Sales Force Compensation Design for Two-Sided Market Platforms

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Abstract

The authors study the use of sales agents for network mobilization in a two-sided market platform that connects buyers and sellers, and they examine how the presence of direct and indirect network effects influences the design of the sales compensation plan. They employ a principal–agent model in which the firm tasks sales agents to mobilize the side of the platform that it monetizes (i.e., sellers). Specifically, the presence of network effects alters the agency relationship between the firm and the sales agent, requiring the platform firm to alter the compensation design, and the nature of the alteration depends on whether the network effects are direct or indirect and positive or negative. The authors first show how the agent's compensation plan should account for different types of network effects. They then establish that when the platform firm compensates the agent solely on the basis of network mobilization on the side cultivated by the agent (sellers), as intuition would suggest, it will not fully capitalize on the advantage of positive network effects; that is, profit can be lower under stronger network effects. To overcome this limitation, the platform should link the agent's pay to a second metric, specifically, network mobilization on the buyer side, even though the agent is not assigned to that side. This design induces a positive relation between the strength of network effects and profit. This research underlines the complexity and richness of network effects and provides managers with new insights regarding the design of sales agents' compensation plans for platforms.

Keywords

two-sided market platform, sales force, incentive design, principal-agent model Online supplement: https://doi.org/10.1177/0022243719825818

Platform businesses have affected diverse industries such as transportation (e.g., Uber), advertising technology (e.g., Google), media (e.g., Twitter), health care (e.g., American Well), social networking (e.g., Facebook), retail (e.g., Amazon), banking (e.g., Credit Karma) and even education (e.g., Coursera). Platform firms like Apple, Google, Facebook, and Microsoft now surpass traditional giants like General Motors, the Coca-Cola Company, and General Electric, not only in terms of brand value, but also in terms of shareholder value. Twosided markets create value differently than other businesses thanks to a mix of direct network effects from interactions between same-side participants (e.g., Facebook users enjoying connecting with their friends) and indirect network effects involving interactions between participants on opposite sides of the market (e.g., OpenTable diners get value when they make reservations at affiliated restaurants, and restaurants derive value from outreach to potential diners). As a result, platform adoption decisions depend crucially on network mobilization, defined as attracting participants on each side of the marketplace, which requires fundamentally different management strategies than are needed by businesses that do not rely on such direct and indirect network effects.

This article focuses on active selling as a key instrument used by managers to conduct business development and secure participation on the platform's network. For instance, Open-Table employs salespeople for the nontrivial task of persuading restaurants to adopt the platform. Kyruus, which provides coordination technology to multipoint health systems, hires sales staff to sign up provider organizations, a challenge amplified by barriers to information technology (IT) adoption in health care. Credit Karma hires sales staff to acquire financial provider firms, rounding out its business objective of serving customers who seek financial products. American Well, an online platform connecting physicians with patients, employs sales agents to reach out to health insurance companies that contract with these physicians. Twitter, like other advertising-oriented



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platforms, employs advertising sales agents to sell advertising space to advertisers. Many software firms embracing a platform approach to innovation (e.g., Atlassian, Intuit, Autodesk, Unity) do so by recruiting external developers (e.g., game developers or software developers) to extend the capabilities of their software products.

Proper design of compensation plans for sales agents is thus vital for platforms. The substantial literature on sales force compensation provides a starting point for plan design. Given the unobservability of an agent's selling efforts, a compensation plan should link the agent's compensation to an observed performance measure (i.e., sales) to achieve a profitmaximizing balance between incentives and risk sharing. A rich literature examines this fundamental issue (Basu et al. 1985; Jain 2012; Krishnamoorthy, Misra, and Prasad 2005; Lal and Srinivasan 1993; Mantrala et al. 2010; Rubel and Prasad 2016; Steenburgh 2008); however, it does not indicate how incentive compensation plans should account for direct and indirect network effects. Meanwhile, while the literature on platforms covers a rich set of issues such as pricing strategies (Liu and Chintagunta 2009), product design (Bakos and Katsamakas 2008), product launch (Lee and O'Connor 2003), seeding strategies (Dou, Niculescu, and Wu 2013), compatibility and competition (Farrell and Klemperer 2007), competition across platforms (Rochet and Tirole 2003), competition between incumbents and entrants (Eisenmann, Parker, and Van Alstyne 2011; Katz and Shapiro 1992), segmentation (Bhargava and Choudhary 2004), timing of product introduction (Bhargava, Kim, and Sun 2013), resource allocation (Sridhar et al. 2011), and business model design (Hagiu 2007; Parker and Van Alstvne 2005), it does not consider the active role of salespeople in selling platforms.

This paper investigates how to optimally incentivize sales agents when network effects drive the value of the product. In doing so, we explore a series of novel questions for which initial intuition seems insufficient: (1) Should network effects alter the design of incentives offered to agents, and how? (2) Should the agent's incentives be based on the traditional metric—sales on the market side that the agent cultivates or, because of cross-market effects, should incentives also account for sales on the side the agent has no responsibility for? (3) Would positive network effects necessarily increase the firm's profit under optimal designs, as predicted by the platform literature, or could stronger network effects, despite being positive, lead to lower profits?

To address these managerial questions, we propose a principal-agent model of platform sales that takes into account direct and indirect cross-market network effects and assumes a utility-maximizing agent, as well as a profit-maximizing platform firm. Our model considers both types of network effects, on both sides of the platform, and each type can be positive or negative. The proposed model reveals that the agency relationship arising in the case of platforms differs from the agency relationship that arises in the case of products without network effects. The reason is that network effects simultaneously affect the agent's selling effectiveness and sales uncertainty. As a result, traditional risk sharing between the principal and the agent, a central topic in the sales force compensation literature (see, e.g., Coughlan and Joseph 2012), is altered. Not only should network effects matter (i.e., they influence optimal incentive design), but a proper managerial response requires a deeper understanding of the type (direct vs. cross-market) and valence (positive or negative) of network effects. Further, the agent and the firm respond differently to network effects. In equilibrium, the optimal commission rate offered to an agent to sell in a two-sided market is always lower than the optimal commission rate offered to sell a product in a market without network effects. Conversely, the agent can work more or less when selling a two-sided market (compared with selling a product without network effects), depending on the valence of network effects.

Furthermore, we produce novel insights regarding the favorable impact of network effects. It is generally understood that positive network effects are good; that is, they increase adoption and value. Thus, making network effects stronger (e.g., by developing better search, discovery, matching, or fulfillment algorithms) should increase the firm's profit. However, we show that when network mobilization requires an active selling effort, some network effects can lower profits, despite being positive. This is because network effects amplify the uncertainty about network mobilization on one side of the market that occurs as a result of demand shocks on the other side. As a result, we propose a compensation plan that is informed by network mobilizations on both sides of the platform to optimally manage the two sources of uncertainty affecting mobilizations. Specifically, the proposed plan links the agent's compensation not only to the metric that the agent affects directly-adoption by sellers-but also to another metric that the agent affects only indirectly: adoption by buyers. The proposed plan then produces the desirable, positive relationship between stronger network effects and profits.

In the next section, we derive platform demands and detail the sequence of the game, the agent's problem, and the manager's optimization program. We then present the equilibrium strategies and profit under traditional compensation plans, which can lead to lower profits. Next, we present a new plan design that extends the agent's compensation to an indirect metric, and we show that it restores optimal risk sharing between the contracting parties when network effects exist. Finally, we generalize our results by considering several extensions.

Model

We start with a general model of platform adoption that considers both direct and cross-market (indirect) network effects. For instance, PayPal displays cross-market network effects between merchants and individuals, and direct network effects between individuals. The platform creates the infrastructure and business rules that enable interactions between the two sides, which we label b (buyers) and s (sellers). One crucial insight from the economics of platforms is that often the optimal strategy for the platform is to subsidize one side of the market while monetizing the other (Eisenmann, Parker, and Van Alstyne 2006; Parker and Van Alstyne 2005). These sides are labeled the "subsidy" (or "free") side and the "paying" side. Commonly, the subsidy side corresponds to buyers, and the paying side corresponds to sellers, while buyers generate value for the firm through indirect network effects (Gupta and Mela 2008). Let p denote the price paid by each seller to join the platform. For expository reasons and to maintain focus on the influence of network effects, we treat the marginal costs of serving sellers as zero.

In addition to the infrastructure and business rules that enable interactions between the two sides, the firm relies on an agent to mobilize one side of the platform. In practice, this side is usually the paying side of the platform (the sellers), whereas network growth on the nonpaying side is primarily achieved organically by word of mouth and because of the inherent value (stand-alone benefits, direct network effects, and cross-network benefits) that customers on this side receive from participation. An example is Credit Karma, which provides consumers with a free credit report, earns revenue by directing them to firms that sell financial products, and employs an in-house sales team to acquire firms. Other prototypical examples to illustrate this idea are the aforementioned firms, such as OpenTable and LinkedIn, which deploy sales agents to recruit members on the paying sides (i.e., restaurants and recruiters, respectively). Similarly, advertising platforms like Facebook or Twitter task advertising agents to sell advertising space to advertisers, not to increase the number of viewers (eyeballs). We subsequently generalize our model to account for platforms that require active selling on both sides.

Influences on Market Formation

We start with the standard model in the literature for market formation under network effects. Microfoundations for the model are presented in Appendix A are based on the assumption that demands on both sides of the platform are determined by early adopters and followers. Let Q_b and Q_s represent sales on the buyer and seller sides, respectively. These sales are affected by stand-alone benefits (V_b and V_s respectively), the price charged to sellers by the platform, the direct network benefits that participants anticipate that they will obtain from the platform (i.e., $\gamma_b Q_b^a$ and $\gamma_s Q_s^a$, where γ_b and γ_s reflect the intensity of direct network effects, and Q_s^a and Q_b^a represent market participants' anticipation about mobilization), and finally the cross-network benefits ($\eta_b Q_s^a$ and $\eta_s Q_b^a$, where η_b and η_s reflect the intensity of cross-network effects). Other influences on Q_b and Q_s are encapsulated in the error terms ε_b and ε_s , respectively, which are unknown at the time of contracting, and which we assume to be normally distributed (with mean 0 and variance σ_b^2 and σ_s^2 , respectively). Specifically, the sales agent's effort, w, exerts positive influence on sales with effectiveness β . Mathematically, these influences are captured by the following model of sales on both sides covered by the platform:



Figure 1. Platform sales.

$$Q_b = V_b + \gamma_b Q_b^a + \eta_b Q_s^a + \epsilon_b \,, \text{ and } \eqno(1a)$$

$$Q_s = V_s + \gamma_s Q_s^a + \eta_s Q_b^a - p + \beta \textbf{w} + \epsilon_s. \tag{1b}$$

This model is graphically depicted in Figure 1.

Next, following the extant platform research, we assume that market participants form rational expectations, which means that participants' anticipations about network size are fulfilled in equilibrium. As is standard in the literature (Katz and Shapiro 1985), the term "expectation" in the expression "rational expectations" means that $Q_b = Q_b^a = q_b$ and $Q_s = Q_s^a = q_s$, which is different from the expectation operator \mathbb{E} in statistics. As a result, we obtain that, at the time of contracting, the equilibrium levels q_s and q_b on the two sides are defined as

$$\begin{split} q_b = & \frac{V_b(1-\gamma_s) + \eta_b(V_s + \beta \mathbf{w} - p)}{(1-\gamma_b)(1-\gamma_s) - \eta_b \eta_s} \\ & + \frac{\epsilon_b[(1-\gamma_b)\gamma_s + \eta_b \eta_s] + \epsilon_s \eta_b}{(1-\gamma_b)(1-\gamma_s) - \eta_b \eta_s} , \text{ and} \end{split}$$

$$\begin{split} q_s = & \frac{V_b \eta_s + (1-\gamma_b)(V_s + \beta \mathbf{w} - p)}{(1-\gamma_b)(1-\gamma_s) - \eta_b \eta_s} \\ & + \frac{\epsilon_b \eta_s + \epsilon_s [(1-\gamma_b)\gamma_s + \eta_b \eta_s]}{(1-\gamma_b)(1-\gamma_s) - \eta_b \eta_s}, \end{split} \tag{2b}$$

both of which incorporate uncertainty and depend on product parameters, as well as the agent's effort level. We assume that parameters satisfy the regularity condition $(1 - \gamma_{\rm b})(1 - \gamma_{\rm s}) - \eta_{\rm s}\eta_{\rm b} > 0$, which is necessary to ensure nonnegative sales. The demand equations (Equation 2) comport with the extant literature on the economics of platforms and two-sided markets. Moreover, when network effects are set to zero, that is, $\gamma_s = \gamma_b = \eta_s = \eta_b = 0$, q_s in Equation 2 provides the classical sales response function used in numerous studies that investigate sales force compensation and moral hazard (see, e.g., Hauser, Simester, and Wernerfelt 1994; Holmstrom and Milgrom 1987; Kalra, Shi, and Srinivasan 2003; Mishra and Prasad 2005; Syam, Hess, and Yang 2016), which will allow us to compare and contrast our results with those obtained for products without network effects. Finally, we do not impose any restriction on the sign of network effects; that is, our model allows network effects to be positive and negative, thus empowering us to analyze different types of platforms.

Contract Design Between the Platform and the Agent

Given the demand system (Equation 2), the manager determines the compensation plan, $S(q_s)$, that incentivizes profit-maximizing effort levels from the agent. Specifically, following the extant sales force literature, we first consider a linear compensation plan whereby incentives are based on revenue generated by the agent's mobilization of sellers (i.e., $p \times q_s$).¹ Thus, the agent's compensation is

$$S(q_s) = \alpha_0 + \alpha_1 \times p \times q_s, \tag{3}$$

where α_0 is the agent's fixed salary and α_1 is the commission rate. Such a structure, which links commission to revenue or profit (here, the same, because we assume cost to be zero for now), is common, for instance, in IT talent companies. These firms connect IT workers (e.g., software engineers) with client companies seeking temporary increases in their IT staff, and they compensate account managers on the basis of the profitability of the businesses they bring in. A similar approach compensating agents on the basis of profitability prevails at an online news platform focused on disseminating information about business schools. Linear contracts have received much attention in the literature because they are employed in practice and are robust (see, e.g., Holmstrom and Milgrom 1987), analytically tractable, and easy to implement. Similarly, we follow the extant literature and model the agent's utility as

$$\mathbf{U}[\mathbf{S}(\mathbf{q}_{s}), \mathbf{w}] = 1 - e^{-\rho[\mathbf{S}(\mathbf{q}_{s}) - \mathbf{C}(\mathbf{w})]},\tag{4}$$

where ρ is the agent's risk aversion coefficient and $C(\mathbf{w})$ is the cost of effort (with $C'(\mathbf{w}) > 0$ and $C''(\mathbf{w}) > 0$). From our discussions with sales leaders at various platform companies, we have found that the profile of sales agents in such companies does not differ from the profile of sales agents in other industries. Thus, we follow the extant analytical (see, e.g., Syam, Hess, and Yang 2016), empirical (see, e.g., Chung, Steenburgh, and Sudhir 2014; Misra and Nair 2011), and experimental (see, e.g., Chen and Lim 2017) literatures on sales force incentives and consider the specific convex function $C(\mathbf{w}) = \mathbf{w}^2/2$, which is also analytically tractable.

The agent's optimal level of effort is determined by maximizing the certainty equivalent of the agent's utility function,

$$\mathbf{w}^{*} = \underset{\mathbf{w}}{\operatorname{arg\,max}} \underbrace{\mathbb{E}[S(q_{s})] - \frac{\rho}{2} \operatorname{Var}[S(q_{s})] - C(\mathbf{w})}_{U_{CE}}.$$
 (5)

The optimal effort \mathbf{w}^* forms the incentive compatibility (IC) constraint in the firm's compensation design problem. Furthermore, the agent's participation or individual rationality (IR) constraint is such that the agent should receive a nonnegative net utility in expectation (as we normalize the value of the agent's outside option to zero without loss of generality). The principal then determines the contract parameters that maximize the expected value of the firm's profit,

$$\mathbb{E}[\Pi] = \mathbf{p} \times \mathbb{E}[\mathbf{q}_{s}] - \mathbb{E}[\mathbf{S}(\mathbf{q}_{s})], \tag{6}$$

subject to the agent's IC and IR constraints. Thus, the sequence of the game is as follows:

- In Stage 1, the principal offers the agent a linear contract, composed of a fixed salary and a commission rate (α₀ and α₁, respectively).
- 2. In Stage 2, the agent accepts or rejects the offer.
- 3. In Stage 3, the agent exerts effort (w).
- 4. In Stage 4, market participants mobilize, and payments are made.

We note that market uncertainty exists when the principal and the agent agree on the compensation contract, and this uncertainty resolves over the time period during which the agent mobilizes the network and participants make decisions. Naturally, therefore, the compensation parameters are chosen with respect to expectations about outcomes in the later stages, and assuming that the agent is fully informed about all the buyer-side response function parameters, including strength and signs of direct and indirect network effects. Finally, the game sequence implies that the contract parameters influence mobilization through the agent's decisions to accept the contract (or not) and to work hard (or not).

Classical Plan: Linking Incentives to Direct Sales

This section explores how network effects influence the optimal configuration of the agent's compensation contract. We first set up the benchmark case by describing the optimal linear compensation plan for products without network effects, for which the sales response function is $\mathbf{q} = \mathbf{V} + \boldsymbol{\beta} \times \mathbf{w}_0 - \mathbf{p} + \boldsymbol{\varepsilon}$, where $\boldsymbol{\beta}$ is the agent's selling effectiveness and $\boldsymbol{\varepsilon}$ represents normally distributed demand shocks with zero mean and variance σ^2 . The agent's optimal effort strategy is then

$$\mathbf{w}_0^* = \boldsymbol{\beta} \times \mathbf{p} \times \boldsymbol{\alpha}_1. \tag{7}$$

In return, the platform achieves optimal expected profit by setting the commission rate to

$$\alpha_1^* = \frac{\beta^2}{\beta^2 + \rho\sigma^2},\tag{8}$$

¹ For now, we treat p as an exogenous fixed price, invariant with q_s , but we relax this restriction later. Another variant is to link the sales commission to the number of adopters excluding early adopters. Such a contract would be hard to implement because at the time of contracting, it is uncertain how many early adopters would remain active users; moreover, this approach does not qualitatively alter our results.

where $\partial \alpha_1^* / \partial \beta > 0$ (see, e.g., Bolton and Dewatripont 2005) and the equilibrium profit is

$$\mathbb{E}[\Pi^*] = \underbrace{(V-p) \times p}_{\text{Core Contribution of Product}} + \underbrace{\frac{p^2 \times \beta^4}{2(\beta^2 + \rho\sigma^2)}}_{\text{Contribution of Selling}}, \quad (9)$$

which can be decomposed into two sources: profit that comes from the stand-alone value of the product (V) and profit that is generated by active selling. Equations 7 to 9 define the benchmark case against which we will compare our results.

Agent's Effort Strategy and Selling Effectiveness with Network Effects

For compensation design under network effects, we start by identifying the agent's optimal effort strategy and the reponsiveness of this strategy to network effects. We develop the following propositions.

P₁: The agent's optimal effort strategy is

$$\mathbf{w}^* = \frac{(1 - \gamma_b)}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s} \times \beta \times \mathbf{p} \times \alpha_1.$$
(10)

As a result, positive network effects (both direct and crossmarket) enhance the agent's selling effectiveness and increase the agent's optimal effort.

We note that Equation 10, with network effects set to zero, yields the optimal effort strategy for nonnetwork goods ($\mathbf{w}^* = \beta \times p \times \alpha_1$, Equation 7). For the richer case of nonzero network effects, we see that network effects do affect the agent's effort and output. The intuition behind P₁ is revealed by rewriting the agent's optimal effort strategy as

$$\mathbf{w}^* = \mathbf{\beta} \times \mathbf{p} \times \mathbf{\alpha}_1,\tag{11}$$

with $\tilde{\beta} = \kappa \times \beta$, where

$$\kappa = \frac{(1 - \gamma_b)}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s}$$

is the multiplier effect that network effects exert on the agent's selling effectiveness. It is easy to confirm that $\kappa > 1$ when all network effects are nonnegative, and that both direct and cross-market network effects enhance the agent's selling effectiveness. Crucially, positive network effects provide not only a direct financial reward to the agent, but also an indirect one due to the feedback loop of sellers on buyers and back to sellers, thus enhancing the value of a marginal seller for the agent. Therefore, at any level of commission α_1 , the agent works more as *any* network effect parameter increases; that is, $\partial \mathbf{w}^* / \partial \gamma_b > 0$, $\partial \mathbf{w}^* / \partial \gamma_b > 0$ and $\partial \mathbf{w}^* / \partial \eta_s > 0$.

Conversely, P_1 reveals that when some network effects are negative, they may exert a downward force on the agent's effort—for instance, in advertising-related platforms, where the number of sellers (advertisers) can have a negative impact on the number of buyers (eyeballs); that is, $\eta_b < 0$. As a result, the agent can work less or more in this scenario than when

selling a product without network effects, depending on the parameters' values. Specifically, under the regularity condition for the model, the agent will work more (κ >1) when $\gamma_{\rm b}(1-\gamma_{\rm s}) > -\eta_{\rm b}\eta_{\rm s}$, and less otherwise. The rationale for this result is that the agent balances two effects: As sellers (advertisers) become more sensitive to the number of buyers (eyeballs), the agent has an incentive to work more due to the direct financial reward in the form of the commission rate. At the same time, the possibly negative cross-market effect of sellers on buyers decreases the agent's incentives to work harder to sell the platform because of the negative feedback loop of sellers on buyers, which could in equilibrium reduce the value of a marginal seller for the agent and thus make the agent work less. From these insights, we now explore how the manager should tune the optimal commission rate as network effects vary, assuming that $\beta = 1$ without loss of generality.

Optimal Commission Rate

The manager chooses the contract parameters to maximize the platform's expected profit, balancing the compensation cost against the reward from the agent's effort. Formally, the manager picks $\alpha_1^* = \arg \max_{\alpha_1} \mathbb{E}[\Pi]$, subject to the agent's IC and IR conditions: $\mathbf{w}=\mathbf{w}^*$ and $U_{CE}(\mathbf{w}^*)\geq 0,$ where $\mathbb{E}[\Pi] = (1 - \alpha_1) \times p \times \mathbb{E}[q_s] - \alpha_0$. From the platform's perspective, and in contrast to the sale of products without network effects, the agent's selling effort has a spillover effect on sales growth due to network effects because of the feedback loop of sellers on buyers and back to sellers. For instance, when network effects are positive, stronger network effects might increase the value of a marginal seller for the platform. As a result, every unit of commission offered to the agent not only generates extra margin from sellers but has a multiplier effect on overall profit. This intuition would suggest that as positive network effects get stronger, the platform should increase the agent's commission rate α_1 because of the multiplier effects of network effects on the agent's selling effectiveness. Our formal analysis provides greater nuance to this intuition.

P₂: The optimal commission rate,

$$\alpha_1^* = \frac{(1 - \gamma_b)^2}{(1 - \gamma_b)^2 (1 + \rho \sigma_s^2) + \rho \eta_s^2 \sigma_b^2},$$
 (12)

is independent of η_b and γ_s and inversely related to γ_b and η_s .

 P_2 , together with P_1 , first demonstrates that the agent and the principal respond differently to network effects. It uncovers in particular that whereas the agent responds to *all* network effects, the platform does not, since $\partial \alpha_1^* / \partial \gamma_s = \partial \alpha_1^* / \partial \eta_b = 0$. Second, and surprisingly, the firm always decreases the agent's commission rate when the intensity of certain network effects increases, since $\partial \alpha_1^* / \partial \gamma_b < 0$, $\partial \alpha_1^* / \partial \eta_s < 0$, even when all network effects are positive. This insight departs from the extant sales force compensation literature, which recommends increases (see, e.g., Table 26.2 in Coughlan and Joseph 2012).

	No Network Effect	Direct Network Effects Only	Indirect Network Effects Only
Effectiveness ²	I	$\left[1/(1-\gamma_s)\right]^2$	$[1/(1-\eta_s\eta_b)]^2$
Risk (sales variance)	$\sigma^2 \times I$	$\sigma_s^2 \times \left[\mathbf{I} / (\mathbf{I} - \gamma_s) \right]^2$	$(\sigma_s^2 + \eta_s^2 \sigma_b^2) \times \left[1/(1-\eta_s \eta_b) \right]^2$
Optimal commission	$1/(1+\rho\sigma^2)$	$I/(I + \rho \sigma_s^2)$	$I/[I+\rho(\sigma_s^2+\eta_s\sigma_b^2)]$

Table 1. Impact of Network Effects on Selling Effectiveness and Risk in Equilibrium.

Note: The expressions are arranged to highlight that direct network effects have a symmetric effect on selling effectiveness and risk (notice the common multiplier $[1/(1 - \gamma_s)]^2$), whereas indirect network effects have an asymmetric effect.

We find the opposite. The reason is that, in contrast to the sale of products without network effects, for which selling effectiveness and sales uncertainty are independent from each other, the presence of network effects causes the agent's effectiveness and sales uncertainty to be correlated, which affects the optimal balance between risk and incentives that the principal seeks to achieve when deciding on the optimal commission rate. This impact, however, is different under direct network effects ($\gamma_s > 0$; $\gamma_b = \eta_s = \eta_b = 0$) versus indirect network effects ($\gamma_s = \gamma_b = 0$; η_s , $\eta_b \neq 0$). This insight can be seen more vividly when we interpret the canonical optimal commission rate $\alpha_1^* = \beta^2/(\beta^2 + \rho\sigma^2)$ as

$$\alpha_1^* = \text{Effectiveness}^2 / (\text{Effectiveness}^2 + \rho \times \text{Risk}).$$

With this canonical form, Table 1 reports the shift in effectiveness and risk components across the three scenarios. Notably, the table reveals that direct network effects have *symmetric* impacts on both the agent's effectiveness and sales uncertainty, that is, $1/(1 - \gamma_s)^2$, and as a result we find that direct network effects, on their own, do not affect the optimal commission rate when indirect network effects are zero. On the contrary, indirect network effects have *asymmetric* impacts on the effectiveness and sales uncertainty; in particular, an additional risk is imported from the buyers' side into the sellers' side because of η_s (i.e., $\eta_s^2 \sigma_b^2$ in the sales variance term). This breakdown highlights that the increase in uncertainty (as indicated in the additional term for risk) is not fully compensated by the increase in effectiveness, causing the optimal commission rate to be lowered as η_s increases.

To summarize, network effects enter the optimal commission rate differently depending on the externalities they generate. Specifically, they enter differently depending on how they simultaneously affect the agent's selling effectiveness and sales uncertainty. These externalities affect the optimal risk sharing that the manager seeks to achieve through the determination of the commission rate. When only direct network effects exist, these externalities occur *within* the same market and balance each other, such that only one source of uncertainty affects the performance metric used to compensate the agent, and this uncertainty is optimally managed by a traditional unidimensional compensation plan. On the contrary, when indirect network effects exist, two sources of uncertainty affect the agent's performance: demand shocks on the sellers' side and demand shocks on the buyers' side, which are imported because of η_s . In this case, the optimal commission rate should account for γ_b and η_s to manage the additional risk originating from the buyers' side. From these insights, we now investigate how the strengths of network effects influence equilibrium profits.

Impact of Network Effects on Profit

Intuitively, and as discussed in the extant platform literature, positive network effects make the platform more attractive as they enhance the value customers derive from it, and therefore, these effects increase profit. Conversely, negative network effects should decrease profits. Moreover, as noted, stronger positive network effects increase the agent's productivity and effort level. As a result, the platform's profit should increase as network effects increase. Surprisingly, however, we find that under the classical approach of compensating the agent solely on the basis of network mobilization on the side of the market that the agent cultivates, the platform may not always be able to leverage stronger positive cross-market network effects into higher profit. To analyze this more precisely, we compute the platform's optimal profit by replacing the agent's optimal effort strategy and the optimal commission rate in the platform's expected profit, which we present in the next proposition. Despite the complex interplay among various network effect parameters, we can separate the profit into two additive components as in Equation 9.

P₃: The platform's total expected profit is separable into two parts, the contribution of the stand-alone benefits V_b and V_s to profit, Λ_1 , such that

$$\Lambda_{1} = \frac{p \times [(V_{s} - p)(1 - \gamma_{b}) + \eta_{s}V_{b}]}{[(1 - \gamma_{b})(1 - \gamma_{s}) - \eta_{b}\eta_{s}]},$$
 (13)

and the selling agent's contribution to the platform's expected profit, Λ_2 , such that

$$\Lambda_{2} = \frac{\left[\mathbf{p} \times (1 - \gamma_{b})^{2}\right]^{2}}{2\left[(1 - \gamma_{b})^{2}(1 + \rho\sigma_{s}^{2}) + \rho(\eta_{s}\sigma_{b})^{2}\right]} \times \frac{1}{\left[(1 - \gamma_{b})(1 - \gamma_{s}) - \eta_{b}\eta_{s}\right]^{2}},$$
(14)
with total expected profit $\mathbb{E}[\Pi^{*}] = \Lambda_{1} + \Lambda_{2}.$

Writing $\Theta = \{\gamma_s, \gamma_b, \eta_s, \eta_b\}$ as the collection of network effects, we compute, for each $\theta \in \Theta$, the sensitivity of the

platform's expected profit to network effects by computing $\partial \Lambda_1 / \partial \theta$ and $\partial \Lambda_2 / \partial \theta$.² As a result, we present the following insights.

Corollary 1 (Negative Impacts on Profit of Positive Network Effects): When all network effects are positive, that is, $\gamma_s > 0$, $\gamma_b > 0$, $\eta_b > 0$, and $\eta_s > 0$, both Λ_1 and Λ_2 are positive. Furthermore, the platform's expected profit can decrease in γ_b and η_s , when

$$\eta_b/(1-\gamma_s) {<} [(1-\gamma_b)\eta_s\rho\sigma_b^2]/[(1-\gamma_b)^2(1+\rho\sigma_s^2)+2\rho\eta_s^2\sigma_b^2]$$

In such a case, $\partial \mathbb{E}[\Pi^*]/\partial \gamma_b < 0$ when $\partial \Lambda_1/\partial \gamma_b < -\frac{\partial \Lambda_2}{\partial \gamma_b}$, and $\partial \mathbb{E}[\Pi^*]/\partial \eta_s < 0$ when $\partial \Lambda_1/\partial \eta_s < -\partial \Lambda_2/\partial \eta_s$.

In all other scenarios, the platform's expected profit increases with stronger network effects.

Corollary 1 provides two important insights. First, it emphasizes that after the mechanics of selling are taken into account, strengthening of network effects γ_b or η_s can have a detrimental impact on profit through Λ_2 . This result contrasts with the general intuition from the literature on platforms that network effects are always good and that they improve adoption and profits. Here, the reverse occurs because these two network effects cause a spillover of buyer-side uncertainty onto sellers' mobilization. As a result, an increase in each of these parameters implies a reduction in the optimal level of sales incentive (α_1 ; see P₂), which cascades all the way into profits, lowering the firm's total profit, since $\partial \Lambda_2 / \partial \gamma_b < 0$ and $\partial \Lambda_2 / \partial \eta_s < 0$. Thus, positive network effects do not always increase profit.

The second insight from Corollary 1 is that this negative impact can be eclipsed when the other two network effect parameters, γ_s and η_b , are sufficiently large. That is, high levels of γ_s and η_b can convert detrimental parameters (γ_b or η_s) into benign ones. Next, we analyze the scenario of platforms whose revenue model is advertising driven (e.g., Facebook). Buyers' sensitivity to the number of advertisers (sellers) in such platforms can become negative (i.e., $\eta_b < 0$), although other network effects remain positive.

Corollary 2 (Advertising-Related Platform): With $\gamma_s > 0$, $\gamma_b > 0$, and $\eta_s > 0$, but $\eta_b < 0$, the firm's expected profit always increases when γ_s and η_b . Conversely, the firm's expected profit increases in γ_b and η_s only when the following conditions hold: $V_b(1 - \gamma_s) + (V_s - p)\eta_b > 0$, and

$$\begin{split} &\partial\Lambda_1/\partial\gamma_b \!>\! - \partial\Lambda_2/\partial\gamma_b \,\, (\text{for } \partial\mathbb{E}[\Pi]/\partial\gamma_b) \quad \text{ and } \\ &\partial\Lambda_1/\partial\eta_s \!>\! - \partial\Lambda_2/\partial\eta_s \,\, (\text{for } \partial\mathbb{E}[\Pi]/\partial\eta_s). \end{split}$$

The additional insight that Corollary 2 provides is that assuming $\eta_b < 0$ (e.g., the readership of a newspaper dislikes advertising) increases the parameter space where some network effects (i.e., γ_b and η_s) have a negative impact on profit, not only through Λ_2 as in Corollary 1, but also through Λ_1 . Specifically, when

 $V_b(1 - \gamma_s) + (V_s - p)\eta_b < 0$, for instance, both Λ_1 and Λ_2 decrease as γ_b and η_s increase, thus implying a reduction in the firm's expected profit. Finally, we discuss the scenario in which all network effects are positive with the exception of γ_s , where the negative value may represent, for instance, congestion effects on the sellers' side (i.e., more sellers brings more competition). In this case, the sensitivity analyses of the firm's expected profit to network effects comport with the results presented in the first corollary.

The Value of Indirect Metrics

The misalignment of the risks and rewards when selling goods with network effects creates a potential for the platform to lose as network effects strengthen. What could it do to reverse this negative impact? To motivate a remedy, we note that under network effects, the agent's effort has a spillover contribution toward adoptions on *both* sides of the platform, even though the agent is hired to mobilize just one side. While the classical plan links the agent's compensation only to sellers, the fact that $\partial q_b / \partial w \neq 0$ (see Equation 2) suggests the benefit of incorporating the indirect metric q_b (adoption by buyers) into the agent's compensation, via a commission rate α_2 , such that $S(q_s, q_b) = \alpha_0 + \alpha_1 p q_s + \alpha_2 q_b$. We show that with this design, the platform obtains higher profit from stronger network effects.

P₄: The platform achieves higher expected profit under the new plan than under the classical plan. Furthermore, when all network effects are positive, $\partial \mathbb{E}[\Pi^*]/\partial \theta > 0$, for all $\theta \in \{\gamma_s, \gamma_b, \eta_s, \eta_b\}$.

How does linking the agent's plan to the indirect metric of buyer-side adoption overturn the negative influence of positive network effects on the platform's profit? The new plan, with its additional indirect metric, enables the platform to reduce and diversify the total compensation risk faced by the agent (with $\alpha_2^* < 0$) while also providing higher incentives than before for mobilizing sellers (see Appendix B for the expressions α_1^*, α_2^* , and $\mathbb{E}[\Pi^*]$). As in the case of teams with observable individual outputs and multiple sources of uncertainty (Bolton and Dewatripont 2005, p. 315), the $\alpha_2^* < 0$ acts as a relative performance evaluation that is used to disentangle the different sources of risk that affect the agent's output and to disentangle how much of the sellers' mobilization is due to the agent's effort versus demand shocks on the buvers' side. With this, the platform (the principal) capitalizes on stronger network effects by incentivizing the agent to work harder as network effects get stronger (i.e., $\mathbf{w}^*|_{\alpha_2^* < 0} > \mathbf{w}^*|_{\alpha_2 = 0}$). From a practical standpoint, where an $\alpha_2^* < 0$ may not be appealing, we show that this plan can be approximated and reinterpreted in terms of a better understood and commonly employed instrument: the use of sales quotas in the compensation plans of sales agents (Raju and Srinivasan 1996). This new contract is

$$S_s(\boldsymbol{q}_s,\boldsymbol{q}_b) = \alpha_0 + \begin{cases} 0 & \text{if } \boldsymbol{q}_s{<}\boldsymbol{z}, \\ \alpha_1 p \boldsymbol{q}_s + \alpha_2 \boldsymbol{q}_b & \text{otherwise} \end{cases}$$

where $z = [\eta_s/(1 - \gamma_s)]\mathbb{E}[q_b] > 0$ at the time of contracting. When mobilization on the sellers' side is lower than the quota

 $^{^2}$ See the proof of P_3 for the expressions of $\partial\Lambda_j/\partial\theta,$ where $j=\{1,2\}.$

 $-\alpha_2/p\alpha_1\mathbb{E}[q_b] = [\eta_s/(1-\gamma_s)]\mathbb{E}[q_b] > 0$, the agent receives a fixed salary only; the agent receives performance-based incentives when mobilization of sellers exceeds the quota. Like the previously suggested plan, this contract enables the platform to account for the two sources of uncertainty that affect network mobilization on the side of the market that the agent cultivates. It fosters more effort from the agent as network effects get stronger and thus preserves the positive influence between stronger network effects and higher platform profit.

Generalizations

This section explores a few generalizations of the previous analyses in order to examine the robustness of our main results. The first question we explore is whether our insights, derived under exogenous pricing (to the seller side) hold when the platform jointly determines both the agent's compensation and the price charged to sellers. We do so under the multiple-metric plan proposed in the previous section. Second, we consider the case where the platform employs two different sales agents, one on each side of the platform, to recruit network participants on both sides of the market.

Endogenous Price

We now examine optimal compensation design while jointly optimizing the seller-side price p. The rules for market formation and agent's behavior remain unchanged. The optimal contract parameters and price, taking into account the agent's IC and IR conditions, are reported in the following proposition.

P₅: Assuming $\gamma_s < 1/2$, the optimal pricing strategy and commission rates are

$$\mathbf{p}^* = \left(\mathbf{V}_{\mathbf{s}} + \mathbf{V}_{\mathbf{b}} \times \frac{\eta_{\mathbf{s}}}{1 - \gamma_{\mathbf{b}}}\right) \times \mathbf{P}_1, \tag{15}$$

and

$$(\alpha_1^*;\alpha_2^*) = \left[\frac{(1-\gamma_b)(1-\gamma_s)}{(1-\gamma_b)(1-\gamma_s) - \eta_s\eta_b} \times \frac{1}{1+\rho\sigma_s^2}; -\frac{\alpha_1^*\times p^*\times \eta_s}{1-\gamma_s}\right],$$
(16)

respectively, where P₁ is reported in Appendix B.

 P_5 , provided that $\gamma_s < 1/2$ to ensure optimality of the firstorder condition, provides the insight that the platform's pricing decision and compensation plan decision can be made independently, with price merely serving as a parameter in compensation plan design. The commission rate linked to recruitment of sellers is independent of price. Meanwhile, the commission rate offered to the agent based on buyers' mobilization is qualitatively the same whether or not the firm strategically chooses the price, although of course its level would vary with the price that the manager chooses. While this analysis confirms the robustness of earlier insights, it also allows us to learn (from numerical simulations reported in the Web Appendix) that the firm increases the price paid by sellers as it expects a stronger mobilization on the buyers' side (due, for instance, to stronger network effects).³

Active Selling for Mobilizing Buyers

Next, we examine compensation plan designs when network participants on each side of the market are recruited, respectively, by sales agents assigned to that side. For instance, a platform like Kyruus might need active selling to recruit both health care provider facilities, on one side, and patient-sourcing sites, such as WedMD, on the other. Denoting the effort exerted on the sellers' side by \mathbf{w}_s and the effort exerted on the buyers' side by \mathbf{w}_b , we can model the network size on either side as

$$Q_b = V_b + \gamma_b Q_b^a + \eta_b Q_s^a + \textbf{w}_b + \epsilon_b, \ \text{and} \eqno(17a)$$

$$Q_s = V_s + \gamma_s Q_s^a + \eta_s Q_b^a - p + \textbf{w}_s + \epsilon_s. \tag{17b}$$

These equations imply, at the time of contracting, equilibrium levels q_s and q_b , such that

$$q_{b} = \frac{(V_{b} + \mathbf{w}_{b})(1 - \gamma_{s}) + [(V_{s} - p) + \mathbf{w}_{s}]\eta_{b}}{(1 - \gamma_{b})(1 - \gamma_{s}) - \eta_{b}\eta_{s}} + \frac{\varepsilon_{b}[(1 - \gamma_{b})\gamma_{s} + \eta_{b}\eta_{s}] + \varepsilon_{s}\eta_{b}}{(1 - \gamma_{b})(1 - \gamma_{s}) - \eta_{b}\eta_{s}}, \text{ and}$$
(18a)

$$\begin{split} q_{s} &= \frac{(V_{b} + \mathbf{w}_{b})\eta_{s} + (1 - \gamma_{b})((V_{s} - p) + \mathbf{w}_{s})}{(1 - \gamma_{b})(1 - \gamma_{s}) - \eta_{b}\eta_{s}} \\ &+ \frac{\epsilon_{b}\eta_{s} + \epsilon_{s}((1 - \gamma_{b})\gamma_{s} + \eta_{b}\eta_{s})}{(1 - \gamma_{b})(1 - \gamma_{s}) - \eta_{b}\eta_{s}}. \end{split} \tag{18b}$$

From our previous results, we consider plans that link agent compensation to metrics on both sides of the market. Thus, the contract offered to the agent tasked to mobilize the sellers' side is $S_s(q_s, q_b) = \alpha_0 + \alpha_1 p q_s + \alpha_2 q_b$, and the contract offered to the agent tasked to mobilize the buyers' side is $S_b(q_s, q_b) = \beta_0 + \beta_1 p q_s + \beta_2 q_b$. Both agents determine their respective levels of efforts so as to maximize their expected utilities; that is, $\mathbf{w}_i^* = \arg \max \mathbb{E}[S_i(q_b, q_s)] - (\rho/2) Var[S_i(q_s, q_b)] - \mathbf{w}_i^2/2$,

for $i=\{b,s\}.$ As a result, the optimal effort strategies are

$$(\mathbf{w}_{s}^{*};\mathbf{w}_{b}^{*}) = \left[\frac{p(1-\gamma_{b})\alpha_{1}+\eta_{b}\alpha_{2}}{(1-\gamma_{b})(1-\gamma_{s})-\eta_{b}\eta_{s}}; \frac{p\beta_{1}\eta_{s}+(1-\gamma_{s})\beta_{2}}{(1-\gamma_{b})(1-\gamma_{s})-\eta_{b}\eta_{s}}\right].$$
(19)

The manager then chooses the commission rates $(\alpha_1, \alpha_2, \beta_1, \beta_2)$ to maximize the firm's expected profit; that is, $\mathbb{E}[\Pi] = (1 - \alpha_1 - \beta_1) p \mathbb{E}[q_s] - (\alpha_2 + \beta_2) \mathbb{E}[q_b] - (\alpha_0 + \beta_0),$ (20)

subject to both agents' IC and IR conditions, which yields the following proposition.

³ We thank an anonymous reviewer for encouraging us to explore this relationship.

P₆: The optimal commission rates for the agent tasked to mobilize sellers are

$$(\alpha_1^*, \alpha_2^*) = \left[\frac{(1 - \gamma_b)(1 - \gamma_s)}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s} \times \frac{1}{1 + \rho \sigma_s^2}; -\alpha_1^* \times \frac{p}{1 - \gamma_s}\right],$$
(21)

whereas the optimal commission rates for the agent tasked to mobilize buyers are

$$(\beta_1^*, \beta_2^*) = \left[-\alpha_1^* \frac{\eta_b \eta_s}{(1 - \eta_b)(1 - \eta_s)}; \alpha_1^* \frac{p \eta_s}{1 - \gamma_s} \right].$$
(22)

P₆ implies that when all network effects are positive, the commissions paid to the two agents based on network mobilizations of the sides they are responsible to cultivate are positive; that is, $\alpha_1^* > 0$ for the agent tasked to recruit sellers and $\beta_2^* > 0$ for the agent tasked to recruit buyers. Conversely, the commissions paid to the two agents based on network mobilizations of the sides they are not responsible to cultivate are negative; that is, $\alpha_2^* < 0$ for the agent tasked to recruit sellers and $\beta_1^* < 0$ for the agent tasked to recruit buyers. The reverse, however, is not true in the case of advertising-related platforms, where $\eta_b < 0$ since for such platforms β_1^* becomes positive while α_2^* remains negative. The literature on moral hazard in teams, whereby the compensation of agent k is contingent on agent k's observed performance as well as on agent l's observed performance (with $1 \neq k$), illuminates the economic mechanism underlying the previous result (see, e.g., Section 8.1.3 in Bolton and Dewatripont 2005). In such situations, basing an agent's compensation on the agent's own performance as well as on the other agent's performance is used as a relative performance scheme intended to filter out the common shocks affecting individual outputs and to reduce agents' risk exposures. Specifically, when common shocks are positively (negatively) correlated, the commission rate based on the other agent's output should be negative (positive). In the case of platforms, when all network effects are positive, common demand shocks on both sides are positively correlated; that is, ε_b enters positively in q_s (see Equation 18), and ε_s enters positively in q_b . Thus, α_2 (β_1) is negative to reduce the exposure of the agent tasked to mobilize sellers (buyers) to the uncertainty coming from the buyers' (sellers') side. Conversely, when $\eta_b < 0$, shocks affecting adoptions are not positively correlated. In such cases, ε_b still enters positively in q_s , whereas ε_s enters negatively in q_b because of η_b (i.e., $\varepsilon_s \times \eta_b$ in Equation 18); thus, the platform should set $\alpha_2^* < 0$ while $\beta_1^* > 0$ to optimally manage agents' asymmetric risk exposures and filter out common noise.⁴

Conclusion

Platforms are an exciting aspect of modern business. The positive feedback created by network effects, the immense popularity of many new platforms, and excellent financial indicators have created enormous interest in this business model. However, setting up platforms and securing participation of key players is difficult and requires concerted selling efforts. To our knowledge, the present study is the first to examine selling strategy and sales force incentives for two-sided markets. Our analysis demonstrates that the existence of network effects indisputably alters the management of sales force compensation plans and, most importantly, that ignoring them when designing performance-based incentives can hurt profits. We offer several results that answer our initial research questions and are pertinent for platform businesses.

Our results underline the complexity and richness of network effects, and their influence on compensation plan design for agents responsible for network mobilization. We explain how direct and indirect network effects influence compensation plan design differently in terms of the correct mix of fixed salary and performance-based incentives. Moreover, our research emphasizes the limitations of the classical compensation plan for two-sided markets, in which the agent's incentive payment is linked only to performance on the agent's assigned task of mobilizing the seller side. We show that this structure is not sufficiently rich for the platform to optimally manage the influence of network effects on the agent's selling effectiveness and compensation risk, and that the platform's profit can decrease when stronger network effects are present. We show that to overcome this limitation, the manager should link the agent's pay to a second metric, network mobilization on the buyer side, despite the fact that the agent is not assigned to that side. This design restores a positive relation between the strength of network effects and profit, through better management of the spillover effects generated by network effects and, most importantly, better management of the agent's exposure to risk.

With these results in place, our work creates possibilities for future research. First, it would be interesting to revisit some of the platform and sales force empirical literature in light of our findings. Second, it would be useful to endogenize the platform's stand-alone quality and the intensity of network effects, to explore the optimal design of platforms that subsequently need to be sold by sales agents under moral hazard. Finally, managers often use other marketing instruments, such as advertising, to grow platforms (Sridhar et al. 2011), often using different instruments on different sides. Thus, considering more than one marketing instrument would provide valuable insights for the design of marketing budgets and allocation strategies.

Appendix A: Microfoundations of Demands

We present how demands (Equation 2) might be derived from microfoundations. Given the direct and cross-platform network effects that are present, the utility obtained by a participant on side $i = \{b, s\}$ of the market is influenced by product characteristics (which affect stand-alone benefit and network

⁴ We furnish two additional extensions in the Web Appendix, with respect to a negative margin on the buyer side and uncertain network effects.

benefits), network and market parameters (which influence the scale of network benefits), and agent effort (if a sales agent is deployed on that side). On either side, market participants may be heterogeneous in the stand-alone benefit they experience. To capture this heterogeneity, the net stand-alone benefit on side i is written as $V_i - tx_i$, where V_i is a proxy for intrinsic product quality, and x_i is a type-index for participants on side i (assumed to be drawn from a uniform distribution). Therefore, with regard to the stand-alone benefit, the utility depends only on the focal user's type rather than on adoption level or market size, similar to the utility of a product without network effects. The second and third components of the utility function are network benefits from direct and/or indirect network effects. These network benefits depend on network sizes (which we call q; and q; respectively) and intensity of direct and cross-network effects (γ_i and η_i respectively). The network size, q_i , on side i is the sum of the number of early adopters (denoted by $M_i = \mu_i + \varepsilon_i$, where ε_i follows N(0, σ_i^2)), participants who are risk-taking innovators not influenced by the agent's effort, and the number of followers, who are influenced by the agent and join if they have nonnegative utility. The decisions of followers are influenced both by the size of the early adopter group and by network sizes on either side of the platform. We denote the size of the follower group as \hat{x}_{b} and \hat{x}_{s} . The set of followers on side i comprises participants whose type index is in $[0, \hat{x}_i]$, with \hat{x}_i being the locations of marginal participants (i.e., whose net utility is zero). The total utility of a representative participant x_b or x_s can then be written as

$$U_b(x_b)=(V_b-t\times x_b)+[\gamma_b(\hat{x}_b+M_b)]+[\eta_b\times (\hat{x}_s+M_s)], \mbox{ and} \eqno(A1)$$

$$\begin{split} U_s(x_s) &= (V_s+w-t\times x_s) + [\gamma_s(\hat{x}_s+M_s)] + [\eta_s\times (\hat{x}_b+M_b)] - p. \end{split} \label{eq:Us}$$
 (A2)

Normalizing the misfit cost to 1 (i.e., t = 1), and equating the two utility functions to zero for marginal participants (i.e., solving $U_b(\hat{x}_b) = 0$ and $U_s(\hat{x}_s) = 0$), we obtain the equilibrium level of the marginal followers on both sides as

$$\hat{x}_b = \frac{(1-\gamma_s)(V_b + \gamma_b M_b) + \eta_b(M_s + V_s + w - p) + \eta_b \eta_s M_b}{(1-\gamma_b)(1-\gamma_s) - \eta_b \eta_s}, \text{ and}$$
(A3)

$$\hat{x}_s = \frac{(1-\gamma_b)(V_s+\gamma_sM_s+\mathbf{w}-p)+\eta_s(M_b+V_b)+\eta_s\eta_bM_b}{(1-\gamma_b)(1-\gamma_s)-\eta_b\eta_s}.$$
(A4)

The firm and the agent negotiate a compensation contract before the agent starts mobilizing network participants, making M_i a random variable at the time of contracting, which we assumed to be normally distributed (mean μ_i and variance σ_i^2),⁵ but which is certain when network participants mobilize. Thus, at the time of contracting, expected demands are $\mathbb{E}[q_b] = \mathbb{E}[\hat{x}_b + M_b]$ and $\mathbb{E}[q_s] = \mathbb{E}[\hat{x}_s + M_s]$, which comports with Equation 2.

Appendix B: Proof of Propositions

Proof of P₁

Differentiating the objective function of Equation 5 and equating the resulting expression to zero yields Equation 10. Furthermore, the second-order derivative of Equation 5 equals -1, which means that the second-order condition for a maximum is met.

Proof of P₂

As is standard in the agency literature, we first replace the agent's optimal effort strategy in the certainty equivalent of the agent's utility function and set α_0 such that the resulting expression equals the value of the outside option. We then replace α_0 and the agent's effort strategy in the firm's expected profit and differentiate the resulting expression with respect to the decision variable (i.e., α_1), once for the first-order condition and twice for the second-order condition. The first-order condition yields a unique solution (Equation 2). The second-order condition for a maximum is met when

$$(2-\gamma_b)\gamma_b {<} \frac{1+\rho(\eta_s\sigma_b^2+\sigma_s^2)}{1+\rho\sigma_s^2}$$

Subtracting 2 from both sides of the inequality yields that the second-order condition is met when

$$(2-\gamma_b)\gamma_b-1{<}\frac{\eta_s^2\rho\sigma_b^2}{1+\rho\sigma_s^2.}$$

For $\gamma_b{<}1,\,(2-\gamma_b)\gamma_b-1{<}0;$ thus,

$$(2-\gamma_b)\gamma_b - 1 {<} 0 {<} \frac{\eta_s^2 \rho \sigma_b^2}{1+\rho \sigma_s^2}$$

which implies that the second-order condition for a maximum is met.

Proof of P_3

Replacing the optimal strategies under the traditional compensation plan in the profit function yields that

$$\mathbb{E}[\Pi] = \Lambda_1 + \Lambda_2$$

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$$A_1 = \frac{p \times [(V_s - p)(1 - \gamma_b) + \eta_s V_b]}{(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s}, \text{ and } (B1)$$

$$\begin{split} \Lambda_{2} = & \frac{\left[p \times (1 - \gamma_{b})^{2} \right]^{2}}{2 \left[(1 - \gamma_{b})^{2} (1 + \rho \sigma_{s}^{2}) + \rho (\eta_{s} \sigma_{b})^{2} \right]} \\ \times & \frac{1}{\left[(1 - \gamma_{b}) (1 - \gamma_{s}) - \eta_{b} \eta_{s} \right]^{2}}. \end{split} \tag{B2}$$

The sensitivities of Λ_1 and Λ_2 to network effects are reported subsequently.

 $^{^5}$ Specifically, $M_i=\mu_i+\epsilon_i$ where ϵ_i is normally distributed with zero mean and variance $\sigma_i^2.$

$$\frac{\partial \Lambda_1}{\partial \gamma_s} = \frac{p(1-\gamma_b)[(V_s-p)(1-\gamma_b)+V_b\eta_s]}{(1-\gamma_b)(1-\gamma_s)-\eta_s\eta_b^2},$$
(B3)

$$\frac{\partial \Lambda_1}{\partial \gamma_b} = \frac{p\eta_s [V_b(1-\gamma_s) + (V_s - p)\eta_b]}{\left[(1-\gamma_b)(1-\gamma_s) - \eta_s \eta_b\right]^2},\tag{B4}$$

$$\frac{\partial \Lambda_1}{\partial \eta_s} = \frac{p(1-\gamma_b)[V_b(1-\gamma_s) + (V_s - p)\eta_b]}{\left[(1-\gamma_b)(1-\gamma_s) - \eta_s\eta_b\right]^2},\tag{B5}$$

$$\frac{\partial \Lambda_1}{\partial \eta_b} = \frac{p\eta_s [(V_s - p)(1 - \gamma_b) + V_b \eta_s]}{\left[(1 - \gamma_b)(1 - \gamma_s) - \eta_s \eta_b\right]^2},\tag{B6}$$

$$\frac{\partial \Lambda_2}{\partial \gamma_s} = \frac{p^2 (1 - \gamma_b)^5}{\left[(1 - \gamma_b)(1 - \gamma_s) - \eta_s \eta_b\right]^3 \left[(1 - \gamma_b)^2 (1 + \rho \sigma_s^2) + \rho \eta_s^2 \sigma_b^2\right]},$$
(B7)

$$\frac{\partial \Lambda_2}{\partial \gamma_b} = -\frac{p^2 (1 - \gamma_b)^3 \eta_s \left\{ (1 - \gamma_b) (1 - \gamma_s) \eta_s \rho \sigma_b^2 - \eta_b \left[(1 - \gamma_b)^2 (1 + \rho \sigma_s^2) + 2\rho \eta_s^2 \sigma_b^2 \right] \right\}}{\left[(1 - \gamma_b) (1 - \gamma_s) - \eta_s \eta_b \right]^3 \left[(1 - \gamma_b)^2 (1 + \rho \sigma_s^2) + \rho \eta_s^2 \sigma_b^2 \right]^2},$$
(B8)

$$\frac{\partial \Lambda_2}{\partial \eta_s} = \frac{p^2 (1 - \gamma_b)^2 \left\{ -(1 - \gamma_b)(1 - \gamma_s) \eta_s \rho \sigma_b^2 + \eta_b \left[(1 - \gamma_b)^2 (1 + \rho \sigma_s^2) + 2\rho \eta_s^2 \sigma_b^2 \right] \right\}}{[(1 - \gamma_b)(1 - \gamma_s) - \eta_s \eta_b]^3 \left[(1 - \gamma_b)^2 (1 + \rho \sigma_s^2) + \rho \eta_s^2 \sigma_b^2 \right]^2}, \text{ and}$$
(B9)

$$\frac{\partial \Lambda_2}{\partial \eta_b} = \frac{\partial \Lambda_2}{\partial \gamma_s} \times \frac{\eta_s}{1 - \gamma_b}.$$
(36)

Proof of P_4

The first-order condition for the agent's effort strategy yields

$$\mathbf{w}^* = \frac{\mathbf{p}(1-\gamma_b)\alpha_1 + \alpha_2 \eta_b}{(1-\gamma_b)(1-\gamma_s) - \eta_s \eta_b}.$$
 (B10)

Furthermore, this is a maximum since the second-order condition for the agent's effort strategy yields -1. We then replace the agent's optimal effort strategy (Equation B10) in the certainty equivalent of the agent's utility function under the twosided compensation plan and set α_0 such that the resulting expression equals the value of the outside option. We then replace α_0 and the agent's effort strategy in the firm's expected profit and differentiate the resulting expression with respect to the decision variables (i.e., α_1 and α_2), once for the first-order condition and twice for the second-order condition. The first-order condition yields the unique solution provided in P_4 . The determinant of the Hessian of the profit function equals

$$\frac{p^2\rho\sigma_b^2(1+\rho\sigma_s^2)}{\left[(1-\gamma_b)(1-\gamma_s)-\eta_b\eta_s\right]^2} {>}0, \tag{B11}$$

while $\partial^2 \mathbb{E}[\Pi] / \partial \alpha_1^2 < 0$ when $(2 - \gamma_b) \gamma_b - 1 < \eta_s^2 \rho \sigma_b^2 / (1 + \rho \sigma_s^2)$, which is true for any $\gamma_b < 1$ as demonstrated in the proof of P_2 . Thus, the commission rates provided in P_4 maximize profit.

We then proceed by replacing the agent's optimal effort strategy in the agent's expected utility function to identify α_0 such that the IR condition is met and then replace the resulting expression for α_0 and the optimal effort strategy in the firm's profit function. We next differentiate the firm's profit with respect to p, α_1 , and α_2 and equate the resulting expressions

Table B1. Impacts of Network Effects on Profits Under the Two-Dimensional Plan.

$\theta \to$	γ_{s}	$\gamma_{\mathbf{b}}$	η_s	η _ь
$\partial \Omega_1 / \partial \theta$	$ \begin{array}{c} m(1-\gamma_b)[(V_s-p)(1-\gamma_b)+V_b\eta_s] \\ /[(1-\gamma_b)(1-\gamma_s)-\eta_s\eta_b]^2 \end{array} $	$\frac{p\eta_s [V_b(1-\gamma_s) + (V_s-p)\eta_b]}{/[(1-\gamma_b)(1-\gamma_s) - \eta_s\eta_b]^2}$	$\begin{array}{c} m(1-\gamma_b)[V_b(1-\gamma_s)+(V_s-p)\eta_b] \\ /[(1-\gamma_b)(1-\gamma_s)-\eta_s\eta_b]^2 \end{array}$	$\begin{array}{c} p\eta_{s}[(V_{s}-p)(1-\gamma_{b})+V_{b}\eta_{s}]\\ /[(1-\gamma_{b})(1-\gamma_{s})-\eta_{s}\eta_{b}]^{2} \end{array}$
$\partial\Omega_2/\partial\theta$	$\begin{array}{l} [p^2(1-\gamma_b)^3/(1+\rho\sigma_s^2)] \\ /[(1-\gamma_b)(1-\gamma_s)-\eta_s\eta_b]^3 \end{array}$	$\begin{array}{l} [p^2(1-\gamma_b)\eta_b\eta_s/(1+\rho\sigma_s^2)] \\ /[(1-\gamma_b)(1-\gamma_s)-\eta_s\eta_b]^3 \end{array}$	$\begin{array}{l} [p^2(1-\gamma_b)^2\eta_b/(1+\rho\sigma_s^2)] \\ /[(1-\gamma_b)(1-\gamma_s)-\eta_s\eta_b]^3 \end{array}$	$ \begin{array}{l} [p^2(1-\gamma_b)^2\eta_s/(1+\rho\sigma_s^2)] \\ /[(1-\gamma_b)(1-\gamma_s)-\eta_s\eta_b]^3 \end{array} \end{array} $

to zero to obtain the equilibrium displaying the firm's expected profit under the new plan,

$$\mathbb{E}[\Pi^*] = \underbrace{p \frac{(1 - \gamma_b)(V_s - p) + \eta_s V_b}{(1 - \gamma_b)(1 - \gamma_s) - \eta_s \eta_b}}_{\text{Contributions of Stand-Alone Values}} + \underbrace{\frac{p^2(1 - \gamma_b)^2}{2[(1 - \gamma_b)(1 - \gamma_s) - \eta_s \eta_b]^2(1 + \rho \sigma_s^2)}}_{\text{Contribution of Selling}}.$$
(B12)

As a result, the difference between the equilibrium profit under this plan and the equilibrium profit under the traditional compensation plan is

$$\begin{split} \delta_{\Pi} = & \frac{[p(1-\gamma_b)\eta_s\sigma_b]^2}{2[(1-\gamma_b)(1-\gamma_s)-\eta_b\eta_s]^2[(1-\gamma_b)^2(1+\rho\sigma_s^2)+\rho\eta_s^2\sigma_b^2]} \\ \times & \frac{\rho}{1+\rho\sigma_s^2} \! >\! 0, \end{split}$$

which is always positive.

Finally, the sensitivity analyses of profit under the new plan with respect to network effects are provided in Table B1, where Ω_1 and Ω_2 are the contributions of the stand-alone values and of selling to the firm's profit, respectively, as provided previously.

Proof of P₅

(B13)

The first-order condition for the agent's effort yields a unique solution, which is a maximum since the secondorder derivative is -1. We then proceed by replacing the agent's optimal effort strategy in the agent's expected utility function to identify α_0 such that the IR condition is met and then replace the resulting expression for α_0 and the optimal effort strategy in the firm's profit function. We then differentiate the firm's profit with respect to p, α_1 , and α_2 and equate the resulting expressions to zero to obtain the equilibrium displayed in P₅, where

$$\mathbf{P}_{1} = \frac{[(1 - \gamma_{b})(1 - \gamma_{s}) - \eta_{b}\eta_{s}](1 + \rho\sigma_{s}^{2})}{(1 - \gamma_{b})\{(1 - \gamma_{b})(1 - 2\gamma_{s}) - 2\eta_{b}\eta_{s} + 2[(1 - \gamma_{b})(1 - \gamma_{s}) - \eta_{b}\eta_{s}]\rho\sigma_{s}^{2}\}}.$$
(B14)

We then proceed in two steps to verify that this point is a maximum. First, we check for the optimality of the commission rates for any price, and then we check for the optimality of the pricing strategy given the equilibrium commission rates. For any p, the second-order conditions for the commission rates are met since the determinant of the Hessian is positive and the second-order derivative of profit with respect to α_1 is negative. Given the optimal commission rates, the second-order condition with respect to p is met when $(1 - \gamma_b)(1 - \gamma_s) - \eta_b \eta_s > (1 - \gamma_b)/[2(1 + \rho \sigma_s^2)]$, which (knowing that $\alpha_1 < 1$ in a valid plan) always holds with $\gamma_s < 1/2$.

Proof of P₆

The agents' first-order conditions yield the effort strategies provided in Equation 19, which are maxima since the second-order conditions for the agents' optimization problems yield -1. The principal's first-order conditions yield the commission rates provided in P₆. Given the mathematical structure

of the principal's optimization problem, we can analyze the optimality of $(\alpha_1^*; \alpha_2^*)$ independently from the optimality of $(\beta_1^*; \beta_2^*)$. The determinant of the Hessian of the profit function with respect to $(\alpha_1^*; \alpha_2^*)$ is positive and $\partial^2 \mathbb{E}[\Pi] / \partial \alpha_1^* \partial \alpha_1^*$ is negative when $(2 - \gamma_b) \gamma_b < [1 + \rho(\eta_s \sigma_b^2 + \sigma_s^2)] / (1 + \rho \sigma_s^2)$, which holds true for $\gamma_b < 1$. Thus, $(\alpha_1^*; \alpha_2^*)$ is a maximum. Similarly, the Hessian of the profit function with respect to $(\beta_1^*; \beta_2^*)$ is positive, and $\partial^2 \mathbb{E}[\Pi] / \partial \beta_1^* \partial \beta_1^*$ is always negative. Thus, $(\beta_1^*; \beta_2^*)$ is a maximum.

Author Contributions

The authors are listed alphabetically and have contributed equally.

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