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BidAnalyzer: A Method for Estimation and Selection of Dynamic Bidding Models

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Online reverse auctions generate real-time bidding data that could be used via appropriate statistical estimation to assist the corporate buyer's procurement decision. To this end, we develop a method, called BidAnalyzer, which estimates dynamic bidding models and selects the most appropriate of them. Specifically, we enable model estimation by addressing the problem of *partial observability*; i.e., only one of N suppliers' bids is realized, and the other $(N-1)$ bids remain unobserved. To address partial observability, BidAnalyzer estimates the latent price distributions of bidders by applying the Kalman filtering theory. In addition, BidAnalyzer conducts model selection by applying multiple information criteria. Using empirical data from an automotive parts auction, we illustrate the application of BidAnalyzer by estimating several dynamic bidding models to obtain empirical insights, retaining a model for forecasting, and assessing its predictive performance in out-of-sample. The resulting one-step-ahead price forecast is accurate up to 2.95% median absolute percentage error. Finally, we suggest how BidAnalyzer can serve as a device for price discovery in online reverse auctions.

Key words: competitive bidding; electronic commerce; Internet auctions; Kalman filtering; reverse auctions; supplier sourcing; e-procurement
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1. Introduction

More than 30% of the major companies who annually purchase more than $100 million use online reverse auctions for procurement (Beall et al. 2003). These buyers belong to various sectors, ranging from manufacturing to consumer goods to high-tech. In these auctions, suppliers bid (instead of buyers), pushing prices down (instead of up) over time and generating real-time bidding data. The resulting data could be used for decision making, but managers need "...statistical estimation techniques...for effectively mining...data generated by auctions to predict behavior and to dynamically improve operational decisions" (Pinker et al. 2003, p. 1,481). To this end, we develop a method, which we call *BidAnalyzer*, for assisting a corporate buyer's procurement decision. Specifically, our research objective is to develop a method for (a) estimating dynamic bidding models and (b) selecting an appropriate one from models with different drift specifications or cross-bidder dependence.

An inherent feature of bidding data from online reverse auctions is *partial observability*; i.e., the buyer observes a bid price from one of the suppliers at a point in time but no bids from the rest of the participating suppliers. Because only one of N suppliers' bids is observable, there are $(N-1)$ missing observations every period. If each supplier's bid price were observed every period, we could have applied standard time-series models (e.g., vector autoregressions) to understand the dynamics of bid prices over time. However, the data-generating process in online reverse auction does not comport with this implicit assumption (of complete observability) made in standard time-series models (namely, all elements of the multivariate dependent variables are observable in every period). Hence, to address partial observability, state-space models provide a reasonable starting point, and so we apply Kalman filtering theory (see, e.g., Durbin and Koopman 2001 for details). Furthermore, we apply information theory (see, e.g., Burnham and Anderson 2002) to conduct nonnested model selection.

To illustrate the application of BidAnalyzer, we use bidding data from an automotive reverse auction in which ten suppliers competed in nine different lots and the total transaction was valued at $8.2 million at the final bid prices. These bidding data serve as inputs to BidAnalyzer, which estimates several dynamic bidding models. Based on the estimated models, we obtain empirical insights as follows: bid prices follow the random walk with a negative drift, revealing that the suppliers' valuations evolve over time rather than remain fixed (i.e., the latent price distribution is nonstationary). In other words, each supplier adjusts its latent price distribution downwards based on the bid prices observed thus far across all the suppliers and not just its own prices. The drift rate within lots can depend on the time elapsed since the start, the interbid hiatus, and the remaining time until the auction ends. The drift rate between lots varies proportional to the average price drop but not the number of bidders. In addition, the bidding patterns exhibit cross-supplier dependence, which can be detected by the BidAnalyzer across a few specific subsets of suppliers.

Finally, to assess predictive performance, we compare one-step-ahead price forecasts with the actual bid prices. Specifically, we estimate the model parameters using bidding data from other lots excluding the data from the lot under consideration, which serves as the holdout sample. Then we generate one-step-ahead price forecasts for each lot iteratively. For lots with high or low valuations, respectively, Figures 2 and 3 present the BidAnalyzer's forecasts and actual prices. Overall, the price forecast is accurate up to 2.95% median absolute percentage error.

Given that BidAnalyzer infers the latent price distributions for every supplier, managers could embed this information in an optimization algorithm for price discovery in online reverse auctions. This capability would enable the buyer to understand whether a given supplier has bid aggressively or held back cautiously. In the latter case, the supplier may become a candidate for postauction discussions. Note that a buyer typically does not ask every bidder to lower the prices further after the auction ends because this action is perceived as unethical. Hence, buyers need to discover a subset of candidate suppliers whose final bids exceed the prices that maximize the suppliers' expected profit and then approach them selectively.

The rest of the paper is organized as follows. Section 2 reviews the literature, and §3 describes the auction format. Section 4 provides a nontechnical overview of how BidAnalyzer works, and §5 presents the mathematical details of distributional inference, parameter estimation, and model selection. Section 6 illustrates an empirical application. Section 7 discusses extensions of BidAnalyzer and identifies avenues for further research.

2. Literature Review

Extant research in the auctions literature tends to ignore price dynamics within the course of the auction, focusing instead on the final bid (for reviews, see Klemperer 1999, McAfee and McMillan 1987) or summary statistics of bidding behavior such as the number of bids (Ariely and Simonson 2003, Wilcox 2000), the rate of bid submission (Häubl and Popkowski-Leszczyc 2006, Katok and Kwasnica 2002, Tuunainen et al. 2001), and price concessions (Ariely and Simonson 2003, Heyman et al. 2004, Ku et al. 2005). These approaches aggregate over the temporal dimension and explain cross-sectional variation across various auctions.

In marketing, research on price dynamics within auctions is emerging. For example, Bapna et al. (2004) apply functional data analysis to online consumer auctions. They estimate the first and second derivatives (i.e., velocity and acceleration) of the price path over time and relate them to auction characteristics at various stages of the auction. For instance, they find that the rate of price increase due to the entry of an additional bidder is smaller at the end rather than the beginning of the auction. In the BidAnalyzer, we will incorporate this role of price velocity via the drift rate, whereas acceleration can be specified by the first difference in drift rate (see Footnote 2).

In a recent study, Park et al. (2003) develop a model to understand consumer bidding behavior in Internet auctions (also see Bradlow and Park 2007). In such consumer auctions, entry is open to any consumer; approximately 44% of the bids arrive after 90% of an auction's duration has passed; and 90% of the auctions have a "buy-it-now" feature (i.e., any bidder can stop the auction immediately and buy the good for the stated price), which becomes one of the key determinants of the final price. Structurally, this auction design and the resulting data generation differ from those of business-to-business reverse auctions, where only qualified suppliers are selected for participation; bids arrive throughout an auction's duration with no snipping because of soft close (i.e., moving end time); and the buy-it-now pricing is not available. Thus, business-to-business reverse auctions are qualitatively different from consumer auctions, and, to best of our knowledge, this study marks the first application of state-space models to predict price dynamics within an online reverse auction.

3. The Online Reverse Auction Context

Jap (2002) provides a general overview of the conditions, structure, and evaluation of online reverse auctions. Several studies describe the implementation of the online reverse auction process from preliminary steps to contract awards (see Emiliani 2000, Mabert and Skeels 2002, Stein et al. 2003).

In this study, the online reverse auction commences after the buyer qualifies—through visits, questionnaires, and research—the short list of viable suppliers. Then the auctioneer, who hosts and manages the event, posts the buyer's request for purchase (RFP) on a website and invites the qualified suppliers to respond to the RFP. The RFP contains the product and delivery specifications, contract expectations, and the rules of the auction event as follows: supplier bids are legally binding, the buyer reserves the right to select the final winner, and the lowest bid price is not guaranteed to win the contract. The buyer also commits not to bid against suppliers in the auction (an unethical practice known as "shilling"). The suppliers do not know who their competitors are or how many suppliers will bid against them. The products are neither pure commodities nor highly customized strategic parts. All the products are used in production or for parts in production. The buyer creates subgroups of multiple items, called lots, which represent a set of items grouped according to the suppliers' capabilities to produce or based on similarities in manufacturing processes or delivery regions. Moreover, the buyer expects that suppliers have the capability to produce all of the items within that lot, and so each lot is auctioned independently. In other words, no bids are accepted for partial lots or individual parts within a lot.

Once the auction starts, suppliers view their competitor's bid price and respond in real time. The auction for each lot ends with a soft close, which means that the closing time extends for a few minutes to allow other suppliers to respond if any bid arrives in the last minute of the closing time (i.e., no snipping). Bidders see only the price bids of competing bidders; they do not see other nonprice factors of competing bidders.[1] After the auction, the buyer evaluates the individual bids and considers their nonprice attributes of interest. In the subsequent four to six weeks, the buyer reviews the bids with other functional units in the organization and may negotiate prices with some subset of suppliers. Finally, the buyer notifies all suppliers whether they won or lost the auction.

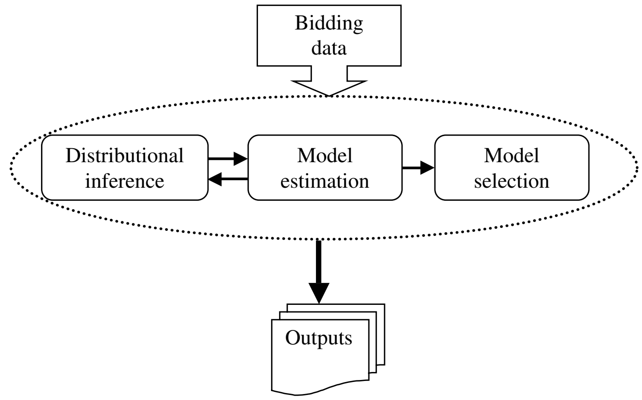
4. How BidAnalyzer Works

Figure 1 presents the schematic representation of BidAnalyzer, which takes the bidding data as inputs, analyzes them via the three modules—distributional inference, model estimation, and model selection—and creates several outputs. Below we explain the three modules in turn without technical details, which are provided in the next section.

4.1. Bidding Data

To characterize the main properties of the data, in Table 1, we display the bid prices from a real online reverse auction, which attracted 19 suppliers, labeled as $\{S_1, S_2, \ldots, S_{19}\}$, who competed for the contract worth more than one million dollars. This auction commenced with an opening bid of \$1,809,970,

Figure 1 Schematic Representation of BidAnalyzer



led to decreasing bid prices over time, generated a total of 45 bids, although not all suppliers submitted their bids, and ended with the minimum bid of \$1,478,880. The drop in price was \$331,090 (i.e., approximately 22%).

The sequence of bid prices exemplifies the fundamental problem of *partial observability*; i.e., only one bidder of the 19 bidders submits a bid at any point in time, t. More specifically, for time point $t = 1$ in Table 1, the fifteenth bidder (i.e., S_{15}) quoted \$1,809,970, while all other 18 suppliers present in this auction at that time did not submit bids. This feature of *partial observability*—we observe only one bid price from some bidder S_j ($j = 1, \ldots, N = 19$) and do not observe bids from the other suppliers—holds at every time point ($t = 1, \ldots, 45$). By denoting $\times$ for no bidding activity, Table 1 shows the bidding pattern from the supplier S_1's point of view. Partial observability is a critical feature of the input data because it impacts our ability to infer the probability distribution from which a supplier's bid price arises, as we explain next.

4.2. Distributional Inference

For clarity, consider the first time point and suppose a supplier j's bid arises from the normal distribution, $N(\mu_j, \sigma_j^2)$. Based on the first observed bid of \$1,809,970, what can we say about the bidder's (i.e., S_{15}) latent price distribution $N(\mu_{15}, \sigma_{15}^2)$ that generated this outcome? Using sample moments as estimators, we can say that $\hat{\mu}_{15} = 1{,}809{,}970$, but we cannot estimate σ_{15}^2 with just one observation. In addition, we can say nothing about the nonbidders' price distributions $N(\mu_{j'}, \sigma_{j'}^2)$ for every j' other than $j = 15$. Thus, because of partial observability, standard time-series techniques such as autoregressive integrated moving average (ARIMA) or vector autoregression (VARs) are unable to infer these price distributions. To resolve this problem, we apply state-space methods (see Durbin and Koopman 2001) to characterize the latent price distribution of every supplier at each point in time (for details, see §5.2).

[1] For research on combinatorial bidding, see Hohner et al. (2003), Metty et al. (2005), Sandholm et al. (2006), and Sheffi (2004).

Table 1 A Typical Bidding Event in an Online Reverse Auction

Time point t	Price (in thousands of dollars)	Supplier 1	Other suppliers
1	1,809.97	×	S_{15}
2	1,710.65	×	S_4
3	1,763.84	×	S_8
4	1,798.19	×	S_2
5	1,769.78	×	S_{11}
6	1,655.45	×	S_{10}
7	1,683.45	×	S_6
8	1,651.32	×	S_4
9	1,808.09	×	S_{14}
10	1,647.19	×	S_{10}
11	1,643.07	×	S_4
12	1,625	×	S_9
13	1,620.94	×	S_4
14	1,689.34	×	S_{11}
15	1,789.97	×	S_{15}
16	1,626.74	×	S_2
17	1,616.84	×	S_{10}
18	1,612.79	×	S_4
19	1,606.35	×	S_9
20	1,602.33	×	S_4
21	1,590	S_1	×
22	1,586.03	×	S_4
23	1,578.44	×	S_{10}
24	1,574.5	×	S_4
25	1,570.56	×	S_{10}
26	1,566.64	×	S_4
27	1,562.72	×	S_{10}
28	1,666.62	×	S_6
29	1,558.81	S_1	×
30	1,554.92	×	S_{10}
31	1,551.03	S_1	×
32	1,658.3	×	S_6
33	1,547.15	×	S_{10}
34	1,543.28	S_1	×
35	1,539.43	×	S_4
36	1,501.26	×	S_{10}
37	1,497.5	S_1	×
38	1,650.02	×	S_6
39	1,493.76	×	S_{10}
40	1,490.02	S_1	×
41	1,486.3	×	S_{10}
42	1,645.89	×	S_6
43	1,482.58	S_1	×
44	1,478.88	×	S_{10}
45	1,641.78	×	S_6
End			

4.3. Model Estimation

The capability to infer distributions is critical for computing the likelihood function required for estimating dynamic bidding models. In other words, model estimation cannot proceed via only the observed bidding data; it depends also on the probability distributions of bid prices. In Figure 1, we show this dependence by the forward arrow (→) between distributional inference and model estimation. Conceptually, the capability to infer the price distributions allows BidAnalyzer to progress by estimating hypothesized dynamic models. To be able to infer price distributions, however, we need to know the parameter values of a hypothesized model. Because we do not know the parameter values a priori, we iterate between the inference and estimation modules as indicated by the backward arrow (←) in Figure 1. At convergence, we obtain both the inferred distributions and the estimated models (for details, see §5.3).

4.4. Model Selection

Applying BidAnalyzer to actual bidding data, buyers can estimate several dynamic models $\{M_1, M_2, \ldots, M_L\}$, where M_l denotes a dynamic bidding model ($l = 1, \ldots, L$). To select a model, we balance the trade-off between parsimony (i.e., simple model) and fidelity (i.e., best-fitting model) by minimizing the Kullback–Leibler (K-L) distance between the candidate and true models (see §5.4). As Figure 1 shows, the module for model selection advances us further from model estimation to the retention of a specific model for decision making.

4.5. Outputs

The three modules—inference, estimation, and selection—together comprise BidAnalyzer, which takes bidding data as inputs and creates the following outputs: (i) price distributions for every bidder in each point in time; (ii) parameter estimates and their significance (see §§5.3 and 6.2); and (iii) a specific retained model (see §§4.4 and 5.4). In practice, managers can embed the latent price distributions from (i) in an optimization algorithm to discover optimal prices for each supplier and then make informed procurement decisions.

5. Method Development

In this section, we present the mathematical details of BidAnalyzer in the following order: dynamic bidding models, distributional inference, model estimation, and model selection.

5.1. Bidding Dynamics

Here we address the partial observability problem inherent in bidding data. Let N suppliers participate in an online reverse auction, and, at any time t, a supplier j submits a bid p_t. This observed price is a noisy signal from the supplier j's latent price distribution π_t^j so that

$$p_t = \pi_t^j + \varepsilon_t, \tag{1}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. We can think of π_t^j as the supplier j's unobserved valuation at time t. We reexpress this equation as

$$p_t = z_t' \alpha_t + \varepsilon_t, \tag{2}$$

where the indicator vector $z_t = (0, \ldots, 1, \ldots, 0)'$ has unity at the jth element (for the bidder j) and zeros elsewhere for the nonbidders, and $\alpha_t = (\pi_t^1, \ldots, \pi_t^N)'$ is the $N \times 1$ vector containing the latent price distributions of all the suppliers.

The zeros in z_t capture the phenomenon of partial observability. Specifically, when the jth supplier submits its bid, its price distribution π_t^j is partly revealed via the observed price, whereas the price distributions of all other suppliers remain hidden because they did not bid at time t. More precisely, the partial observability of vector α_t means that one element of α_t is observed, while the other $(N-1)$ elements are not. Furthermore, the composition of vector z_t is time-varying because different suppliers bid at different instants.

Because observed price declines over time in reverse auctions, we formulate the following specification (which can be modified as shown in the appendix):

$$\pi_t^j = \pi_{t-1}^j + \delta + \nu_t^j, \quad j = 1, \ldots, N, \tag{3}$$

where δ is the drift rate and the error vector $\nu_t \sim N(0, \Sigma_\nu)$. The specification $\Sigma_\nu = \sigma_\nu^2 I$ implies independence across bidders; we capture correlated bidding by relaxing off-diagonal elements to be nonzero (see appendix).

We expect the drift rate δ to be negative in reverse auctions. However, if the drift in bid prices is not constant, we can replace it by heterogeneous drifts across lots or by time-varying drift within a lot.[2] For example, bidding can be brisk at the start of an auction, meanders in the middle, and becomes aggressive near the end. BidAnalyzer can incorporate such dynamic effects (see appendix).

Equation (3) constitutes a set of multiple time-series, not just one scalar equation. So we reexpress (3) as the system of N equations

$$\begin{bmatrix} \pi_t^1 \\ \vdots \\ \pi_t^j \\ \vdots \\ \pi_t^N \end{bmatrix} = \begin{bmatrix} \pi_{t-1}^1 \\ \vdots \\ \pi_{t-1}^j \\ \vdots \\ \pi_{t-1}^N \end{bmatrix} + \begin{bmatrix} \delta \\ \vdots \\ \delta \\ \vdots \\ \delta \end{bmatrix} + \begin{bmatrix} \nu_t^1 \\ \vdots \\ \nu_t^j \\ \vdots \\ \nu_t^N \end{bmatrix},$$

which permits the compact vector representation

$$\alpha_t = \alpha_{t-1} + \delta + \nu_t. \tag{4}$$

[2] Note that the drift rate, which equals the change in latent prices, can be viewed as velocity (Bapna et al. 2004). Consequently, the acceleration, which equals the change in velocity, can be specified by $\delta_t = \delta_{t-1} + X'\gamma + \omega_t$, and these additional dynamics can be subsumed within the state-space model by augmenting the state vector to be $\tilde{\alpha}_t = (\alpha_t', \delta_t')'$.

Next, we explain how the buyer learns about the suppliers' evolving valuation (i.e., the latent price distributions) as the auction progresses.

5.2. Inferring Price Distributions

The bidding activity generates information on the bid amount and bidder's identity, which are denoted by (p_t, z_t'), respectively. As the auction progresses, this information accumulates in the information set $\Im_t = \{(p_1, z_1'), (p_2, z_2'), \ldots, (p_t, z_t')\}$. How should we use this information in $\Im_t$ to infer price distributions in $\alpha_t = (\pi_t^1, \ldots, \pi_t^N)'$?

To this end, let $\alpha_0 \sim N(\mu_0, \Sigma_0)$ denote the initial distribution, where μ_0 and Σ_0 are the vector of means and covariance matrix, respectively. Before the first bid arrives, at $t = 1$, the moments of α_1 can be obtained from (4) as follows:

$$\begin{aligned} \mu_{1|0} &= \mathrm{E}[\alpha_1 \mid \Im_0] = \mu_0 + \delta_0 \\ \Sigma_{1|0} &= \mathrm{Var}[\alpha_1 \mid \Im_0] = \Sigma_0 + \Sigma_\nu, \end{aligned} \tag{5}$$

where $(\mu_{1|0}, \Sigma_{1|0})$ are the means and covariance of α_1 based on the information set $\Im_0$.[3]

After the first bid arrives at $t = 1$, we learn the bid amount and the bidder's identity, (p_1, z_1), which we use to update the above moments of α_1

$$\begin{aligned} \mu_{1|1} &= \mathrm{E}[\alpha_1 \mid \Im_1] = \mu_{1|0} + \lambda_1 (p_1 - z_1' \mu_{1|0}) \\ \Sigma_{1|1} &= \mathrm{Var}[\alpha_1 \mid \Im_1] = (I - \lambda_1 z_1') \Sigma_{1|0}, \end{aligned} \tag{6}$$

where $\Im_1 = \Im_0 \cup (p_1, z_1)$ is the augmented information set, and $\lambda_1 = \Sigma_{1|0} z_1 / (z_1' \Sigma_{1|0} z_1 + \sigma_\varepsilon^2)$ is an $N \times 1$ vector.

The intuition for this updating is as follows: Adjust the means $\mu_{1|0}$ proportional to the difference between observed and expected bid prices $(p_1 - z_1'\mu_{1|0})$. The vector λ_1 contains the proportionality constants, which follow from the Bayes theorem (see, e.g., Durbin and Koopman 2001).

At the end of the first time point, the price distribution for all of the suppliers—not just the one who submitted the bid—is given by $\alpha_1 \sim N(\mu_{1|1}, \Sigma_{1|1})$. Before the second bid arrives, the moments of α_2 conditional on the known information in $\Im_1$ are given by

$$\begin{aligned} \mu_{2|1} &= \mathrm{E}[\alpha_2 \mid \Im_1] = \mu_1 + \delta_1 \\ \Sigma_{2|1} &= \mathrm{Var}[\alpha_2 \mid \Im_1] = \Sigma_1 + \Sigma_\nu. \end{aligned} \tag{7}$$

As the new bid arrives at $t = 2$, (p_2, z_2), we update the moments via Bayes theorem as follows:

$$\begin{aligned} \mu_{2|2} &= \mathrm{E}[\alpha_2 \mid \Im_2] = \mu_{2|1} + \lambda_2 (p_2 - z_2' \mu_{2|1}) \\ \Sigma_{2|2} &= \mathrm{Var}[\alpha_2 \mid \Im_2] = (I - \lambda_2 z_2') \Sigma_{2|1}, \end{aligned} \tag{8}$$

where $\Im_2 = \Im_1 \cup (p_2, z_2)$ and $\lambda_2 = \Sigma_{2|1} z_2 / (z_2' \Sigma_{2|1} z_2 + \sigma_\varepsilon^2)$.

[3] In the subscript $1 \mid 0$, the index before the vertical slash refers to the time point for the random variable α, and the index after the slash indicates the time point for the information set $\Im$ used in evaluating the expectations.

Over the course of the entire auction, bids arrive at time points $t = 1, \ldots, T$, and we process them recursively to *learn* about the mean and covariance of α_t via the four equations

$$\begin{aligned}\mu_{t\mid t-1} &= \mathrm{E}[\alpha_t \mid \Im_{t-1}] = \mu_{t-1} + \delta_{t-1}\\ \Sigma_{t\mid t-1} &= \mathrm{Var}[\alpha_t \mid \Im_{t-1}] = \Sigma_{t-1} + \Sigma_\nu\\ \mu_{t\mid t} &= \mathrm{E}[\alpha_t \mid \Im_t] = \mu_{t\mid t-1} + \lambda_t(p_t - z_t'\mu_{t\mid t-1})\\ \Sigma_{t\mid t} &= \mathrm{Var}[\alpha_t \mid \Im_t] = (I - \lambda_t z_t')\Sigma_{t\mid t-1},\end{aligned} \tag{9}$$

where $\lambda_t = \Sigma_{t\mid t-1} z_t / (z_t'\Sigma_{t\mid t-1} z_t + \sigma_\varepsilon^2)$. This set of four equations in (9) yields the distributional inference on α_t for all t.

In sum, as the reverse auction progresses, we receive information sequentially on the bid amount and bidder identity $\{(p_t, z_t')\colon t = 1, \ldots, T\}$. Using (9), we infer the sequence of moments $\{(\mu_{t\mid t}, \Sigma_{t\mid t})\colon t = 1, \ldots, T\}$, thus learning about the latent price distributions for all of the suppliers. Thus, for each supplier in every period, a supplier's valuation (i.e., its latent price distribution) depends on the *bids of all of the suppliers thus far* and not just its own bids. To see this point, note that the conditioning in (9) uses the information set $\Im_t$, which equals $\Im_t = \Im_{t-1} \cup (p_t, z_t')$, and, on recursive substitution, it yields the entire history of prices and bidders: $\Im_t = \Im_0 \cup \{(p_t, z_t'), (p_{t-1}, z_{t-1}'), \ldots, (p_2, z_2'), (p_1, z_1')\}$.

In practice, the buyer does not know the magnitudes of the initial means, the drift rates, or the error variances, and hence BidAnalyzer estimates them using actual bidding patterns observed in a given auction, as we describe next.

5.3. Model Estimation

A typical auction consists of several lots, $k = 1, \ldots, K$. A lot represents a grouping of individual parts created by the buyer based on factors such as similarities in manufacturing processes, size, or location. In each lot k, suppliers' bidding activity generates the price trajectory $(p_{1k}, p_{2k}, \ldots, p_{Tk})'$. Let $f(p_{1k}, p_{2k}, \ldots, p_{Tk})$ denote the joint density function and $f(p_{tk} \mid (p_{1k}, \ldots, p_{t-1,k})) = f(p_{tk} \mid \Im_{t-1,k})$ be the conditional density function.[4] Then, the likelihood function is given by

$$\begin{aligned}L_k &= f(p_{1k}, p_{2k}, \ldots, p_{Tk})\\ &= f(p_{1k}) f(p_{2k} \mid p_{1k}) f(p_{3k} \mid (p_{1k}, p_{2k}))\\ &\quad \cdots f(p_{Tk} \mid (p_{1k}, \ldots, p_{T-1,k}))\\ &= f(p_{1k} \mid \Im_{0,k}) f(p_{2k} \mid \Im_{1,k}) f(p_{3k} \mid \Im_{2,k}) \cdots f(p_{Tk} \mid \Im_{T-1,k})\\ &= \prod_{t_k=1}^{T_k} f(p_{tk} \mid \Im_{t-1,k}),\end{aligned} \tag{10}$$

where T_k is the total number of bids in lot k, and the set $\Im_{t,k} = \{(p_{1k}, z_{1k}'), (p_{2k}, z_{2k}'), \ldots, (p_{tk}, z_{tk}')\}$ contains information up to time t in lot k. The information set $\Im_{0k}$ contains the initial means μ_{0k}, which differ across the nine lots but are constant across bidders.

Next, to find the moments of $f(p_{tk} \mid \Im_{t-1,k})$, we make use of the result (9) and model (2). Specifically, the mean and variance, respectively, are given by $\mathrm{E}[p_{tk} \mid \Im_{t-1,k}] = z_{tk}'\mu_{t\mid t-1,k}$ and $\mathrm{Var}[p_{tk} \mid \Im_{t-1,k}] = z_{tk}'\Sigma_{t\mid t-1,k} z_{tk} + \sigma_\varepsilon^2$, which fully characterizes the normal density $f(p_{tk} \mid \Im_{t-1,k})$. Hence, we can compute the likelihood L_k via (10), which we sum across all the lots to compute the total log-likelihood:

$$L(\Theta) = \sum_{k=1}^{K} \mathrm{Ln}(L_k), \tag{11}$$

where the parameter vector Θ contains

$$\{(\mu_{01}, \ldots, \mu_{0K})', \delta, \sigma_\varepsilon^2, \Sigma_\nu\}.$$

Finally, we maximize (11) with respect to Θ to obtain the maximum-likelihood estimates

$$\hat{\theta} = \arg\max_{\Theta} L(\Theta). \tag{12}$$

The standard errors are given by the square root of diagonal elements of the covariance matrix

$$\mathrm{Var}(\hat{\theta}) = \left[-\frac{\partial^2 L(\Theta)}{\partial\Theta\,\partial\Theta'}\right]^{-1}_{\Theta=\hat{\theta}}, \tag{13}$$

where the Hessian of $L(\Theta)$ is evaluated at the estimated values $\hat{\theta}$. We close this section by describing BidAnalyzer's model selection module.

5.4. Information-Theoretic Model Selection

A buyer can specify several dynamic models $\{\mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_L\}$ and estimate each one of them via the above estimation approach. To retain a specific model, we compute the Kullback–Leibler (K-L) distance metric and select the model that attains the smallest value (for further details, see Burnham and Anderson 2002). An approximation of the K-L metric is given by Akaike's information criterion $\mathrm{AIC} = -2L^* + 2p$, where L^* is the maximized log-likelihood value and p is the number of parameters in Θ. As model complexity increases, both L^* and p increase; thus, AIC balances the trade-off between goodness-of-fit and parsimony.

[4] The proposed approach can be adapted to estimate continuous-time models using discrete-time data by integrating the transition equations in (4) forward in time until the data are observed; for details, see Naik et al. (2005, §4, p. 29).

Table 2 Parameter Estimates for the Constant Drift Model

Parameters	Estimates	t-values
Drift δ	−13.13	−13.5
Initial means		
Lot 1, π_{01}	1,346.73	59.5
Lot 2, π_{02}	136.96	2.55
Lot 3, π_{03}	2,668.67	61.4
Lot 4, π_{04}	108.94	2.12
Lot 5, π_{05}	1,446.61	27.6
Lot 6, π_{06}	113.28	2.15
Lot 7, π_{07}	3,484.63	77.3
Lot 8, π_{08}	230.80	5.43
Lot 9, π_{09}	3,577.44	73.0
Error variance, σ_ε	0.51×10^{-3}	0.08
Error variance, σ_ν	35.65	28.4

Because AIC ignores the sample size $B = \sum_k T_k$, we use its bias-corrected version for finite samples (Hurvich and Tsai 1989) and the Bayesian information criterion.[5] By using multiple criteria, we gain convergent validity when these criteria retain the same model, thus enhancing confidence. If multiple criteria retain different models, however, then AIC_C is preferred when the p/B ratio is large (i.e., many parameters or small sample sizes) and BIC when the p/B ratio is small (see Naik et al. 2007, Remark 2 and Table 2). The next section illustrates the application of BidAnalyzer to real bidding data.

6. Empirical Illustration

6.1. Bidding Data

A major parts manufacturer in the automotive industry provided the bidding data for this study. This company operates in 25 countries with sales revenue exceeding $15 billion. It wanted to buy metal parts for use in its production process and invited ten qualified suppliers to participate in the online auction. These ten suppliers competed in nine different lots and generated 403 bids, and this transaction was valued at $8.2 million at the final bid prices.

6.2. Estimation of Model Parameters

We first estimate the model specified by Equations (2) and (3), assuming bidders are identical and independent (i.e., $\Sigma_\nu = \sigma_\nu^2 I$). We later consider the general case of correlated bidding dynamics (see appendix). Table 2 reports the parameter estimates and t-values, which were obtained using Equations (12) and (13). All parameter estimates, except for one of the error variances, are significant at the 95% confidence level. For example, the initial means $\hat{\pi}_{0k}$ for each lot k are significant; they represent the bidders' common valuation for each lot. In addition, the estimated drift δ is negative and significant in a one-tailed test. That is, bid prices decrease by a fixed amount as the auction evolves such that the longer the bidding lasts, the lower the final bid prices. Given the average bid price of $1,314,312, this estimated drift indicates that bid prices drop by 1% for every additional bid. Thus bid prices follow the random walk with a negative drift, revealing that the suppliers' valuations evolve over time rather than remain fixed.

6.3. Cross-Validation of Price Forecasts

To assess BidAnalyzer's predictive performance, we reestimate the constant drift model for the high-value lots (>$1 million) and low-value lots (<$100,000). Specifically, for each lot type (low or high value), we use bidding data from all similar lots except the one under consideration, which serves as the holdout sample. In other words, we generate bid-price forecasts in the lot under consideration without using bidding data from that lot to estimate the model's parameters. The initial mean for a lot is its opening bid (or one could set it to the current price of an incumbent supplier). The forecasted price is the weighted average of the expected prices from all bidders, where the weights depend on their bidding frequency[6] and the expected prices are based on the prior mean $\mu_{t|t-1}$ given in Equation (9). We also forecast the 95% price interval based on the prior covariance matrix $\Sigma_{t|t-1}$ given in Equation (9).

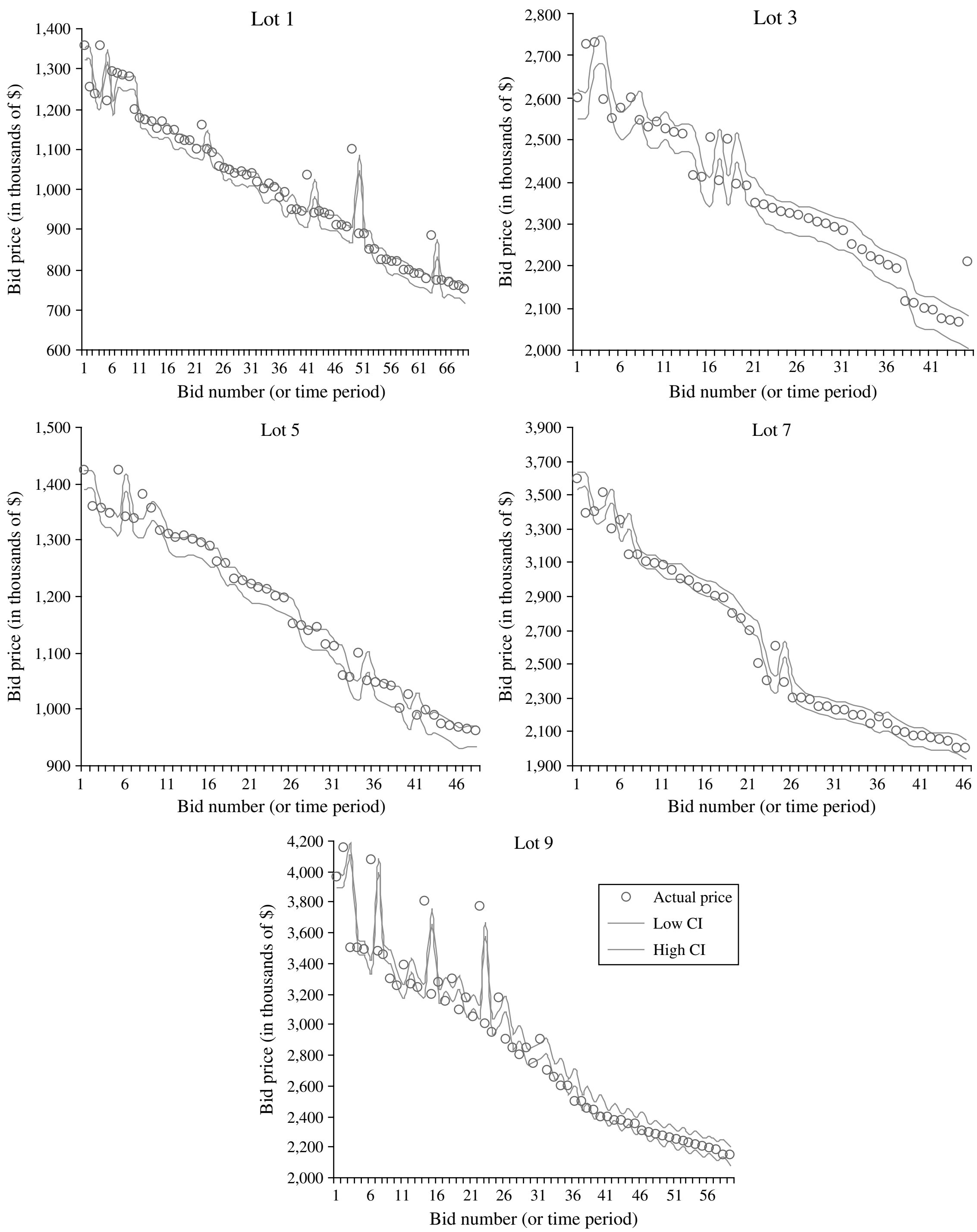
Using the reestimated model, we next forecast the price we expect to observe. As the actual bid arrives, we realize the inaccuracy of BidAnalyzer's forecast. So we update the price distribution for each bidder (using Equation (9)) and forecast again both the bid price and price interval for the next time point. We repeat the two steps—forecast and update—until the bidding in that lot ends.

Figures 2 and 3 display the predicted 95% price interval *before* the actual bid was observed for high- and low-value lots, respectively. For comparison, we also show the actual bid price. Initially, BidAnalyzer misses a few bids, from which it learns; later on, it tracks even the turning points quite well. It is important to emphasize that, when estimating the model parameters, we did not use the price information in the lot under consideration. Yet the actual bid prices are usually within the predicted price interval. Moreover, the median absolute percentage deviation between the forecasts and actual prices is 2.95%. Finally, to obtain empirical insights, BidAnalyzer can

[5] The bias-corrected $\text{AIC}_C = -2L^* + B(B+p)/(B-p-2)$ and $\text{BIC} = -2L^* + p\text{Ln}(B)$.

[6] The weights are given by $w_{jt} = (s_{jt}+1)/(n_t+10)$, where s_{jt} is the number of times the bidder j cast a bid out of the total number of bids n_t observed thus far at time t, $t = 1, \ldots, T_k$.

Figure 2 BidAnalyzer's Out-of-Sample Forecasts in High-Value Lots

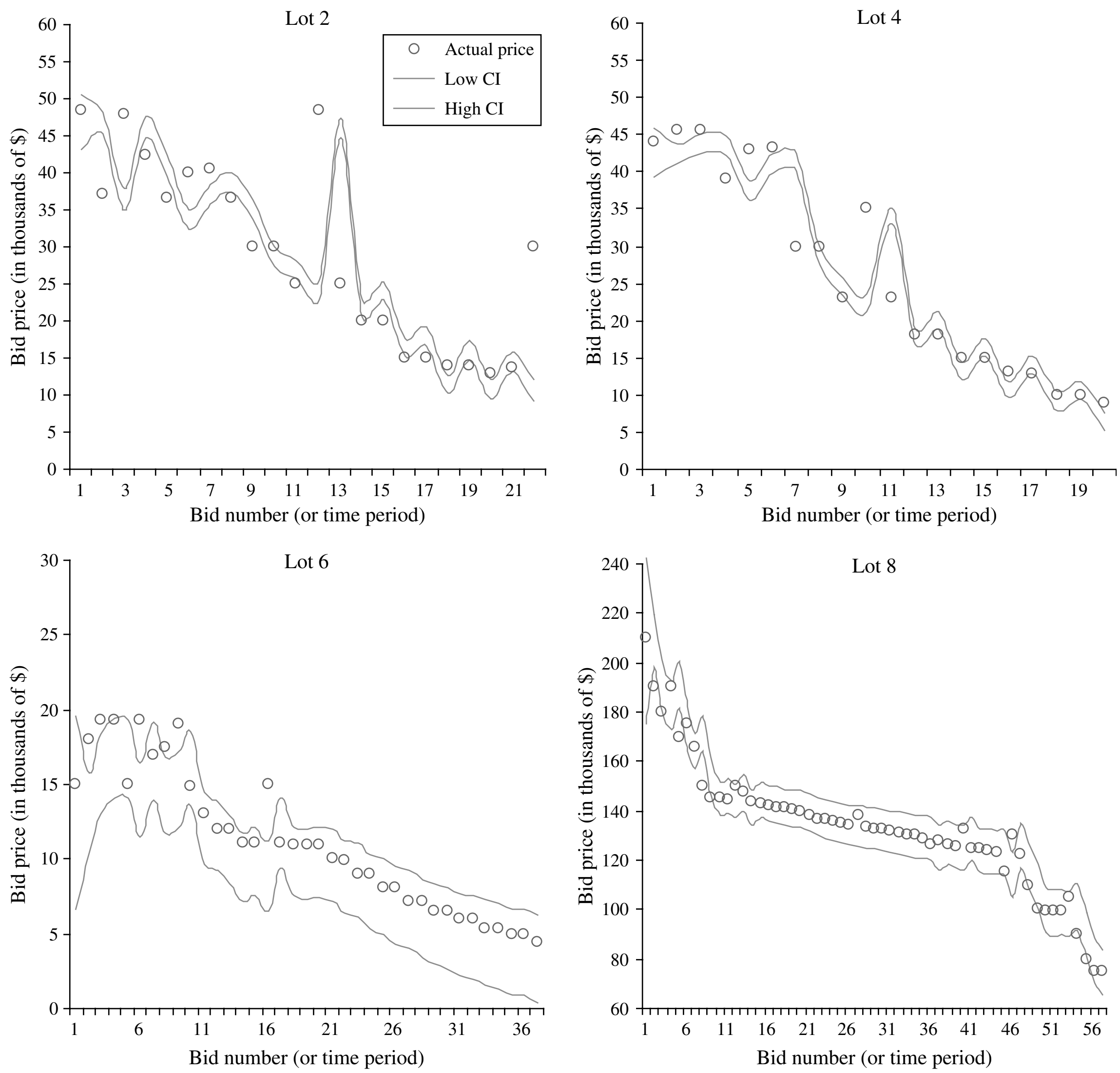


incorporate various drift specifications or cross-bidder dependence, which we explain in the appendix.

7. Discussion and Conclusions

This paper presents an approach to estimate and select dynamic bidding models using point-by-point data from reverse auctions. The methodological development marks an important and necessary step for improving the buyer's price discovery process in online reverse auctions. Specifically, using the estimated latent price distribution π_T^j from BidAnalyzer, the buyer can compute the chance that the supplier j wins the contract if it offers the price b_j (i.e., $P(\pi_T^j > b_j)$). If b_j is too high, the supplier would make a large profit but risks losing the contract (because its selection probability decreases). On the other hand, if b_j is too low, the supplier increases its selection probability but would earn less profit.

Figure 3 BidAnalyzer's Out-of-Sample Forecasts in Small-Value Lots



By formally optimizing this trade-off, the buyer may be able to identify the price that maximizes the supplier's expected profit. We encourage further research to extend BidAnalyzer to serve as a device for price discovery in online reverse auctions.

We note that the state-space model specified via Equations (2) and (4) is hierarchical by construction, where the first level describes the observations (e.g., actual bid prices) and the second level speci-

Table 3 Parameter Estimates for Heterogeneous Drift Model

	Drift parameters, δ_k		Initial means, π_{0k}	
Parameters	Estimates	t-values	Estimates	t-values
Lot 1	−7.55	−4.98	1,327.03	70.6
Lot 2	−1.59	−0.35	49.93	0.91
Lot 3	−11.96	−5.37	2,662.56	72.6
Lot 4	−1.98	−0.39	50.19	1.01
Lot 5	−9.70	−4.54	1,419.71	31.5
Lot 6	−0.44	−0.07	20.02	2.70
Lot 7	−35.69	−15.1	3,560.26	95.9
Lot 8	−2.01	−0.89	190.65	5.42
Lot 9	−29.43	−13.8	3,633.35	90.6
Error variance, σ_ε	0.50×10^{-3}	0.11		
Error variance, σ_ν	28.76	28.4		

Table 4 Estimated Correlations Among 10 Bidders

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_1	1.0									
S_2	**0.95**	1.0								
S_3	0.68	0.48	1.0							
S_4	−0.47	−0.66	−0.01	1.0						
S_5	−0.25	0.02	−0.60	−0.73	1.0					
S_6	**0.88**	**0.93**	0.28	−0.47	−0.08	1.0				
S_7	0.57	0.54	0.68	−0.65	0.17	0.20	1.0			
S_8	−0.27	−0.29	0.30	−0.14	0.22	−0.63	0.61	1.0		
S_9	−0.23	−0.24	0.31	−0.18	0.24	−0.59	0.64	**0.99**	1.0	
S_{10}	−0.23	−0.26	0.32	−0.17	0.22	−0.61	0.64	**0.99**	**0.99**	1.0

Table 5 Information Criteria Values for Dynamic Bidding Models

Bidding dynamics	Specification	L^*	Parameters	AIC	AIC$_c$	BIC
Constant drift	Equation (4)	−1,918.97	12	3,861.94	4,267.88	3,869.20
Heterogeneous drift	Equation (A1)	−1,832.54	20	3,705.08	4,112.50	3,717.19
Time-varying drift	Equation (A2)	−1,824.47	16	3,680.94	4,087.53	3,690.62
Correlated bidders	Equation (A3)	−1,565.22	67	3,264.44	3,697.54	3,304.99

fies the dynamics (e.g., suppliers' evolving valuation). Additional hierarchies can be introduced, or example, serial correlation in errors (see Naik and Raman 2003, pp. 384–385); such hierarchies also are *subsumed within the Equation* (4) *itself* by properly augmenting the state vector α_t (e.g., see Footnote 2). In other words, the two-level state-space models not only accommodate multi-level hierarchies (e.g., see Frühwirth-Schnatter 2006, Ch. 13) but also offer many other advantages (for details, see Dekimpe et al. 2008).

Three methods exist to estimate hierarchical dynamic models: maximum likelihood (e.g., Naik et al. 1998), EM algorithm (Shumway and Stoffer 1982), and Bayesian approach (e.g., Bass et al. 2007). The maximum-likelihood method uses the Kalman filter algorithm, whereas both the EM and Bayesian methods require the Kalman filter and Kalman smoother algorithms. The Kalman filter uses price information *thus far* (i.e., it does not require future prices until the auction ends), whereas Kalman smoother uses *all* the price information up to the final bid p_T at every *intermediate* time point t ($t = 1, \ldots, T-1$). In other words, the Kalman (forward) filter infers the suppliers' valuations of the lot given the past and current prices and generates price forecast as the auction progresses in real time; the Kalman (backward) smoother interpolates the suppliers' valuations given the past, current, and future prices after the auction ends. Hence, we apply maximum-likelihood estimation so that users have the flexibility to update price forecasts during the course of the auction, and we encourage researchers to investigate other merits of the alternative estimation approaches (e.g., shrinkage).

We close by identifying additional avenues for research. BidAnalyzer can be used to understand how supplier characteristics such as manufacturing capabilities, past history with the buyer, or dedicated investments affect the bidding strategy. It can be extended to estimate the latent price distributions by conditioning on the supplier being a bidder in the t-th time point. Analytical researchers can introduce the role of competition across sequential lots to determine the optimal price path (e.g., Oren and Rothkopf 1975, Zeithammer 2006) or study the suppliers' decisions not to bid between different lots, making the entry decision endogenous (e.g., Seshadri et al. 1991). We acknowledge the limitation of our research, which assumes that the rules of online reverse auctions are pre-determined (Shugan 2005); future research may investigate how such rules themselves get set or should be reset for different target groups or desired goals (e.g., liquidity). We believe that such efforts would further improve the theory and practice of online procurement and marketing.

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Appendix. Alternative Models and Empirical Findings

The following table presents three model specifications as well as their descriptions, findings, and model selection.

Model	Specification	Description	Findings
Heterogeneous drift (A1)	$\alpha_{kt} = \alpha_{k,t-1} + \delta_k + \nu_t$	Relaxes the constant drift across all lots in Equation (4) and allows between-lot variations. Table 3 presents the estimated lot-specific drift rates.	Drift rates range from −0.44 to −35.69; average drift rate = −11.15, which compares well with the estimated constant drift of −13.13 in Table 2.

Appendix. (cont'd.)

The following table presents three model specifications as well as their descriptions, findings, and model selection.

Model	Specification	Description	Findings
Time-varying drift (A2)	$\delta_{tk} = \gamma_1 X_{1t,k} + \gamma_2 X_{2t,k} + \gamma_3 X_{3t,k}$ $X_{1t,k} = (t - t_{0k}) =$ the elapsed time since the start of an auction, t_{0k}; $X_{2t,k} = (T_k - t) =$ the remaining time until bidding ends, T_k; $X_{3t,k} =$ the hiatus in two successive bids.	Incorporates the covariates to drive bidding dynamics. Explains *within-lot* variation instead of the between-lot variation.	Price dynamics depends collectively on time elapsed since the start, the remaining time, and the interbid hiatus.[7]
Correlated bidders (A3)	$\Sigma_\nu = L \times L'$; $L = \begin{bmatrix} l_{11} & 0 & \cdots & 0 \\ l_{21} & l_{22} & & \vdots \\ \vdots & & \ddots & 0 \\ l_{10,1} & l_{10,2} & \cdots & l_{10,10} \end{bmatrix}$; $l_{ij} \in (-\infty, \infty)$; $\delta_k = \beta_1 N_k + \beta_2 \Delta P_k$, where $(N_k, \Delta P_k)$ are the number of bidders and the average rate of price drop in lot k.	Relaxes cross-bidder independence;[8] drift varies because of competitive factors. Table 4 presents the estimated correlations across 10 bidders.	The subsets of suppliers (S_1, S_2, S_6) and (S_8, S_9, S_{10}) exhibit high correlations, suggesting the potential for information cascades (Bikhchandani et al. 1992); drift rate depends on the average price drop ($\hat{\beta}_2 = -0.85$; $t = -14.25$), but not on the number of bidders ($\hat{\beta}_1 = 0.03$; $t = 0.27$).
Model selection	AIC, AIC_C, BIC	Table 5 displays the scores on information criteria.	Correlated bidders model attains the smallest value.

[7] Estimates and t-values are as follows: $\hat{\gamma}_1 = 0.0035$ (1.78), $\hat{\gamma}_2 = -0.0778$ (−1.03), $\hat{\gamma}_3 = 0.0007$ (0.03). Although the individual effects are insignificant when considered separately (via t-values), the joint effect of the three time-varying covariates is significant compared to the heterogeneous drift model (in which the drift remains constant within each lot) because the latter model attains higher scores on the information criteria (see Table 5).

[8] We must ensure that (i) every correlation is a positive or negative fraction (i.e., $\rho_{ij} \in (-1, 1)$); (ii) the correlation matrix is positive definite; (iii) and the correlation matrix is symmetric (i.e., $\rho_{ij} = \rho_{ji}$). The specification $\Sigma_\nu = L \times L'$ satisfies these three properties in every trial parameter value of the iterative search process (i.e., not just at the final converged estimates).

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