

Bundling for Flexibility and Variety: An Economic Model for Multi-Producer Value Aggregation

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ABSTRACT

Many markets feature an economic structure in which value is co-created by multiple producers and aggregated into a common bundle by a producer-consortium or independent firm. Examples include in-home video entertainment, technology goods and services, multi-sourced data platforms, and patent pools. This paper develops an economic model to study demand, production choices, revenue-sharing, and relative market power in such markets. Producers in these markets are not rivalrous competitors in the usual zero-sum sense, because output of each casts an externality on production decisions of others and total market demand expands with total output, albeit with diminishing returns. This property allows multiple producers to flourish in equilibrium (vs. just one with the most favorable technological or cost structure), and more so when the market expands less quickly with total output. Equilibrium production quantities of competitors are strategic complements, yet competition between producers does manifest itself, e.g., if one acquires better production technology (i.e., makes value units at lower cost) then the equilibrium production levels of other producers are reduced. Insights are also derived for alternative market structures, e.g., producers have more output and earn higher profit when organized into a distribution consortium (e.g., Hulu, or consortia of zoos or museums) vs. relying on a separate retailer. Mergers between producers have similar effect. The formulation enables us to rigorously answer economic questions ranging from pricing, revenue sharing, and production levels in a static setting, to market dynamics covering both the causes and effects of changes in industry structure.

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1 Introduction

Many markets feature an economic structure in which value is co-created by multiple producers and their outputs are collected and sold as a common bundle by a producer-consortium or by a separate and independent firm, a retailer. A motivating example is in-home video entertainment consumed on TVs and other personal devices, where a bundler (cable and satellite TV providers, or streaming services such as *Quibi*) combines movies and TV shows from an oligopoly of content providers such as studios and programming networks (Ulin, 2019). The collection is offered as a one-price bundle from which buyers can pick and consume arbitrary items sourced from multiple producers (e.g., crime thrillers, sitcoms, documentaries). This structure is favored when buyers want variety in available goods and flexibility in choosing what and how much they consume at any instant, or when economic or technological considerations make it difficult for producers to sell directly to consumers. For instance, aggregators such as *Gympass*, *Apple News+*, *Spotify* and, previously, *Netflix's* DVD rental service, give their users subscription-based “pick your own” access to large bundles. Other examples include multi-source patient data platforms (Ohno-Machado et al., 2014), patent pools (Lerner and Tirole, 2004), software platforms with add-on and plug-in tools (Jansen and Cusumano, 2013), and season/ground passes for various events (Holmgren et al., 2016). The bundling entity may be a third-party retailer, aggregator, platform, community organizer, or a consortium of producers. We will primarily refer to it as a *bundler* or *retailer*, depending on context.

The economic interplay between producers in co-production markets contrasts that in standard multi-producer markets which feature *Cournot* (quantity) or *Bertrand* (price) competition. First, because buyers value variety and quantity, saturation market size (i.e., bundle demand at zero price) is elastic and increasing (although with diminishing returns) in total bundle output, rather than constant. Second, every producer “serves” all customers who purchase the bundle (though each buyer may actually consume only a subset of items in it) vs. competing with other producers

to capture customers. Hence, although producers do compete for a share of revenues and make production decisions in self-interest, they are also non-rivalrous collaborators because the output of one producer benefits others by increasing the bundle’s price potential. This distinctive mix of traditional competition and implicit collaboration under the umbrella of a bundled product, along with the distinct role of the bundling entity, creates a need for suitable theory that captures the essential economic forces and describes decision making, outcomes, and industry transformation, in these markets.

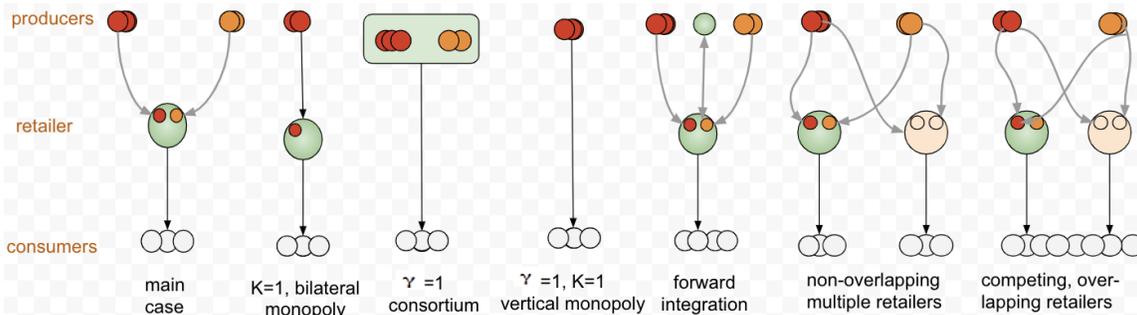


Figure 1: Some market structure variations for multi-producer bundle goods. K is the number of independent producers, γ is the total revenue share of producers, while $1-\gamma$ that of the retailer. This paper does not analyze the final two structures involving competition between retailers.

This paper develops an economic model for analyzing multi-producer value aggregated bundles and to answer the following questions. What is the equilibrium output level of each producer, and how does it depend on the producer’s and its competitors’ characteristics? How do equilibrium outcomes—including output levels, surplus and welfare—vary under different market structures (see Fig. 1) or due to actions such as mergers or alliances between producers? How does revenue sharing between the bundler and producers affect equilibrium outcomes, and what are its implications for industry structure? While the main ideas reflect a structure in which distinct firms fulfill the production and bundling roles, two other structures—labeled “consortium” and “bilateral monopoly” in Fig. 1—are obtained as simplifications of the main result: i) the bundling role is performed by a consortium of producers (like Hulu for in-home video entertainment), and ii) all bundle components are made by the same producer and sold directly (e.g., *Adobe Creative*

Cloud). Another structure of interest is backward integration by the bundling firm into the production layer. For instance, *Netflix* started as a pure aggregator but has become a significant content producer. Similarly, the team productivity tools *Slack* and *Trello* both offer basic software features besides also sourcing dozens of plug-ins or features from various software developers (covering capabilities such as polling, task management, graphic communication, etc.) and offering these to buyers under a collective single price.

We formulate a model with three types of players: a large market of heterogeneous end-user consumers who value size, quality and variety in a bundle, multiple producers who each pick their output levels and are heterogeneous in production technologies, and a retailer (e.g., aggregator or platform) who sources and combines these outputs into a bundle, decides a market price for it, and shares bundle revenue with producers. We show that although producers compete to define and share rewards from total output, multiple producers can flourish in equilibrium (vs. just one with the most favorable technological or cost structure). Surprisingly, the number is higher when the saturation market size expands less slowly with total output. Notably, equilibrium production quantities of competitors can be strategic complements, yet competition between producers does manifest itself; if one acquires better production technology (i.e., makes value units at lower cost) then the equilibrium production levels of other producers are reduced. Insights are also derived for alternative market structures, e.g., producers have more output and earn higher profit when organized into a distribution consortium vs. relying on a separate retailer. Mergers between producers have similar effect.

The rest of the paper is organized as follows. §2 discusses three perspectives from relevant literature, covering models of competition, value co-creation in platforms, and the challenges in modeling bundle demand. §3 describes the economic game among market participants, and introduces a reduced-form specification for bundle demand which respects a wide spectrum of bundling scenarios. §4 describes equilibrium market outcomes, including demand, production and revenue-sharing. Next, §5 examines the drivers and consequences of changes in market structures and how

these affect market outcomes. §6 summarizes results and identifies topics for additional research.

2 Perspectives and Related Literature

We cover three relevant perspectives. First, §2.1 discusses models of competition for homogeneous goods, differentiated goods, complementary and composite goods, and systems. Next, §2.2 discusses value co-creation in platforms. §2.3 discusses the literature on bundling, covering the challenges in specifying bundle demand and optimal bundle design, and concluding with key insights from the literature which we build on to develop a bundle demand specification.

2.1 Competition

Firms that make a homogeneous good (i.e., outputs are substitutes) compete directly by choosing quantity (*Cournot* competition) and/or price (*Bertrand* competition). Market price depends on total output, reduces as output increases, and the lower-cost firm gets higher output and profits (Varian, 1992, Ch. 16). Firms' price responses move in the same direction as competitor's price (i.e., $\frac{\partial P_j(P_i)}{\partial P_i} > 0$) whereas their production quantities move in opposite direction ($\frac{\partial Q_j(Q_i)}{\partial Q_i} < 0$), i.e., they are *strategic substitutes*. This fundamental property holds under variations such as horizontally or vertically differentiated goods (Shaked and Sutton, 1982; Gabszwick and Thisse, 1986; Ferreira and Thisse, 1996) and in models of non-localized competition (Chen and Riordan, 2007). It also applies when a common intermediary makes transactions and volume decisions between networks of buyers and sellers (Nguyen and Kannan, 2019). In contrast, value co-production markets can exhibit *strategic complementarity* in output levels (i.e., $\frac{\partial Q_j(Q_i)}{\partial Q_i} > 0$ in some region).

The form of competition more closely related to the present paper arises between producers of complements, with the extreme case being that of *composite goods* (Cournot (1929), Fig. 2a). Unlike with substitutes, competing firms are co-producers of the composite good, combining outputs of different producers raises the market price, and price increase by one firm weakens demand and

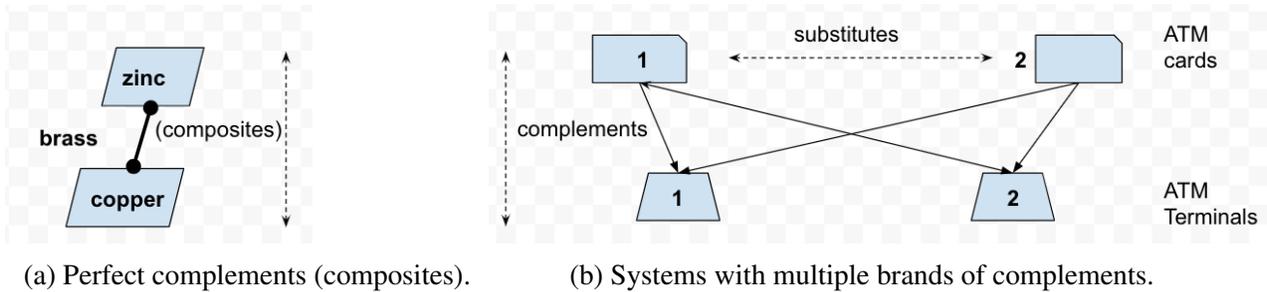


Figure 2: Production and competition with composite goods and systems. For panel (b) there are two producers 1 and 2 who each make two products, ATM cards and ATM terminals.

profits for the other. A generalization of this structure appears in “systems competition” (Fig. 2b) where components are complements but there are multiple brands of each component good (i.e., they compete directly), for instance ATM cards that require, and interoperate on, ATM machines (Katz and Shapiro, 1994). Research in this area has focused on issues such as price and quantity decisions (Economides and Salop, 1991), systems vs. mix-and-match component-level competition between producers (Farrell et al., 1998; Matutes and Regibeau, 1988), usually with two exogenously given component types and two producers of each; producers in such markets are rivals in capturing customers unlike in co-produced bundles where all producers serve all buyers. Lerner and Tirole (2004) examined welfare implications of co-produced bundles in the context of patent pools and technology licensing. The setting where an independent retailer creates a bundle from exogenously given multi-producer outputs was examined by Bhargava (2012), who described how vertical and horizontal conflicts between these firms adversely affected the use of bundling. The present paper extends past work by focusing on a broad set of questions related to co-production bundles, including production output levels, revenue-sharing, and the consequences and drivers of alternative market structures.

2.2 Platforms and Value Co-Creation

Platforms provide infrastructure, rules and incentives that *enable* multiple groups of entities (say, users and producers) to congregate, discover, and conduct value-creating transactions with each

other (Choudary et al., 2016; Cusumano et al., 2019). For example, *OpenTable* connects diners and restaurants, helping diners make reservations or discover restaurants, and helping restaurants attract diners or have personalized interactions with them. Users are attracted to platforms because of the incredible variety of products and producers whom they can discover and transact with (Hagiu, 2009). Although several platforms enable bilateral connections in which individual producers decide product prices (e.g., *Amazon*, *Airbnb*, *Google Play Store*), other platforms essentially combine contributions from their partner-producers into a single co-created bundle and are most attractive when users value flexibility in what and how much they consume at any instant. For instance, the medical research platform *pSCANNER* combines patient data from multiple hospitals, *Gympass* offers access to its partner network of fitness facilities, *Quibi* offers short-form video content sourced from multiple producers, similarly *Waze* offers real-time traffic intelligence that is supplied by numerous members. Since the end-user product in these settings is an all-in-one bundle, its price is set by the platform, however producers have decision autonomy on whether, what, and how much, to contribute into the bundle based on revenue-sharing rules and their production characteristics.

Ceccagnoli et al. (2012) empirically examine the effect on small producers' performance when they participate in a platform's value co-creation ecosystem. Foerderer et al. (2018) examine the effect on complement-provision and innovation when platform owners also make complements. Nocke et al. (2007) examine the effects of platform ownership—e.g., concentrated into a single entity or dispersed among a club of producers or suppliers or other entities—on participation, volume, and welfare. Demirezen et al. (2018) examine collaboration between two firms who are jointly responsible for some output, when one of them can lead and define a contract for the contribution of the other. Adner et al. (2016) build a “frenemies” model of competition between platforms that also make apps and decide whether to offer their apps on competing platforms. While these papers examine important issues in platforms and value co-creation, their goals and results are distinct from those of this paper. More generally, although there is a substantial and growing liter-

ature on platforms, existing papers have primarily considered micro-level decisions (e.g., business model design, level of openness, product line expansion, salesforce compensation). In contrast, the present paper aims to jointly examine (for platforms such as *Slack* that bundle third-party apps with the platform) a wide spectrum of issues including platform pricing, producers' output decisions, revenue-sharing with producers, and the effects of alternate market structures.

2.3 Bundling

Product bundling, one of the simplest and widely practiced business strategies, improves seller profits with little extra effort especially when component goods have low marginal costs. Rao et al. (2018) offer a recent overview of bundling concepts and literature. This paper extends analysis of bundling in two main ways. First, while past literature primarily considers only bundling of its own components by a single firm (e.g., a *MS Office* bundle), this paper is concerned with multi-producer bundles that are put together potentially by a separate firm (Bhargava, 2012). Second, rather than examine bundling for an exogenous set of component goods, the present paper is concerned with the provision of bundle components, the effect of market structure on provision, producer participation in the bundle, and the inter-dependencies between producers. Doing so requires a closed-form expression of bundle demand and a richer consideration of bundle settings.

Algebraic expression of bundle demand is challenging because it involves convolution of the statistical distributions that underline demand for individual goods, and even messier when valuations are sub-additive or super-additive or when demand functions for individual goods are correlated (Venkatesh and Kamakura, 2003). However, the bundling literature underlines a crucial and general property that bundling lowers the heterogeneity in consumer valuations, so that bundle demand is “flatter in the middle” and even more so as bundle size increases (Stigler, 1963; Adams and Yellen, 1976; Schmalensee, 1984; McAfee et al., 1989) For instance, if the individual component goods had linear, independent and additive demand, combining them into a bundle would yield a sideways-S demand curve (Bakos and Brynjolfsson, 2000, Fig. 1). This behavior is confirmed

via simulation for more complicated settings involving correlation and sub- or super-additivity by Olderog and Skiera (2000, Fig. 2-5). We employ these insights from literature in developing a bundle demand function in §3.4.

3 Modeling a Co-Created Bundle Economy

This section develops a model design to fit a cross-producer bundle economy in which producers make the bundle components that consumers value, but lacking direct reach into the consumer market must rely on a specialist firm, a retailer, to sell to consumers. For ease of exposition, the discussion will frequently employ a concrete setting of “TV bundles” that are offered to consumers most recently by entrants such as *Quibi* and traditionally by communications firms (cable operators, telecom, satellite service providers) using content sourced from multiple studios and programming networks. Consumers evaluate such bundles based on quantity (more is better, though at diminishing rate), variety (e.g., for a TV bundle buyers want a mix of movies, TV shows, political thrillers, children-oriented content, comedy, and so on, that comprise many genres and appeal across many moods, age groups, tastes etc.), and quality (creative aspects, star talent, production quality, unique special effects, etc.). The collection of outputs from various producers may comprise items that are potentially substitutes (e.g., crime thrillers from multiple producers) or items from different categories (e.g., crime thrillers and romantic dramas in a TV bundle), although the perception of substitution itself may differ across different consumers. Further, these items may be highly different in the aggregate market interest they generate (e.g., a niche show about a regional cuisine vs. a highly popular sitcom). Individual item demands could be correlated and might be sub-additive or super-additive, again with the level of additivity varying across consumers in a large market. Keeping in mind this multi-dimensionality complexity of the cross-producer bundle, our goal is to develop a framework that captures relevant concepts—aggregate demand, marginal demand, total supply, marginal supply, and revenue sharing in the industry—in a consistent way. The framework

comprises four key elements discussed below.

3.1 Bundle Value Units

The first element in the framework is to employ a canonical “value unit” metric for measuring the product—the bundle and its contributing components—as a combination of variety, quantity, and quality. The bundle of Q value units is offered to consumers under an unlimited-use price P and is an aggregation of outputs Q_i from multiple producers ($i = 1 \dots \mathcal{I}$). Producer i ’s contribution Q_i is a proxy for the incremental market demand from including this output in the bundle. Producers are heterogeneous in their ability to create value units, and this is captured via exogenously specified cost parameters c_i which represent a producer i ’s cost of supplying one value unit. The cost parameter reflects the costs of producing output relative to the potential demand or revenue resulting from it. For instance, producers who have excellent studio facilities, contracts with superstars or A-list directors, or hold production rights to highly popular content, will have a low c_i if their content fetches disproportionately high revenues relative to the cost of producing it (e.g., superheroes and cinematic universes vs. say romantic dramas). In general, a producer’s cost parameter will depend on their fixed assets such as contracts with production talent, licensing and production rights to stories, characters and scripts, studio facilities, and other intellectual or physical property. In this way, the cost vector also accommodates production of different categories of content.

3.2 Economic Actors and Structure of the Game

The second element of the framework is the specification of relationships and decisions set up as a static game of complete information among multiple actors in the economy—consumers, the retailer or platform, and multiple producers. The retailer sources bundle components of aggregate value Q from producers and uses its distribution infrastructure, built at fixed cost F , to market the bundle at a per-subscriber cost of $w^R(Q)$. F will play no role in the main optimization prob-

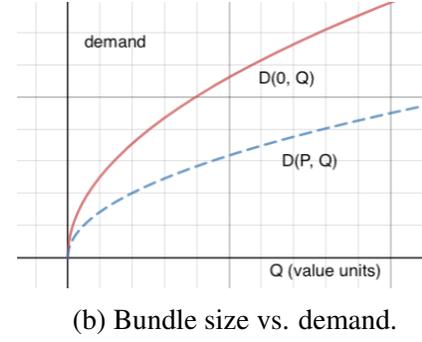
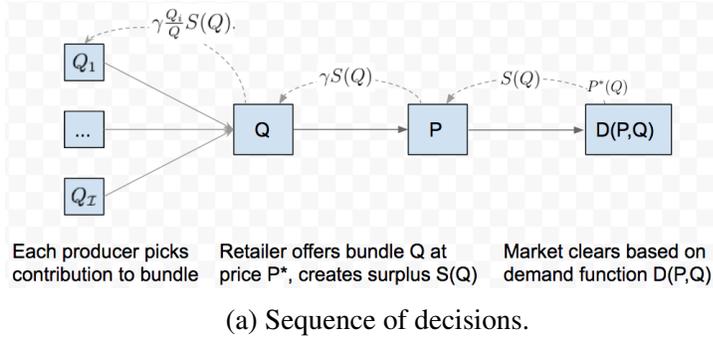


Figure 3: Modeling a cross-producer bundle economy.

lem, however F can explain why multi-producer outputs are served as a bundle (e.g., economies of scope in distribution, and consumer preferences for size, variety, flexibility) rather than each producer’s goods separately (e.g., as in a supermarket). The variable costs $w^R(Q)$ include, for instance, market research, price determination, digital transmission and account management costs.

The retailer sets bundle price P^* to maximize its profit $\Pi_R(Q) = \max_P (1-\gamma) ((P-w^R(Q))D(P, Q))$, creating demand $D(P^*, Q)$ and surplus $S(Q) = (P^* - w^R(Q))D(P^*, Q)$. We refer to $S(Q)$ as the *sharable* industry surplus, i.e., it ignores fixed costs F of the retailer’s distribution infrastructure and producers’ one-time costs $c_i(Q_i)$. This surplus is shared between the retailer and producers according to their relative market power. For example, for in-home entertainment, producers have some power because content drives consumer demand (as expressed in the oft-stated maxim “content is king”), while the retailer’s power is driven by expertise and technology for delivering content (e.g., holding the conduit to deliver content into homes). The revenue-share between producers and the retailer will vary, e.g., based on the level of concentration within each layer. We start by assuming that the retailer is a monopolist who can extract $(1-\gamma)$ fraction of bundle revenue, so that its profit is $(1-\gamma)S(Q)$. The remainder $\gamma S(Q)$ becomes the total revenue available to producers, split proportional to the value-units they provide, i.e., $\gamma \frac{Q_i}{Q} S(Q)$. Fig. 3a depicts the production-distribution (bundling) relationship between industry players as well as the sequence of decisions made by them.

3.3 How do Bundle Costs and Value Increase with Bundle Size?

The third element is the specification of growth rates in production costs, distribution costs, and sharable surplus, as Q increases. Without loss of generality at this point, let production costs be linear in Q , with $c_i(Q_i) = c_i Q_i$ and i 's arranged in ascending order of c_i . To ensure that the problem does not become unbounded and vacuous, we impose a regularity requirement on rate of growth in sharable surplus $S(Q)$.

Requirement 1 (Bounded $S(Q)$). *The costs of producing bundle components and of distributing the bundle increase with Q at a faster rate than bundle demand and sharable surplus $S(Q)$. That is, i) $\frac{\partial^2 D(P,Q)}{\partial Q^2} < 0$ (with $\frac{\partial D(P,Q)}{\partial Q} \geq 0$), and $\frac{\partial^2 S(Q)}{\partial Q^2} < 0$ (with $\frac{\partial S(Q)}{\partial Q} \geq 0$), and ii) the retailer's distribution costs $w^R(Q)$ should increase at a diminishing rate, although faster than $S(Q)$.*

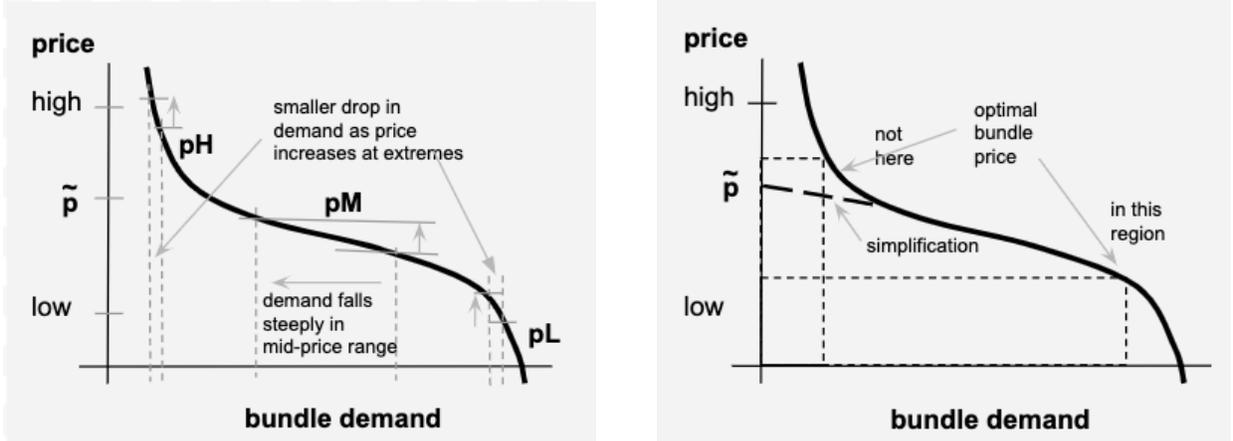
As indicated in Fig. 3a, producers choose output level simultaneously, with producer i picking Q_i to maximize its profit Π_i which is its share of the surplus less its own production costs $c_i(Q_i)$. Let Q_{-i} denote aggregate output of all producers other than i , then $\pi_i(Q_i, Q_{-i}) = \gamma \frac{Q_i}{Q} S(Q) - c_i(Q_i)$, where $Q = Q_i + Q_{-i}$. The individual rationality (IR) constraint for producers is that they make positive (or zero) profit, i.e., that $\frac{\gamma S(Q)}{Q} \geq c_i$ for producer i (i.e., average revenue exceeds average cost). In equilibrium, producers with cost parameter higher than $\frac{\gamma S(Q)}{Q}$ have no output, while the rest choose Q_i that maximizes own profit. This yields the system of equations,

$$\text{active producers} \quad K = |\{i : c_i \leq \frac{\gamma S(Q^*)}{Q^*}\}| \quad (1a)$$

$$\text{optimality conditions} \quad \forall i \in 1 \dots K : \gamma \left[\frac{S(Q^*)}{Q^*} - \frac{Q_i^*}{Q^*} \left(\frac{S(Q^*)}{Q^*} - \frac{\partial S(Q)}{\partial Q} \Big|_{Q^*} \right) \right] = c_i. \quad (1b)$$

$$\text{adding them up over } i : \quad K \left(\frac{\gamma S(Q^*)}{Q^*} - \bar{c}_{(K)} \right) = \gamma \left(\frac{S(Q^*)}{Q^*} - \frac{\partial S(Q)}{\partial Q} \Big|_{Q^*} \right). \quad (1c)$$

where K is the number of producers with positive output (i.e., $i = 1 \dots K$) and $\bar{c}_{(K)}$ is the average of cost parameters for those producers. Eq. 1b suggests the plausible result that output levels of producers are inversely related to their cost parameters. However, its assertion requires computation of the equilibrium value Q^* (and K), and the model needs further precision in order to establish



(a) Shape of bundle demand.

(b) Shape simplification.

Figure 4: Inverse bundle demand. Bundle demand is “flatter in the middle,” more so as bundle size increases (left panel). Because the optimal bundle price is *not* in the high price region, the demand curve can be simplified as shown in the right panel.

and identify a unique or globally optimal solution.

3.4 Bundle Demand Function

The literature on bundling underlines that reservation prices for a bundle are more homogeneous than for component goods, causing the bundle demand curve to be flatter in the middle section (see §2.3). Fig. 4a depicts this insight, specifically that as one moves from a low price (marked **pL** in Fig. 4a) to a medium priced region (**pM**), bundle demand declines more rapidly as price increases (i.e., $\frac{\partial^2 D}{\partial P^2} < 0$, due to bunching of valuations in the middle) upto a price \tilde{p} after which again demand declines at a slow rate in the high-price range **pH** (because only a few consumers have extreme valuations). However, the mechanics of bundling ensure that the high-price region is never optimal, because bundling works by homogenizing valuations and leading to high-demand optima. This allows a crucial simplification of the bundle demand curve: as shown by the dashed region in Fig. 4b, we can assume that bundle demand function satisfies $\frac{\partial^2 D}{\partial P^2} < 0$ throughout.

We leverage these insights (i.e., that $\frac{\partial^2 D}{\partial P^2} < 0$, besides $\frac{\partial D}{\partial P} < 0$) and Requirement 1 (i.e., that surplus (and demand) increase in Q but at diminishing rate, i.e., $\frac{\partial D}{\partial Q} > 0$ and $\frac{\partial^2 D}{\partial Q^2} < 0$) to specify

bundle demand as a function of Q value units, $D(P, Q) = \sqrt{AQ^\theta - bP}$ (with $\theta \in (0, \frac{2}{3})$). The exponent θ represents consumers' propensity for "bigger" bundles, i.e., the inverse of their budget constraint for consuming bundles (higher θ implies a more elastic constraint). Formally, letting $M(Q) = D(0, Q) = \sqrt{AQ^\theta}$ represent market saturation level for a bundle of Q value units, θ equals the elasticity of M^2 to Q). Along with this specification of bundle demand, and in order to satisfy Requirement 1, we write the retailer's distribution cost function as $w^R(Q) = cQ^\theta$.

The bundle demand function can be understood as follows: i) start with a linear demand function $A - bP$, ii) introduce the effect of Q (market saturation demand increases in Q at diminishing rate) replacing A with AQ^θ , and then iii) incorporate the demand smoothing effect of bundling by altering it to $\sqrt{AQ^\theta - bP}$. The equation can be generalized to $D(P, Q) = (AQ^\theta - bP)^\alpha$ (where $\alpha \in (0, 1)$), however $\alpha = \frac{1}{2}$ simplifies the exposition. While the qualitative results would still follow from other demand functions—such as linear, quadratic, negative exponential or constant elasticity demand functions—the above function is chosen because of how it captures the demand-side behavior of bundling (Table in Appendix lists other functions that were considered and why they were rejected). Finally, while the demand function $D(P, Q)$ is defined over an abstract measure of value units, Q can be estimated via its effect on the maximum level of market demand, i.e., $Q^\theta = (D^{-1}(0))^2 / A$. The framework ensures both that the retailer enjoys economies of scale and also that as Q increases, cost increases more rapidly than demand.

4 Equilibrium Analysis

The sequence of decisions in this multi-firm economy is that heterogeneous producers (with production costs $c_i Q_i$ and collective market power γ which is encoded into a revenue-sharing parameter with the retailer) choose their Q_i 's, the retailer aggregates these outputs into a bundle $Q = \sum_i Q_i$ in exchange for transfer prices F_i , and the retailer distributes the bundle (incurring additional cost cQ^θ) at market price P . We solve the problem in backward sequence, first identifying optimal

P which maximizes the retailer's profit given Q , then determining Q_i 's while satisfying the aggregation constraint ($Q = \sum_i Q_i$) and producer's participation constraints ($\pi_i(P^*(Q), Q_i, Q_{-i}) \geq 0$, where Q_{-i} is the vector of all Q_i 's except Q_i). The worth of this modeling framework is in the results it produces: ease of generating them, what they cover, how meaningful they are, and their credibility. Lemma 1 describes the industry equilibrium solution, which we develop and explain in the rest of this section.

Lemma 1 (Equilibrium Solution). *With producers' costs c_i per value unit arranged in ascending order, the equilibrium set of producers $i = 1 \dots K$ with positive output, their magnitude of value units produced, and the market price set by the retailer are*

$$K = \max\left\{i : c_i \leq \left(\frac{c_1 + \dots + c_i}{i - (1-3\theta/2)}\right)\right\} \quad (2a)$$

$$\forall i = 1 \dots K : Q_i^* = \left[1 - c_i \left(\frac{K - (1-3\theta/2)}{c_1 + \dots + c_K}\right)\right] \frac{2Q^*}{(2-3\theta)} \quad (2b)$$

$$\text{with } Q^* = \sum_{i=1}^K Q_i^* = \left[\frac{2\gamma}{b} \left(\frac{K - (1-3\theta/2)}{c_1 + \dots + c_K}\right)\right]^{2/(2-3\theta)} \left(\frac{A-bc}{3}\right)^{3/(2-3\theta)} \quad (2c)$$

$$P^*(Q) = \frac{2A+bc}{3b} Q^\theta. \quad (2d)$$

When Eq. 2a yields $K=1$ (i.e., $c_2 > \frac{2c_1}{3\theta}$), then $Q_1^* = Q^* = \left(\frac{3\gamma\theta}{bc_1}\right)^{\frac{2}{2-3\theta}} \left(\frac{A-bc}{3}\right)^{\frac{3}{2-3\theta}}$.

4.1 Pricing

Price determination is straightforward and done the usual way. Given the total available content Q , the retailer sets the optimal price to maximize profit ($P^*(Q) = \arg \max_P \Pi_R(P, Q)$), which is

its bundle revenues less the cost of sourcing content from producers,

$$\Pi_R(P, Q) = (1-\gamma)(P-cQ^\theta)D(P, Q) = (1-\gamma)(P-cQ^\theta)\sqrt{AQ^\theta-bP} \quad (3a)$$

which yields $P^*(Q) = \frac{2A+bc}{3b}Q^\theta \quad (3b)$

$$D^*(Q) = \sqrt{\frac{A-bc}{3}Q^\theta} \quad (3c)$$

and $\Pi_R^*(Q) = \frac{2}{b}(1-\gamma)\left(\frac{A-bc}{3}Q^\theta\right)^{3/2} \quad (3d)$

with $S^*(Q) = \frac{2}{b}\left(\frac{A-bc}{3}Q^\theta\right)^{3/2}. \quad (3e)$

The final term $S^*(Q)$ is the overall industry surplus when a bundle of magnitude Q is offered to consumers at P^* . Notice that the surplus and profit terms increase with Q (unlike Cournot quantity competition where price and profit would fall as supply increased), and less than linearly (with $\theta \in [0, \frac{2}{3}]$). This suggests that Q is better thought of as *quality* than *quantity*. The equilibrium level of demand also increases with Q but at a diminishing rate, while price-per-unit- Q falls with Q . Eq. 2d suggests the interpretation of θ as the elasticity of optimal bundle price $P^*(Q)$ to bundle size Q . The analysis would be the same if the retailer's profit function were set up (instead of Eq. 3a) as a constant fraction of net revenues (with producers getting the rest).

4.2 Production

Producers pick their output levels simultaneously. The equilibrium levels of output are such that no producer gains by unilaterally deviating from chosen output level, given the choices of other

producers. Each producer's Q_i is chosen to maximize own profit π_i given Q_{-i} .

$$Q_i^* = \arg \max_{Q_i \geq 0} \pi_i : \quad \pi_i(P, Q_i, Q_{-i}) = \left[\frac{2}{b} \left(Q^\theta \frac{A-bc}{3} \right)^{3/2} \right] \frac{Q_i}{Q} \gamma - c_i Q_i \quad (4a)$$

$$\text{set } \frac{\partial \pi_i}{\partial Q_i} = 0 : \quad c_i = \frac{2\gamma}{bQ^2} \left(Q^\theta \frac{A-bc}{3} \right)^{3/2} \left(Q - \frac{(2-3\theta)Q_i}{2} \right) \quad (4b)$$

$$\Leftrightarrow \quad Q_i = \left[2 - \frac{bc_i Q}{\gamma} \left(\frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{3/2} \right] \frac{Q}{(2-3\theta)} \quad (4c)$$

$$\text{share of production} \quad \frac{Q_i}{Q} = \left[2 - \frac{bc_i Q}{\gamma} \left(\frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{3/2} \right] \frac{1}{(2-3\theta)} \quad (4d)$$

$$\text{IR constraint : } \forall i \quad Q_i \geq 0 \Leftrightarrow c_i \leq \frac{2\gamma}{bQ^{\frac{2-3\theta}{2}}} \left(\frac{A-bc}{3} \right)^{3/2} \quad (4e)$$

$$\left(Q = \sum_i^K Q_i \right) \quad \Leftrightarrow Q \leq \left(\frac{2\gamma}{bc_K} \right)^{\frac{2}{2-3\theta}} \left(\frac{A-bc}{3} \right)^{\frac{3}{2-3\theta}} \quad (4f)$$

where K is the highest i for which the RHS of Eq. 4e holds. For each i , Eq. 4c yields the optimal output level Q_i^* given the levels Q_{-i} of other “feasible” producers (i.e., $i=1\dots K$). In traditional competition (e.g., Cournot, or Hotelling competition with covered market), producers are pure competitors. Higher output by producer i forces lower output by j (not doing so would drive market price down), and production levels are *strategic substitutes*. With cross-producer bundling (i.e., as long as $\theta > 0$), higher output by any producer improves market demand for (and revenue from) the bundle, creating a “rising tide lifts all boats” potential for higher revenue to competing producers. However, because each producer's revenue is proportional to its contribution, other producers must also increase production to capitalize on this potential. Producer i 's optimal output can increase as output of other producers increases, yielding a distinctive aspect of inter-producer competition in this value co-creation setting.

Remark 1 (Output levels as strategic complements). *The optimal output response of producer i may increase with competing producers' output Q_{-i} when Q_{-i} is not too high.*

The collection of Eq. 4c for feasible producers defines the industry-level supply equilibrium,

however it is an implicit condition stated in terms of $Q = \sum_i Q_i$ (Eq. 4f). Next, to figure out the equilibrium output levels, repeat and add up Eq. 4c for $i=1\dots K$, and let $\bar{c}_{(K)} = \sum_i c_i$ denote the average cost parameter for content production. This yields

$$Q = \frac{KQ}{(2-3\theta)} \left[2 - \frac{b\bar{c}_K}{\gamma} Q^{\frac{2-3\theta}{2}} \left(\frac{3}{A-bc} \right)^{3/2} \right] \quad (5a)$$

$$\equiv Q \left(\frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{3/2} = \frac{\gamma}{b\bar{c}_{(K)}} \left(2 - \frac{2-3\theta}{K} \right) \quad (5b)$$

$$\equiv Q^* = \left[\frac{\gamma}{b\bar{c}_{(K)}} \left(2 - \frac{(2-3\theta)}{K} \right) \right]^{2/(2-3\theta)} \left(\frac{A-bc}{3} \right)^{3/(2-3\theta)} \quad (5c)$$

Combining Eq. 4e, for each i , with above equations (Eq. 5b is the most useful) yields that participation is limited to producers with the following cost parameters.

$$\text{feasible cost vector : } (c_1, \dots, c_K) \text{ such that } c_K \leq \frac{\bar{c}_K \cdot K}{K - (1 - 3\theta/2)}. \quad (6)$$

Note that if $i (> 1)$ is in the feasible vector then $i-1$ is too (see proof of the Lemma). Among all i 's in the feasible vector, the highest i for which the feasibility condition is satisfied is denoted as K in Lemma 1. Procedurally, K can be identified by testing the condition first with all \mathcal{I} producers; if it fails then the highest c_i is removed from the vector, successively, until the condition is satisfied, yielding K and the set of feasible producers with non-negative Q_i . Once this is done, Eq. 5c describes the product bundle, given the various parameters of the problem, and combining this with Eq. 4c produces Q_i the value supplied by each producer. Producers with index higher than K have no production.

4.3 Equilibrium Properties

Fig. 5 displays example computations with an exogenously given cost vector, showing Q_i^* and K for different levels of γ and different levels of θ . First, in Fig. 5a, note that output levels of

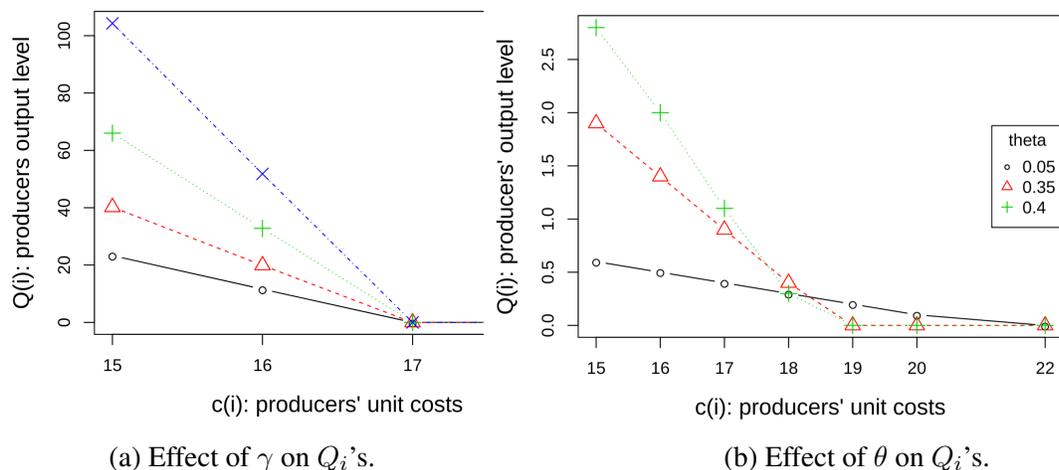


Figure 5: Impact of cost structure, propensity for bigger bundles (θ) and revenue-sharing (γ) on number of producers and production levels. The left panel has $\theta=0.5$, while the right panel has $\gamma=0.6$, and both panels have A, b, c such that $\frac{A-bc}{3}=10$.

producers are higher for producers with better production technology (lower unit cost) or when producers get a higher revenue-share (γ) of the sharable surplus. This is intuitive and confirmed in Proposition 1, because higher revenue-share (or lower cost) implies a higher output level at which marginal revenue from incremental output (given output levels of other producers) equals marginal cost. Second, both panels demonstrate that the number of viable producers with positive output is neither just one (the best-cost producer) nor all producers (Proposition 2). Third, from Fig. 5b, note that Q_i^* increases with θ for some producers (with lower cost per value unit), but it drops for other (i.e., higher-cost) producers as θ increases. This result is more surprising and nuanced, and discussed in Proposition 3. The behavior of Q_i 's against c_i 's, and against the revenue-sharing parameter γ , is presented first.

Proposition 1 (Production levels vs. costs). *Output levels of producers are (i) higher for lower-cost more efficient producers (i.e., $(c_i < c_j)$ implies $Q_i > Q_j$), and (ii) increase with revenue-share γ for all producers ($\frac{\partial Q_i}{\partial \gamma} \geq 0$).*

The linear production cost in our model ($c_i Q_i$) raises the possibility of a corner-point equilibrium solution in which only the most efficient or lowest-cost producer has positive production level while others have zero. This is because producers “draw from the same well” for revenue and

marginal revenue is linked to total bundle size rather than how much each producer has made, giving the lowest-cost producer an advantage for every next unit of production (i.e., marginal revenue less cost is highest for producer 1), regardless of existing production levels. Part (i) of Proposition 1 is consistent with this argument. Our analysis reveals, however, that it does not imply the corner solution.

Proposition 2 (Multi-producer output). *Multiple producers have positive output (i.e., $K \geq 2$) so long as the cost gap between the top two most-efficient producers is not too high, i.e., $(c_2 \leq \frac{2c_1}{3\theta})$. Equivalently, a single-producer outcome is obtained when the ratio $\frac{c_1}{c_2} < \frac{3\theta}{2}$. In general, producer i has positive output only if*

$$c_i \leq \frac{\bar{c}_{(i-1)}(i-1)}{(i-1) - (1-3\theta/2)}, \quad (7)$$

where $\bar{c}_{(i-1)}$ is the average cost of producers $1 \dots i-1$.

Why is the single-producer corner solution avoided in general despite the linear cost functions, allowing higher-cost producers sustain positive production? First, although producer 1 has the best cost for making the next unit at every level of Q , producers' output decisions are made independently and concurrently within a complete-information static game. Thus, several producers stake positive output with positive profit, knowing that the lower marginal revenue will cause others to constrain their production. Second, with $\theta = \frac{2}{3}$ in $D(P, Q)$, producer 1's diminishing marginal gains from higher Q constrains its own output, leaving room for some higher-cost producers to make positive profit at lower levels of output. This effect is weaker when θ is high, and producer 1 is the sole contributor to the bundle as θ approaches its maximum value $\frac{2}{3}$. It is stronger when bundle demand increases less slowly with Q (i.e., θ is small (however, the model loses meaning at $\theta=0$ because then the only motivation for making output is that revenue corresponds to share of output)). In general, Eq. 2a in Lemma 1 trivially leads to the corollary that the number of active producers in the market (K) is inversely proportional to the intensity θ with which demand increases against Q , i.e., $\frac{\partial K}{\partial \theta} < 0$ (strictly speaking, this is a weak inequality because K is discrete integer valued). The effect of θ on producer output and market participation is deeper, as explained

in the next result.

Proposition 3 (Effect of θ on output levels and market concentration). *Increase in θ (demand propensity for bigger bundles) increases market concentration in the producer layer. Q_i 's (and profit) increase for producers with lower c_i and for the retailer, while Q_i 's can fall for producers with higher costs, and those with highest c_i may be forced to exit the market (with $\frac{\partial K}{\partial \theta} < 0$).*

Recall that θ represents, approximately, consumers' greed or propensity for "bigger" bundles. Low values of θ restrain the output levels of low-cost producers (because gains are low), allowing additional producers to survive (see Eq.7). Stronger demand propensity for bigger bundles makes efficient producers more aggressive at capturing a greater share of the bundle pie, leaving less room for other producers and increasing the level of market concentration (see Fig. 5b). At the limiting value ($\theta < \frac{2}{3}$), only producer 1 can survive, no matter how close c_2 is to c_1 , because producer 1 has unbounded gain from greater output. In general, high θ shifts output to more efficient (lower-cost) producers and eliminates some high-cost producers.

Next, consider the effect on output levels when one producer i is able to reduce its production cost c_i , e.g., by acquiring improved talent or technology. Intuitively, producer i can raise output because of this lower cost, but what happens to output levels of other producers? The result highlights a manifestation of competition in this value co-production setting.

Corollary 1 (Cross-producer conflict). *Reduction (improvement) in one producer's cost forces others to reduce production and can cause some producers to exit. Formally, $\frac{\partial Q_j^*}{\partial c_i} > 0$.*

Because i produces more, other producers j observe a lower marginal revenue at the existing level Q_j (because of diminishing marginal gains from higher Q) and must ramp production down to the point where marginal revenue equals marginal cost. In equilibrium, with lower c_i , producer i makes more and other producers make less. This result holds when cost changes within a range that K remains the same, i.e., no Q_j^* crosses the boundary $Q_j \geq 0$ (at the boundary, if some producers are driven out of the market, then it is possible for other producers to have higher output than before.) The result further illuminates the intuitive understanding that while producers are engaged

in collaborative production in this setting, a competitive effect emerges because consumer spending is shared across them collectively.

5 Market Structure and Revenue-Sharing

The previous two sections have analyzed a reduced-form model and developed equilibrium outcomes when a single retailer offers a bundle comprising outputs collected from multiple producers (shown in the left panel of Fig. 1). Collectively, the results presented thus far confirm the distinctive aspects of this market structure and also validate the reduced-form demand structure employed in setting up the model. Alternative market structures can readily be analyzed through variations of this model. For instance, setting both $K=1$ and $\gamma=1$ corresponds to a vertically integrated monopoly, and setting just $\gamma=1$ (with $K>1$) represents a production consortium where the consortium makes pricing decisions (rather than a separate retailer who shares revenues). Varying K affects the number of producers in the market, with $K=1$ representing a bilateral monopoly comprising a single producer and single retailer.

5.1 Impact of Producer Ecosystem

Total output in the cross-producer bundle economy is dependent on the cost or technological characteristics (c_i) of producers. Consider, now, how output levels would vary in two scenarios with identical average cost but distributed among more, or fewer, producers. At the extreme, in two scenarios with equilibrium values $K_1=1$ and $K_2>1$, but with $\bar{c}_{K_1} = \bar{c}_{K_2}$, which would lead to higher output? Intuitively, since market demand $D(P, Q)$ is responsive to Q rather than K , this would suggest higher output under a single producer than if production and profits were shared among multiple producers. More generally, the nature of competition implied in Corollary 1 also suggests that output would be higher under fewer producers. However, computing the expression $\frac{\partial Q}{\partial K}$ using Eq. 5c, we find that higher K leads to greater supply of content.

Proposition 4 (Oversupply with more producers). *Ceteris paribus, higher K leads to higher output (i.e., $\frac{\partial Q}{\partial K} > 0$), and greater market coverage for the bundle. Total output would be lower under a single producer (with unit cost same as \bar{c} of existing producers).*

This unusual result is obtained because cross-producer bundling and revenue-sharing has an effect analogous to the productivity- and production-enhancing effect of technology. Generally, a producer's output level is set to equate marginal cost of making more output (here, a constant c_i) with its marginal revenue. However, under multi-producer bundling, producer i 's benefit from every dollar spent on production (cost c_i) gets amplified. Producer i benefits from the higher market demand and price that arises due to the Q_j 's of other producers, however it can capture these gains only proportional to its share of content. As all producers evaluate their output decisions this way, the result is an oversupply of output. Together, Corollary 1, and Proposition 4 explain the interplay of collaboration-competition in this market structure: one, producers do compete because higher production by one crowds out others, but conversely each producer's production also creates some gains for others. Notice that the result arises purely on account of co-production externality, rather than due to any dependence of K on either bundle price or the level of revenue-sharing with the retailer. Analysis of individual-level output decisions leads to the next result.

Proposition 5 (Mergers between producers). *A merger between producers, such that the new entity has cost parameter equal to average of merged producers, causes all other producers to make less output, but the merged entity earns higher profit.*

How do mergers or splits, or changes in K , affect profit-sharing between the retailer and producers? From Proposition 4, reduction in K causes lower Q , hence lowers the retailer's profit. Moreover, while γ is exogenous in the model, over time such shifts in the producer layer can cause market power to move towards producers. A lower K would make it difficult for the retailer to shut out a producer with whom it can't reach a profit sharing agreement; at the extreme, $K=1$ makes the producer(s) more consequential to the retailer's survival, and the retailer must surrender a higher share (γ) of bundle revenues to the producer(s). Hence, mergers and acquisitions among producers have a doubly harmful effect on the retailer, who earns lower revenues on account of lower Q and

higher γ . This result is another peculiarity of the bundling structure inherent in selling in-home entertainment content. It contrasts industries where producers compete for individual customers (through a retailer), and consolidation among producers generally leads to higher prices and higher margins for both producers and the retailer.

Proposition 6 (Horizontal mergers among producers). *Mergers and acquisitions between producers, and other actions that reduce K , reduce the retailer's profits, $\frac{\partial \Pi_R}{\partial K} > 0$.*

Next, consider equilibrium outcomes when producers can organize into a bundling consortium and directly offer the bundle to the market (vs. revenue-sharing with a retailer). This structure occurs, for instance, in technology patent licensing (Lerner and Tirole, 2004) and it roughly describes Hulu's formation as a multi-producer distributor of in-home entertainment (initially as a joint venture between The Walt Disney Company, AT&T Warner Media, and Comcast-NBC). To examine this ($\gamma=1$ and $K>1$), suppose that the consortium faces the same additional cost of distribution cQ^θ as would a separate retailer. More generally, consider the effect of γ on equilibrium outcomes. Now, given Q , the consortium would set price exactly as the retailer would. because γ linearly affects the retailer's profit, it does not directly impact bundle price, hence P^* is as given in Lemma 1. However the higher γ motivates producers to create more output, increasing Q more than linearly in γ .

Corollary 2 (Consortium vs. a Retailer (Proposition 1)). *Producers supply more content when selling content bundles as a consortium rather than through a separate retailer. Generally, producers make more content when they can get higher share of content subscription revenues, i.e., $\frac{\partial Q}{\partial \gamma} > 0$.*

5.2 Backward Integration and Revenue-Sharing

Corollary 2 describes an extreme case of "backward integration" where producers, collectively, can directly reach consumers without needing the retailer. Now consider the reverse structure where the retailer can also double as producer. For instance, in the entertainment industry, both broadcast networks and cable firms have substantial stakes in content production or licensing. More recently,

Netflix shifted from being a pure aggregator of third-party content to production of original or first-party content. To address this in a general way, let c_R denote the unit production cost for the retailer, capturing the retailer's hurdle for producing bundle value units (similar to the c_i 's for other producers). Let $Q^* = \sum_i Q_i^*$ be the equilibrium output in the absence of backward integration, yielding the retailer a profit Π_R^* . Under backward integration, let Q_R represent the retailer's output level, with total production cost $c_R Q_R$. Let Q still denote the value units made by *other* producers, so that bundle size is now $\hat{Q} = Q + Q_R$.

Hagi and Spulber (2013) study tradeoffs in two-sided market platforms when the platform offers first-party content, and Zhu and Liu (2018) offer an empirical analysis focusing on Amazon.com's competition with first-party products against its third-party complementors. A full analysis of backward integration in our setting would require extensive development to understand how it would impact existing producers' output levels, however we offer here a limited high-level intuition to describe the process and its effects. The retailer's role as producer differs from that of other producers because it can coordinate production and bundle-selling decisions, while maximizing profit $\left(\gamma Q_R S(\hat{Q}) / \hat{Q} \right) - c_R Q_R + (1 - \gamma) \left(S(\hat{Q}) - c \hat{Q}^\theta \right)$. Depending on the nature of entry (and fixed costs for creating production systems), the Q_R decision could be understood as either simultaneous or preceding the Q_i decisions of producers. In either case, the retailer's entry will force a reduction in existing producers' output ($Q_i < Q_i^*$) and their share of bundle revenue, while possibly causing some high c_i producers to exit the market. What is more notable, though, is that this shift would also lower the surplus that the retailer obtains from aggregating output of other producers (because $Q < Q^*$ would cause $S(Q) < S(Q^*)$), mitigating some of the benefits of backward integration. Taking all factors into account, let \hat{c}_R denote a cost threshold such that entry into production is profitable (i.e., $Q_R^* > 0$) if and only $c_R < \hat{c}_R$.

Should the retailer capitalize on the opportunity for backward integration if $c_R < \hat{c}_R$ and if long-term incremental profits from production exceed fixed costs of entry? Moreover, does the mere *possibility* (vs. the *reality*) of backward integration affect the competitive relationship between

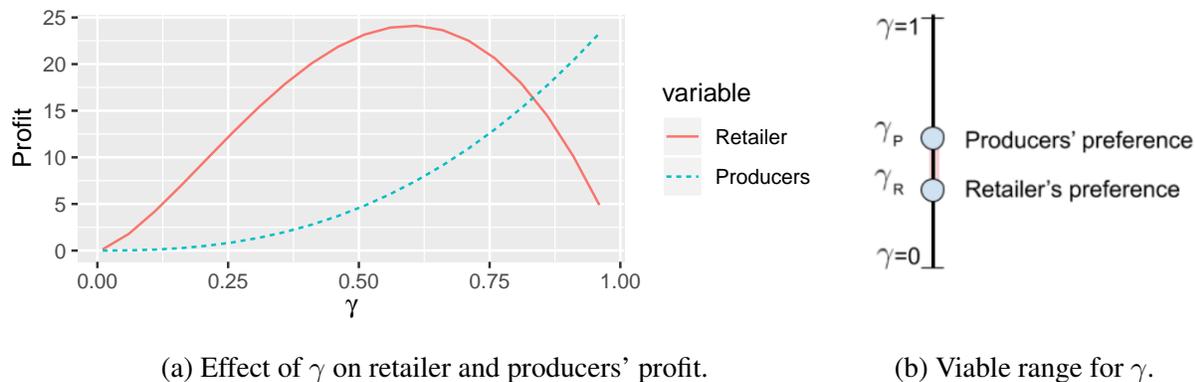


Figure 6: Forces affecting choice of revenue-sharing parameter γ .

producers and the retailer? We frame this question in terms of the choice and process of determining the revenue-sharing parameter γ . For the retailer, higher γ induces more output and sharable surplus but gives the retailer less of it, whereas low γ would imply lower Q from producers and hence low profit for the retailer (and lower overall industry profits). Therefore, the retailer is naturally constrained from demanding a very high share of revenue (i.e., low γ), and is willing to accept an interior value γ_R , as shown in Fig. 6. For producers, a superficial analysis might suggest an unrestrained high γ because that would lead to both higher output and higher share of surplus (Fig. 6b). However, now the potential backward integration by the retailer becomes significant. It follows intuitively from our model (and its implicit extension to backward integration) that i) $\frac{\partial Q_R}{\partial \gamma} > 0$ (conditional on entry, higher γ would motivate the retailer to make more output), and ii) $\frac{\partial \hat{c}_R}{\partial \gamma} > 0$, i.e., that high γ would increase the cost threshold and make it more likely that the retailer would pursue backward integration, causing substantial profit loss to producers. It is this potential for loss, then, that should mitigate producers' desire for high γ to a more realistic value γ_P that guides the revenue-sharing negotiation (see Fig. 6b).

Remark 2 (Negotiating the revenue-sharing parameter). *The bargaining range for the revenue-sharing parameter (producers' revenue-share γ) is bounded from above because high γ makes backward integration by the retailer, which is detrimental to producers, more likely; and from below because low γ would cut producers' bundle contributions and reduce the retailer's profit.*

Producers would receive higher revenue-share in equilibrium if the retailer faces strong entry

barriers for backward integration, or if producers have an “outside option” such as competing retailers or the possibility of direct distribution to consumers. Producers’ share would be lower if the retailer is well placed for moving into production. If γ_R and γ_P are too far apart, then it is likely that the retailer would exercise its opportunity for backward integration. An example is the industry dynamic that led to Netflix’s entry into content production around 2010 when, in a reflection of “carriage fee disputes” producers demanded much higher license fees for their content than Netflix was willing to grant, causing cancellation of streaming licenses and leading Netflix to invest heavily in original content. This backward integration caused deep harm to producers and, over the years, led to significant industry turmoil including mergers between producers and the introduction of new direct-to-consumers services as producers increasingly saw the aggregator Netflix as a competitor. The insight from the overall analysis is that it is important for the retailer to at least possess, if not execute on, some capability for backward integration in order to manage producers’ exercise of market power for their output.

6 Conclusion

This paper has developed a model for analyzing markets in which outputs are sourced from multiple producers and offered as a bundle. While such markets have existed for long (e.g., sports tournaments, auto shows, technology exhibitions, county fairs, carnivals, and arts and music festivals), information technology and platform business models have made them more prominent by facilitating the merging of cross-producer outputs, especially when buyers want variety rather than just quantity and quality. Building on existing perspectives on competition, bundling and value co-creation, our goal was to model the entire economic system, explaining the retailer or platform’s pricing as well as producers’ output decisions, modeling revenue-sharing between them, and exploring the effects of alternate market structures. Given the well-known challenges in representation and analysis of bundle demand, we achieve these goals by first developing a reduced-form

specification for bundle demand which fits and respects the characteristics of bundling across a wide spectrum of bundling scenarios, and then deriving market outcomes under alternative market structures as well as the drivers and consequences of changes in market structures.

While the primary focus of this paper was to understand and explain the economic forces and their effects in a co-producer market setting, our results also have useful applications and lessons for managers and practitioners. First, for producer firms, the findings regarding equilibrium output levels provide guidance regarding how the output decisions of each producer affect, and depend on, output decisions of others, and on market demand propensity for bigger bundles (θ). Moreover, the results establish that having the best cost or technology is not a necessary condition for positive production to be viable. Counter-intuitively, this property is stronger when propensity for big bundles (θ) is lower. Another informative and counter-intuitive finding is that output levels may be strategic complements. Further, the effect of θ on market concentration and sharable surplus suggests that the retailer and the most powerful producer would have incentives to take actions (e.g., advertising) that increase market desire for bigger bundles. Our results also provide guidance regarding the effects of mergers between producers, which would cause output levels to fall and make the merged entities more competitive both against other producers and against the retailer. The insights regarding backward integration and how its mere possibility affects revenue-sharing can be helpful in resolving the revenue-sharing parameter in practice.

The modeling framework of this paper is limited in the sense that it does not accommodate individual-level demand preferences for specific products or producers, or for combinations of them, nor does it explore or advise regarding what specific outputs are made by each producer. However, it captures higher-level requirements in a way that is analytically tractable and leads to meaningful conclusions, and as a foundation for analysis of additional market structures. The main setting—that all outputs are combined into a universal single bundle offered to all buyers—works best when outputs of individual producers are of roughly comparable value, e.g., oligopoly with a few large and powerful producers. However, the model can be stretched to understand

related structures. For instance, a market with hundreds of tiny or niche producers, in addition to a few big oligarchs, could be approximated by treating Q as an aggregation only across the large i producers (i.e., undercounting Q a little) and assigning very high c_i 's to the niche producers, with the understanding that they have alternative motivations (e.g., advertising) rather than revenue-share from the retailer. Alternately, if some producers are powerful outliers with extremely high-value, then these high-value items (e.g., HBO) can be separated out of the main bundle and offered as an “add-on” (see partial bundling, e.g., Bhargava (2013)).

The main model assumed a separation of production and bundling roles, but these often overlap in practice when not prohibited by regulation. It would be useful to extend the model to allow for a comprehensive analysis of both direct-to-consumer strategies by producers and backward integration by the retailer (§5.2 offers a limited analysis of the latter). Similarly, the main model assumed a constant homogeneous revenue-sharing agreement between the producers and retailer, however it would be useful to consider heterogeneous revenue-sharing contracts wherein more powerful producers can extract higher revenue shares and be less fearful of backward integration by the retailer. Another useful extension involves multiple competing and overlapping retailers, with analysis of single- vs multi-homing behavior of consumers as well as exclusivity contracts between producers and retailers, or competing but non-overlapping retailers (with distinct market regions) and an analysis of mergers between retailers and the implications on the vertical competitive relationship between producers and retailers. All these are exciting prospects for research for which this paper lays out a formal and relevant modeling and analytical framework.

$D(P, Q)$	unit cost	P^*	$D(P^*, Q)$	$\frac{\partial^2 D}{\partial P^2} < 0?$	$\frac{\partial(\frac{P^*}{Q})}{\partial Q} < 0$ at $c=0$	$\frac{\partial D(P^*, Q)}{\partial Q} > 0?$	$\frac{\partial^2 D}{\partial P \partial Q} > 0?$
$A - b \frac{1}{Q} P$	$c \ln Q$	$\frac{AQ}{2b} + \frac{c \ln Q}{2}$	$\frac{A}{2} - \frac{bc \ln Q}{2Q}$	No	No	Yes	Yes
$A - b \frac{1}{\ln Q} P$		$\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$	$\frac{A}{2} - \frac{bc}{2}$	No	Yes	No	Yes
$A Q^\theta - b P$	$c \ln Q$	$\frac{AQ^\theta}{2b} + \frac{c \ln Q}{2}$	$AQ^\theta - \frac{bc}{2} \ln Q$	No	Yes	?	?
$A \ln Q - b P$		$\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$	$(A - \frac{bc}{2}) \ln Q$	No	Yes	Yes	?
$A \ln Q - b P^2$		$\frac{A \ln Q}{2b} + \frac{c \ln Q}{2}$	$(A - \frac{bc}{2}) \ln Q$	Yes	Yes	Yes	?
$\sqrt{A \ln Q - b P}$	$c \ln Q$	$\frac{2A+bc}{3b} (\ln Q)$	$\sqrt{\frac{A-bc}{3}} (\ln Q)$	Yes	Yes	Yes	Yes
$\sqrt{A Q^\theta - b P}$	$c Q^\theta$	$\frac{2A+bc}{3b} Q^\theta$	$\sqrt{\frac{A-bc}{3}} Q^\theta$	Yes	Yes	Yes	Yes
$A Q^\theta e^{-\frac{P}{b}}$	$c \ln Q$	$b + c \ln Q$	$AQ^\theta e^{-\frac{b+c \ln Q}{b}}$	No	No	Yes	No
$A \ln Q e^{-\frac{P}{b}}$		$b + c \ln Q$	$A \ln Q e^{-\frac{b+c \ln Q}{b}}$	No	No	Yes	No
$A e^{-\frac{P}{b Q^\theta}}$	$c \ln Q$	$bQ^\theta + c \ln Q$	$A e^{-\frac{bQ^\theta + c \ln Q}{b}}$	No	Yes	Yes	No

Table 1: Alternate ways to model demand for content at bundle price P . The exponent θ above is assumed to lie in $(0, 1)$ and further restricted to $(0, 2/3)$ in some cases. The “?” indicates that the property may hold or not depending on certain parameter values.

A Appendix

Table 1 lists multiple demand models that were examined, including multiple ways to incorporate Q into the demand function. As evident from the table, several of the standard demand formulations are not well great at capturing the bundle structure of demand, suggesting the chosen demand model, $D(P, Q) = \sqrt{A Q^\theta - b P}$, (with $\theta \in [0, 2/3]$).

A.1 Technical Details and Proofs

Proof of Lemma 1. The proof is divided into two parts representing the retailer’s price optimization problem, and the provision decisions Q_i^* of producers jointly with computation of Q^* and K , the number and set of active producers.

Retailer's optimal price P^* : The pricing problem is of usual form, given Q as an input. The retailer maximizes its payoff function, which is a $1-\gamma$ fraction of total surplus (Eq. 3a), and optimality conditions yield the price P^* . Equilibrium levels of demand and $S(Q)$, given Q , re obtained by substitution.

Production decisions (Q_i) and K : Each producer chooses Q_i to maximize $\gamma \frac{Q_i}{Q} S(Q) - c_i Q_i$ where Q is the aggregation of Q_i 's for active producers (with the IR constraint, $c_i \leq \frac{\gamma S(Q)}{Q}$). Optimality conditions yield a series of equations across all active producers, $Q_i = \left[2 - \frac{bc_i Q}{\gamma} \left(\frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{3/2} \right] \frac{Q}{(2-3\theta)}$ (i.e., Eq. 4c). Let K be the number of active producers. Then, (i) because of the ascending order on cost, the set of IR constraints reduces to $c_K \leq \frac{\gamma S(Q)}{Q}$, and (ii) adding the series of equations (4c, using $Q = \sum_{i=1}^K Q_i$) yields Eq. 5a and 5b. The total output level (Eq. 5c) is obtained by algebraic rearrangement.

For the feasibility condition (IR constraint), note that if some i satisfies $c_i \leq \frac{c_1 + \dots + c_{i-1} + c_i}{i - (1-3\theta/2)}$ then so does $i-1$ (for $i \geq 2$). The first condition can be rearranged to $c_i(i-1) - c_i(1-3\theta/2) \leq (c_1 + \dots + c_{i-1})$. Further, $LHS(i-1) < LHS(i)$, i.e., $(c_{i-1}(i-1) - c_{i-1}(1-3\theta/2)) \leq (c_i(i-1) - c_i(1-3\theta/2))$ (for $i \geq 2$), and hence $c_{i-1} \leq \frac{c_1 + \dots + c_{i-1}}{i-1 - (1-3\theta/2)}$.

Write $Z = \left(\frac{1}{Q^\theta} \frac{3}{A-bc} \right)^{3/2}$, which is the common term between $c_K \leq \frac{\gamma S(Q)}{Q}$ and Eq. 5b. Rewriting $c_K \leq \frac{\gamma S(Q)}{Q}$ using Eq. 3e yields (i) $\frac{QZb}{\gamma} \leq \frac{2}{c_K}$, while Eq. 5b yields (ii) $\frac{QZb}{\gamma} = \frac{1}{\bar{c}_{(K)}} \left(2 - \frac{(2-3\theta)}{K} \right)$. Combining the two yields the feasibility condition in the form $c_K \leq \frac{\bar{c}_{(K)}}{K - (1-3\theta/2)}$. Now, because producers are arranged in ascending order of cost, the set of active producers is exactly the set $\{1 \dots K\}$ where K is the highest K which satisfies the feasibility condition. Plugging Eq. 5b into Eq. 4c yields Q_i in the required form (Eq. 2b in Lemma 1), alternately write Eq. 4c using Z and substitute $\frac{QZb}{\gamma} = \frac{1}{\bar{c}_{(K)}} \left(2 - \frac{(2-3\theta)}{K} \right)$. Plugging Eq. 5b into Eq. 4d yields each producer's fractional share of total product value in equilibrium, specifically $\frac{Q_i}{Q} = \left(2 - \frac{c_i}{\bar{c}} \right) \left(\frac{2}{2-3\theta} - \frac{1}{K} \right)$. The equilibrium expressions in Lemma 1 for P^* , D^* , and the profits of retailer and producers can be derived similarly. ■

Proof of Remark 1. To see how the best response function for Q_i behaves against the output of competing producers Q_{-i} , rewrite Eq. 4c as

$$\underbrace{2Q_{-i} + \frac{3\theta}{2}Q_i}_{\text{LHS}(Q_i)} = \underbrace{Zc_i(Q_i+Q_{-i})^{2-\frac{3\theta}{2}}}_{\text{RHS}(Q_i)}, \quad \text{where } Z = \frac{b}{\gamma} \left(\frac{3}{A-bc} \right)^{\frac{3}{2}}$$

For any given Q_{-i} the best response Q_i^* lies at the intersection of the two increasing functions LHS and RHS above. Note that LHS is linear with slope less than 1 and zero curvature, while RHS has positive curvature (because $2-\frac{3\theta}{2} > 1$). When Q_{-i} is small enough, increasing it will yield higher Q_i^* , for any θ , Z and c_i . An interior solution $Q_i^* > 0$ occurs if $Zc_iQ_{-i}^{1-3\theta/2} < 2$. Taking derivatives (against Q_{-i}) on both sides of above equation, we see that LHS rises faster than RHS—causing increase in Q_i^* —when Q_{-i} is small (and this is more likely for small Z and small c_i), while the reverse occurs for high Q_{-i} . Hence, we get a strategic complementarity in outputs where best-response output functions are increasing in competitor’s output at lower levels of output. ■

Proof of Proposition 1. The result follows trivially because Q_i is inversely related to c_i , see Eq. 2b. The second result is the consequence of Q^* being monotonic in γ (see Eq. 2c). ■

Proof of Proposition 2. Write $c_1+\dots+c_i = (c_1+\dots+c_{i-1}) + c_i$, which equals $c_i + (i-1)\bar{c}_{i-1}$. Plugging this into Eq. 2a, yields the result after algebraic simplification, and also generates the special case for $i=2$. ■

Proof of Proposition 3. First, computing $\frac{Q_i^*}{Q^*}$ using Eq. 2b-2c reveals that the dispersion in share of output becomes more extreme as θ increases (within a range where K remains the same); this is because higher θ gives more weight to the term that multiplies $-c_i$. Hence increase in θ increases the output shares for low c_i producers. Second, for how θ affects K , Eq. 2a trivially yields $\frac{\partial K}{\partial \theta} < 0$, except that the inequality is, strictly speaking, weak because K is discrete. Compute

$\frac{\partial Q_i^*}{\partial \theta}$ using Eq. 2b. For any K denote the term $\left(\frac{K-(1-3\theta/2)}{c_1+\dots+c_K}\right)$ as Z_K . Then, after a couple of steps of differentiation and rearrangement,

$$\frac{\partial Q_i^*}{\partial \theta} = \underbrace{\frac{2Q^*}{(2-3\theta)} \left(1 - c_i \frac{\partial Z_K}{\partial \theta}\right)}_{1. \text{ decreasing in } c_i} + \underbrace{\frac{2(1-c_i Z_K)}{(2-3\theta)} \left((2-3\theta) \frac{\partial Q^*}{\partial \theta} - 3Q^*\right)}_{2. \text{ same sign for all } i}.$$

Now, trivially, $\frac{\partial Q^*}{\partial \theta} > 0$ (see Eq. 2c), therefore $\frac{\partial Q_i^*}{\partial \theta} > 0$ for at least some i . Since, as marked above, the first term is decreasing in c_i (and the other is the same sign for all i) $\frac{\partial Q_i^*}{\partial \theta}$ has more chance to be negative for high i , leaving two possibilities, either (i) $\frac{\partial Q_i^*}{\partial \theta} > 0$ for all i , or (ii) $\frac{\partial Q_i^*}{\partial \theta} > 0$ for low i and $\frac{\partial Q_i^*}{\partial \theta} < 0$. Possibility (ii) is confirmed (hence (i) ruled out) by counterexample, as displayed in Fig. 5b. The positive effect of θ on the retailer's profit is trivial (from the profit function) and also consequently explains that profit increases for θ for those producers who experience higher Q_i^* . ■

Proof of Propositions 4 and 5. Consider the equilibrium when the economy has K active producers. Suppose producers $\{i, j\}$ (with $j > i$ and both $\leq K$) merge yielding a new entity whose cost parameter is the mean of c_i and c_j . Now, from Proposition 2, this merger has no effect on the participation decisions of producers with index higher than j , because average cost of their superior producers remains the same. Hence we only need to consider the change in Q_i 's of all active producers. This effect is obtained from Eq. 2b and Eq. 2c in Lemma 1, but these must be considered simultaneously because they yield interrelated outputs Q_i 's and Q . For convenience rewrite Eq. 2b as $Q_i = \left(\frac{2}{2-3\theta} - \frac{c_i}{\bar{c}} \frac{1}{K}\right) Q$. With this format—and, for the moment, ignore any change in Q itself—it is evident that for all active producers other than $\{i, j\}$, the only impact on output decision occurs on account of lower K , i.e., they have less output in equilibrium. For producers i and j the reduction effect is starker; this can be seen by comparing their new output with the sum of previous output of i and j , and recognizing that $\frac{2}{2-3\theta}$ lies between 1 and infinity. There are two effects to note while comparing. (i) The previous joint output of $\{i, j\}$ has a higher term $2\frac{2}{2-3\theta}$ vs. just $\frac{2}{2-3\theta}$ of the merged entity; (ii) the merged entity has a greater term $-\frac{c_i+c_j}{2}$ vs. c_i+c_j of the joint

output. However, part (ii) is multiplied by $\frac{1}{\bar{K}}$ hence has a smaller effect than part (i). Therefore, the only viable effect on Q is that the post-merger value is lower. This lower value (which is a multiple in Eq. 2b) ensures that every producer has lower output. If the merged producers have a better cost structure than the average of c_i and c_j then it is possible that the merged entity has higher output. However, the merger has positive impact on profit of the merged producers even with the lower output, because this lower output veers more towards the efficient level for them. ■

Proof of Proposition 6. From Proposition 4, post-merger total output is lower. From Eq. 3e, retailer's total profit in equilibrium (and, similarly, $S^*(Q)$) is an increasing function of Q (assuming the same level of γ), hence the merger reduces the retailer's profit. ■

Proof of Corollary 2. Consider the general case first, with $\gamma < 1$. Because γ linearly affects the retailer's profit, it does not directly impact bundle price, and P^* is as given in Lemma 1. For production decisions, though, a higher γ gives producers higher marginal gain for each unit of cost, leading to higher Q_i and Q . It is evident from Eq. 2c that this effect is more than linear in Q , because $(2-3\theta) < 2$. For the consortium case, with $\gamma=1$, the profit function Eq. 3a is replaced by simply $(P - cQ^\theta)D(P, Q)$ (since there is no retailer), and the rest of the apparatus can be executed to yield the results for this extreme case. ■

Source file : co-creation-MS-final-2020.tex, March 14, 2020.

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