it is less tractable. Yet the model lends itself for numerical analysis and yields results consistent with Proposition 1: It is not profitable for a monopolist to produce more than $\tau$ products. For example, consider the case where $\tau = 4$, $N = 10$, $v = 30$, $t = 1$, $F = 0.1$, $\omega = 0.2$, and $A(n, \omega) = 5n\omega$. Now if the monopolist offers $n = 4$ products, it earns $\pi_M = 0.440$. However, if the monopolist increases its product line length to $n = 5$ and, thereby, clutters consumers’ attention span, then its profits decline to $\pi_M = 0.405$. Thus, the monopolist makes more profits by offering products no greater than $\tau$. This finding, in turn, could prompt another question: If consumers’ attention span were not constrained, how will it impact the monopolist’s product line length. One might naturally expect the monopolist to offer more products if attention span is not limited. Using a baseline model, where consumers’ attention span is not limited, we show that this intuition is valid. To illustrate, consider the case where consumers’ attention span is not limited, $N = 10$, $v = 30$, $t = 1$, $F = 0.1$, $\omega = 0.2$, and $A(n, \omega) = 5n\omega$. Now the monopolist will raise its product line length to $n = 6$ and earn $\pi_M = 0.453$. Further details on this baseline model of product advertising can be seen in the Appendix. Next, we advance to examine how cognitive limitation affects another form of advertising that firms use to inform their consumers.

3. A MODEL OF BRAND ADVERTISING

We first outline a model of brand advertising and then discuss the trade-off that firms make between brand advertising and product advertising.

Brand Advertising. Consider a consumer who thinks of the brand Swatch when she contemplates watches. Similarly, when she thinks of cereals, thoughts about Kellogg’s brand come to her. Which specific product will come to this consumer’s mind when she thinks of
Swatch or Kellogg’s? The answer to this question depends on the strength of associations between the brand and its numerous products. In fact, if too many products are offered under a brand name, the linkages between a brand and its products become weak. We refer to this reduction in the accessibility of a brand’s individual products as *brand dilution.* There is a need to balance this potential for brand dilution against a firm’s desire to add another new product under its brand umbrella.

In theory, brand advertising can play different roles. A firm could use brand advertising to signal the unobserved quality of the products under its brand umbrella (e.g., Milgrom and Roberts 1986), or add image to its products and thereby directly augment the utility derived from its product portfolio (e.g., Becker and Murphy 1993). In contrast to these persuasive effects of brand advertising but in keeping with the informative advertising models of Grossman and Shapiro (1984), we focus on advertising that explicitly communicates about a brand’s existence and some features. For example, Bajaj Auto, the leading manufacturer of two-wheelers in India, traditionally promoted all its products using an umbrella advertising campaign. We see Jaguar informing consumers about its portfolio of products using a composite print advertisement. Similarly, Toyota uses a composite print advertisement to inform consumers about the fuel efficient vehicles under the Lexus umbrella. Thus, although brand advertising can play both persuasive and informative roles, we analyze informative brand advertising to understand its theoretical implications.

To fix ideas, consider the simple case where a firm uses a composite advertisement to inform consumers about all its products, and let \( \phi \) be the probability of consumers having an opportunity to see the firm’s brand advertising. We capture the notion of brand dilution by letting the probability that consumers can recall the advertised information about a product at the time of purchase be \( \frac{\phi}{f(n)} \), where \( n \) is the number of products under a firm’s brand umbrella. We assume that \( f(1) = 1 \), implying that recalling product information at the time of purchase is not an issue if a firm advertises only one product under a brand umbrella. But, if a firm advertises more products under a brand umbrella, consumers face greater difficulty in recalling information about any given product under the brand umbrella,\footnote{There is an extensive body of literature suggesting that consumers organize their memory as a network of nodes connected by links (e.g., Raaijmakers and Shiffrin 1981, Cowley and Mitchell 2003) This spreading-activation model has been helpful in studying brand associations (e.g., Keller 1991, 1998). There is also related literature on trademark infringement that studies how counterfeits and related products may lead to trademark dilution (see Pullig, Simmons and Netemeyer 2006).}
implying $\frac{\partial f(n)}{\partial m} > 0$. Hence, the probability of a consumer being informed about a product at the time of purchase can be lower than the reach of brand advertising. Using this formulation, we next examine how the likelihood of brand dilution may influence the number of products offered by a firm.

**Multiproduct Monopoly.** To facilitate exposition, let $\Phi \equiv \frac{\phi}{f(n)}$. Further, denote the elasticity of brand dilution with respect to the number of products offered by a monopolist by $\eta_f \equiv \frac{\partial f(n)/f(n)}{\partial m/n}$. The elasticity of brand dilution gives the percentage increase in difficulty that consumers face while recalling the products offered by a firm, if the firm increases its productline length by one percent. The increased difficulty leads to errors in retrieving information from memory and thereby decreases the probability of being informed about the products under the brand umbrella at the time of purchase. Now the fraction of consumers who will have access to the two products in their consideration set and can recall information about both of them at the time of purchase is given by:

$$\Phi^2 \frac{n(n - 1)}{N(N - 1)}. \quad (15)$$

The fraction of consumers whose two preferred products are offered by the monopolist but can recall information about only one of them is:

$$2\Phi(1 - \Phi) \frac{n(n - 1)}{N(N - 1)}. \quad (16)$$

The monopolist may not offer all the varieties preferred by consumers, as $n \leq N$. Consequently the monopolist may supply only one of the two products preferred by some consumers. The fraction of the market, that has any one of its two preferred products available and is informed about it at the time of purchase, is given by:

$$2\Phi \frac{n(N - n)}{N(N - 1)}. \quad (17)$$

Like in the previous section on product advertising, we assume that the fixed cost of introducing a new product is high enough to ensure competition within a firm’s product and that the constant marginal cost is $c = 0$. The total fixed cost of introducing $n$ products is $nF$ and the fixed cost per consumer is $\frac{2nF}{N}$. Now, the monopolist’s profits are given by:

$$\pi_M \left( \Phi^2 \frac{n(n - 1)}{N(N - 1)} \Phi^2 + 2\Phi(1 - \Phi) \frac{n(n - 1)}{N(N - 1)} + 2\Phi n (N - n) \right) \frac{p_M}{N} \frac{2nF}{N} - A(\phi), \quad (18)$$

where $A(\phi)$ is the cost of providing $\phi$ fraction of the consumers an opportunity to see the monopolist’s product advertisement. We assume that $\frac{\partial A(\phi)}{\partial \phi} > 0$ and $\frac{\partial A}{\partial \phi} = 0$, implying that
the advertising cost function is linear. As the monopolist uses a composite advertisement to inform consumers about all its products, the cost of advertising reach need not depend on the number of products advertised. Hence, we make the simplifying assumption that \( \frac{\partial A_\phi}{\partial n} = 0 \). Furthermore, we initially treat advertising reach as exogenous to the model so that it is easier to appreciate the key results. The monopolist first determines the optimal number of product varieties, and then sets its prices.

On examining the pure-strategy equilibrium of the game, we find the monopolist’s product line length increases with advertising reach when \( \eta_f < 1 \). Note that when \( \eta_f < 1 \), increasing the products under a brand umbrella leads to a less than proportionate reduction in consumer’s ability to recall information about the products under the brand umbrella. Thus only when brand dilution is weak the monopolist will increase the number of products under its brand umbrella. To understand this result, note that if the monopolist adds another product to its portfolio, it increases its ability to offer the preferred products of some more consumers. At the same time, the additional product makes it more difficult for consumers to recall information about any given product at the time of purchase. Consequently, the monopolist needs to balance two countervailing forces: improved market coverage and brand dilution. The finding suggests that a monopolist will offer an additional product only if the elasticity of brand dilution is below a threshold. If the elasticity exceeds the specified threshold, then a monopolist will offer fewer products as advertising reach increases. Next, we investigate how brand dilution affects the product line of competing firms.

**Multiproduct Duopoly.** For ease of exposition, let \( \Phi_i \equiv \frac{\phi_i}{f(k_i)} \). Also denote the elasticity of brand dilution with respect to product line length by \( \tilde{\eta}_f \equiv \frac{\partial f(k_i)/f(k_i)}{\partial k_i/k_i} \). Here again the elasticity of brand dilution gives the percentage increase in difficulty that consumers face in recalling the products offered by a firm, if the firm increases its product line length by one percent. As in the case of product advertising, the demand for firm \( i \)'s products emanates from five segments of consumers, but the demand expressions are different. Firm \( i \) could cater to consumers whose two preferred brands are produced by it, and who can recall information about both the products at the time of purchase. The size of this segment is given by:

\[
\frac{\Phi_i^2 k_i (k_i - 1)}{N (N - 1)}.
\]

Firm \( i \) could also sell its product to consumers whose two preferred products are made by it although consumers can recall information about only one of the products at the time of
purchase. The demand from this second segment of consumers is:

\[ 2\Phi_i (1 - \Phi_i) \frac{k_i (k_i - 1)}{N (N - 1)}. \]  

(20)

Consumers, who have only one of firm \( i \)'s products in their consideration set and can recall information about it, could also contribute to the firm’s sales. For some of these consumers, their other preferred variety may be offered by firm \( j \), and the demand from this third segment of consumers is given by:

\[ 2\Phi_i (1 - \Phi_j) \frac{k_i k_j}{N (N - 1)}. \]  

(21)

For certain other consumers, their other preferred variety may not be offered by any firm. The demand for firm \( i \)'s product from this fourth segment of consumers is:

\[ \frac{2\Phi_i k_i}{N} \left( 1 - \frac{k_i + k_j - 1}{N - 1} \right). \]  

(22)

Finally, the consideration set of some consumers includes one product from each of the two firms and they can recall information about both the products. The demand for firm \( i \)'s product from this fifth segment of consumers depends on the prices of the two competing firms, and it is given by:

\[ \frac{2\Phi_i \Phi_j k_i k_j}{N (N - 1)} \left( 1 + \frac{p_j - p_i}{2t} \right). \]  

(23)

Now the profits of firm \( i \) can be expressed as follows:

\[ \pi_i = \left[ \frac{\Phi_i^2 k_i (k_i - 1)}{N(N-1)} + 2\Phi_i (1 - \Phi_i) \frac{k_i (k_i - 1)}{N(N-1)} + 2\Phi_i (1 - \Phi_j) \frac{k_i k_j}{N(N-1)} \right] p_i - \frac{2k_i F}{N} - A(\phi_i). \]  

(24)

The game proceeds in two stages with the firm choosing the number of products to offer in the first stage and setting its prices in the second stage. In equilibrium, we find the duopolist’s product productline length increases with the reach of brand advertising when \( \eta_f > 2 \). Note that \( \eta_f > 2 \) implies that the duopolists will increase their productline length if brand dilution is strong. Thus, the duopoly result is qualitatively different from the monopoly result discussed earlier. To see the intuition for this finding, notice that when consumers are more informed about the products offered by the two firms, there is likely to be a higher level of cannibalization within a firm’s productline as well as increased competition between the duopolist’s productlines. If the elasticity of brand dilution is above a critical threshold and a firm adds a product to its portfolio, consumers will find it demanding to recall information
about the products under its brand umbrella. The resulting poor retrieval of advertised information from memory makes it difficult for consumers to engage in product comparison, thereby reducing the cannibalization and competitive effects of advertising and encouraging each firm to increase the number of products it supplies. This is the reason why when the elasticity of brand dilution is above a threshold, the number of products offered by a duopolist increases. The following proposition summarizes how the effect of brand dilution can vary with market structure.

**Proposition 2** The monopolist’s optimal number of product varieties increases with advertising reach when \( \eta_f < 1 \), whereas the duopolist’s optimal number of product varieties increases with the reach of brand advertising when \( \tilde{\eta}_f > 2 \)

**Discussion.** An insight drawn from the above analysis is that, although brand dilution has a direct negative effect on the effectiveness of brand advertising, it has a positive strategic effect: It reduces cannibalization within a firm’s own product line and softens competition between the product lines of the competing firms. Consequently, if the level of brand dilution is above a threshold, namely \( \tilde{\eta}_f > 2 \), the duopolists will increase their product line length as the reach of brand advertising increases. This may lead one to ask the question: In the absence of brand dilution, what will be effect of advertising reach on a firm’s product line length? To answer this question, we analyzed a baseline model with no brand dilution. We find that in the absence of brand dilution the monopolist offers more products whereas a duopolist offers fewer products, as the reach of advertising increases. To save space, the analysis of this baseline model of brand advertising is presented in the Appendix. Thus far we have considered situations where a firm could use either product advertising or brand advertising to inform consumers about its products. In reality, however, firms have the option of using both product advertising and brand advertising to inform consumers about their products. This poses yet another question: How much emphasis a firm should place on product advertising compared to brand advertising. We address this issue in the following section.

**4. TRADE-OFF BETWEEN BRAND AND PRODUCT ADVERTISING**

To understand this trade-off, consider a situation where firms can engage in product and brand advertising. Specifically, the firm can advertise each of its products in a separate
product advertisement, and it also has the option of advertising its products multiple times in a composite brand advertisement. The key decision facing the firm is, what fraction of its advertising reach should come from brand advertising?

Multiproduct Monopoly. Consider a monopolist who offers \( n \) products. As before, we assume that consumers process only \( \tau > 2 \) advertisements in a window of time. Furthermore, let \( 0 \leq g(n, \tau) \leq 1 \) capture the notion of brand dilution when the monopolist can release its brand advertisement \( n \) times and consumers’ attention is limited to \( \tau \) advertisements.\(^4\)

Now, if \( 0 \leq \delta \leq 1 \) is the relative emphasis that the firm places on brand advertising and \( \omega \) is the fraction of consumers who have an opportunity to see the brand advertisement, the probability of consumers being informed about a given product through brand advertising is given by \( g(\tau, n)\delta\omega \). Further, as the emphasis given to product advertising is \((1 - \delta)\), the probability of consumers being informed about a product through product-specific advertising is \( \lambda(n, \tau)(1 - \delta)\omega \). Hence the probability of consumers being informed about a given product at the time of purchase either through the firm’s brand advertising or its product advertising is given by \( \Upsilon = g(\tau, n)\delta\omega + \lambda(n, \tau)(1 - \delta)\omega \). The resulting monopolist’s profit function can be written as (similar to equations 8 and 18):

\[
\pi_M = \left[ \frac{n(n-1)\Upsilon^2}{N(N-1)} + \frac{2\Upsilon(1 - \Upsilon)n(n-1)}{N(N-1)} + \frac{2\Upsilon n(N-n)}{N(N-1)} \right] p_M - \frac{2nF}{N} - A(n, \omega). \tag{25}
\]

To analyze the relative emphasis a monopolist places on brand advertising, we further assume that \( \lambda(n, \tau) \) is \( \frac{\tau}{n} \) if \( n > \tau \) but becomes 1 if \( n \leq \tau \). As originally noted by Van Zandt, this formulation has several desirable properties. Specifically, when \( n > \tau \) we have \( 0 < \lambda(n, \tau) < 1 \), but when \( n \leq \tau \) we have \( \lambda(n, \tau) = 1 \). Furthermore, \( \frac{\partial \lambda(\cdot)}{\partial \tau} > 0 \) and \( \frac{\partial \lambda(\cdot)}{\partial n} < 0 \) suggesting that the effective advertising reach increases with consumers’ attention span but decreases with productline length.

On analyzing the monopolist’s advertising strategy, we find that it is more profitable for a monopolist to use only brand advertising to promote its products. To see the intuition for this result, recall that, if a monopolist uses product advertising, the number of products offered by the firm will not exceed \( \tau \). Because of consumers’ limited attention span, adding

\(^4\)Recall that in the previous section, we modeled brand dilution by \( \frac{1}{f(n)} \). Now we allow for the possibility that a firm may release its brand advertisements \( n \) times, and this repetition could lead to some improvement in recalling information about the products under its brand umbrella. Furthermore, consumers’ attention span is limited. All these three effects are captured in the function \( g(n, \tau) \), which helps to appropriately scale down the effective reach of a firm’s brand advertising.
another product to its portfolio exerts a negative externality on the monopolist’s entire set of products. Specifically, it reduces the probability of consumers becoming aware of any of its products. Brand advertising, on the other hand, chunks information about all the products under a brand umbrella into a composite advertisement and in the process better handles the limited attention span of consumers. Thus brand advertising can help the monopolist to offer more products and earn more profits by better meeting the diverse needs of its customers. As this positive effect is more powerful than the negative brand dilution effect, we find that as indicated in the proposition, a monopolist engages only in brand advertising. However, we obtain a different result in the presence of competition.

**Multiproduct Duopoly.** In a duopoly, as noted earlier, each firm does not fully internalize the negative effects of the information overload caused by its advertising and hence remains motivated to offer more products. To understand how the presence of competition tempers the optimal level of brand advertising, we focus attention on the case where \( k_i + k_j > \tau \). In this case, we can view \( k_i + k_j - \tau \) as a measure of the information overload induced by advertising. If the emphasis placed on brand advertising is \( \delta \), the probability of customers being informed of a given product at the time purchase through the duopolist’s product and brand advertising is:

\[
0 \leq g_i (\tau, k_i, k_j) \delta_\omega_i + \lambda (k_i, k_j, \tau) (1 - \delta_i) \omega_i \leq 1. \tag{26}
\]

To simplify notation, let \( \Upsilon_i \equiv g_i (\tau, k_i, k_j) \delta_\omega_i + \lambda (k_i, k_j, \tau) (1 - \delta_i) \omega_i \). Now we can express the profits of firm \( i \) as follows:

\[
\pi_i = \left[ \Upsilon_i + \frac{\Upsilon_i k_i (N(N-1))}{N^2} \left( 1 - \frac{k_i + k_j - \tau}{N(N-1)} \right) + \frac{\Upsilon_i k_i (N(N-1))}{N^2} \left( \frac{1}{2} + \frac{p_j - p_i}{2\tau} \right) \right] p_i - \frac{2k_i F}{N} - A (k_i, \omega_i). \tag{27}
\]

We further assume that \( \lambda (k_i, k_j, \tau) = \frac{\tau}{k_i + k_j} \). Note that \( \lambda (k_i, k_j, \tau) = 1 \) when \( k_i + k_j \leq \tau \), but \( 0 < \lambda (k_i, k_j, \tau) < 1 \) when \( k_i + k_j > \tau \). Also, \( \frac{\partial \lambda}{\partial \tau} > 0 \) and \( \frac{\partial \lambda}{\partial k_i} < 0 \), implying that the effective reach of a firm’s product advertising reduces with its productline length but increases with consumers’ attention span.

Our analysis shows that in equilibrium, a duopolist uses both brand advertising and product advertising. Furthermore, the optimal level of brand advertising decreases with \( \tau \) when \( k_i + k_j - \tau > \frac{1 - g_i(\tau, k_i, k_j)}{\partial g_i(\tau, k_i, k_j)/\partial \tau} \). To follow this result recall that, unlike a monopolist, a duopolist does not fully internalize the negative externalities induced by its advertising. Consequently,
a duopolist is more motivated to supply the preferred products of more customers and also engage in product advertising. But only engaging in product advertising will significantly overload consumers’ limited attention span. This congestion, however, can be attenuated by engaging in brand advertising, wherein all the products are advertised through a composite advertisement. Hence, unlike a monopolist, the duopolists are motivated to use both product advertising and brand advertising to inform consumers about their products.

We also find that a firm may decrease its use of brand advertising when consumers’ attention span increases. Recall that $g_i(\tau, k_i, k_j)$ captures brand dilution, and that $\partial g_i(\tau, k_i, k_j)/\partial \tau$ shows how much the effectiveness of brand advertising improves with $\tau$. Hence, $1-g_i(\tau, k_i, k_j)$ can be viewed as a measure of the ineffectiveness of brand advertising. So $\frac{1-g_i(\tau, k_i, k_j)}{\partial g_i(\tau, k_i, k_j)/\partial \tau}$ indicates the extent to which an improvement in consumer’s attention span reduces the ineffectiveness of brand advertising. Our analysis shows that if this reduction is less than information overload, namely $k_i + k_j - \tau$, then firms should reduce the emphasis placed on brand advertising.\(^5\) The following proposition summarizes the key findings on the trade-off between product advertising and brand advertising.

**Proposition 3** *It is more profitable for a monopolist to use only brand advertising to promote its products. A duopolist, however, uses both brand advertising and product advertising, with the optimal level of brand advertising decreasing with $\tau$ when $k_i + k_j - \tau > \frac{1-g_i(\tau, k_i, k_j)}{\partial g_i(\tau, k_i, k_j)/\partial \tau}$.

5. CONCLUSIONS

The purpose of this paper was to understand how limited attention span and the prospect of brand dilution affect multiproduct competition in horizontally differentiated markets. Toward this goal, we developed a novel model of multiproduct competition and informative advertising. Our analysis offers some insights into how consumers’ cognitive limitations may shape a firm’s productline decisions.

1. How does consumers’ limited attention span affect a firm’s productline? Empirical research on advertising suggests that consumers’ attention span is limited and consumers

\(^5\)For another interpretation of the condition $k_i + k_j - \tau > \frac{1-g_i(\tau, k_i, k_j)}{\partial g_i(\tau, k_i, k_j)/\partial \tau}$, consider the following. Denote the hazard of brand dilution by $h(\tau) \equiv \frac{\partial g_i(\tau, k_i, k_j)}{1-g_i(\tau, k_i, k_j)}$. Thus, the condition implies that if this hazard is above a threshold $h^*$, then the emphasis placed on brand advertising reduces. Furthermore, $h^* = \frac{1}{k_i + k_j - \tau}$ implying that the threshold reduces as information overload increases.
tend to handle information overload by attending to only a small fraction of advertisements bombarded at them. On analyzing a model of product advertising, we find that the number of products advertised by a monopolist will not violate consumers’ cognitive constraint under weak conditions. Specifically, if a one percent increase in the monopolist’s productline length congests consumers attention span and reduces the effective reach of its advertising by at least one percent, our finding holds. In the presence of competition, however, the total number of products advertised by duopolists could exceed consumers’ attention span and there will be advertising clutter. To appreciate the intuition for this finding, note that a monopolist internalizes all the negative consequences of congesting consumers’ attention span. A duopolist, however, does not fully internalize the negative effects, and hence offers more products and clutters consumers’ attention span.

2. **What is the effect of brand dilution on a firm’s productline?** To explore this issue, we examined a simple form of brand advertising where a firm releases a composite advertisement for all the products within its brand umbrella. A common problem with brand advertising is that, if too many products are promoted under a brand umbrella, it could potentially lead to brand dilution — a reduction in the cognitive accessibility of individual products. However, the additional products may help a firm to offer the preferred products of more consumers. In a monopoly, if the elasticity of brand dilution is below a threshold, it is profitable for a firm to offer more products as advertising reach increases. The results, however, are reversed in a duopoly. Here, if the elasticity of brand dilution is above a threshold, the duopolists will be motivated to offer more products as advertising reach increases. The rationale for this result is that, although brand dilution directly reduces the effectiveness of brand advertising, it exerts a positive strategic (indirect) effect by attenuating cannibalization and softening price competition in a duopoly.

3. **What is the optimal trade-off between product advertising and brand advertising?** In contrast to a monopolist, a duopolist may use both brand and product advertising. A monopolist prefers brand advertising as it helps to better handle the cognitive constraints of consumers. As a duopolist does not fully internalize the effects of its advertising clutter, it is motivated to offer more products and engage in product advertising.
We find that the optimal level of brand advertising increases if consumers’ attention span decreases.

Although our analysis is based on a stylized model, it helps us to better understand product markets. It is known that heterogeneity in consumers’ preferences and low cost of introducing a new product lead to product proliferation. It is well known that firms use advertising to inform consumers about the characteristics of their products. What is less recognized is that a firm’s desire to precisely communicate the position of its products could in itself affect the firm’s productline length and lead to counterintuitive results. Specifically, motivated by its very desire to keep consumers informed about its products at the time of purchase, a duopolist could end up offering too many products and, thereby, induce congestion of consumers’ attention span and brand dilution. For example, Byron reports that 352 varieties of toothpastes are available in retail outlets (Byron 2011). Similarly, we see too many varieties of laundry detergents, cereals, and chips in grocery stores. Our analysis raises the possibility that in addition to factors such as diversity in consumer taste and cost factors, the product proliferation observed in these competitive categories could be related to consumers’ limited cognitive abilities.

Limitations and directions for further research. In developing our model, we attempted to capture the notion of limited attention span and brand dilution. Future theoretical research can attempt to model further nuanced features of information processing, such as selective information processing and screening, and examine their strategic implications. Also our analysis focused on the informative role of brand advertising. However, brand advertising can help improve the image of a brand (e.g., Becker and Murphy 1993) or signal the quality of its product portfolio (e.g., Milgrom and Roberts 1986). Researchers can consider modeling these persuasive effects and analyze the trade-off between informative and persuasive brand advertising.

While we analyzed a horizontally differentiated market, future research can explore how different types of advertising influence the product-line decisions of multiproduct firms in a vertically differentiated market. Several emerging advertising technologies help firms to better target their communication and potentially reduce cannibalization within its productline and also save costs. Hence there is need to study how targeted advertising affects multiproduct competition.
In our model we assumed that a firm offers multiple products in the same category. Consider, for example, Porsche offering several variants of cars, or P&G offering multiple laundry detergents. We can study the behavior of such firms using our framework. Now consider Porsche offering an SUV in addition to its cars. An important implication of this situation is that consumers preferring cars are more likely to choose among the different cars available in the market rather than make a trade-off between cars and SUV. Consequently, Cayenne, the new Porsche SUV, is likely to draw more sales from other SUVs in the market. This type of substitution pattern within a firm’s product portfolio is beyond the scope of the current model. As in logit models, however, a nested structure may help capture the essence of the interrelationships within this firm’s product line. For example, Anderson and de Palma (2006) analyze a nested logit model where nests are symmetric and the variants within a nest are symmetric. There is an opportunity to develop an analytically tractable model of multiproduct competition that allows for asymmetric inter-relationship within a firm’s product line. If consumers were to organize products in their memory using a nested structure, it raises an additional theoretical question. Will adding a new product to a nest have asymmetric brand dilution effects across the different nests in memory? For example, Porche recently introduced Panamera, which is a four-door car. Will this new product have asymmetric effect on consumers’ memory of Porche SUV models vis a vis Porche two-door cars. The theoretical implications of such asymmetric brand dilution is yet to be explored.

In our formulation, we assumed that consumers are uninformed of a product unless they are exposed to its advertisement. In the marketplace, however, consumers obtain information not only from advertisements but also from other sources, such as word of mouth and online blogs. Consequently, firms may have different levels of baseline awareness because of these information sources. For example, firm $i$’s products can be endowed with a higher level of baseline awareness, $b \in (0, 1)$ while competing firm $j$’s baseline awareness can be normalized to 0. There is an opportunity to understand how the baseline awareness affects product line decisions and price competition. Furthermore, consumer’s search behavior could shape firm’s communication strategy. Using a signaling framework, Mayzlin and Shin (2011) show that a high-quality firm may use advertising devoid of attribute information to signal its high quality. Future research can further explore the differential impact of product advertising and brand advertising on consumer search and in turn a firm’s product line length.
There is also a need to confront our model’s predictions with field and experimental data (e.g., Ratchford et al. 2003, Draganska and Jain 2005, Amaldoss and Jain 2010). Recall that Proposition 1 suggests that a monopolist’s productline length will not exceed consumers’ attention span. One can perhaps directly test the behavior of a monopolist in a laboratory. To do this one can consider using a demand formulation as outlined in our model, and assessing whether participants choose a productline length as predicted by the equilibrium solution. Alternatively, one can consider doing an event analysis with field data on emerging markets (longitudinal study) and see how a firm’s productline length changes with the advent of competition. For example, Bajaj Auto in India had a very small productline for decades when it was in a near monopoly position. However, with the advent of competition, the firm substantially increased its productline length; and moreover shifted from using only brand advertising to increasing its emphasis on product advertising with the advent of competition. There exists an opportunity to measure consumers’ attention span using eye tracking studies and see how it is correlated with firm’s productline length and the level of competition. Proposition 2 suggests that the monopolist’s productline length increases with advertising reach only if the elasticity of brand dilution is below 1; whereas a duopolist’s productline length increases with advertising reach if the elasticity of brand dilution is above 2. In the laboratory one can study the supply side predictions of the model, keeping the demand side fixed as outlined in our model. In a field setting, one can venture to measure the elasticity of brand dilution in a given category by assessing the accuracy of consumer’s ability to freely recall products for a given brand in the category. Then one can examine how well this measure of elasticity of brand dilution is correlated with the average productline length in the category. Finally, incorporating behavioral regularities into normative models and understanding their implications for firm behavior is a fruitful area for management research (e.g., Parco et al 2005, Ho, Lim and Camerer 2006, Cui, Raju and Zhang 2007, Chen, Joshi, Raju and Zhang 2009, and Chen, Iyer and Pazgal 2010).
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