Inequality and Aggregate Demand

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Abstract

We explore the effects of transitory and persistent increases in income inequality on the level of economic activity in the context of a Bewley-Huggett-Aiyagari model in its Keynesian regime of constant real interest rates. A temporary rise in inequality lowers output modestly because the covariance between changes in income and marginal propensities to consume is negative but small in the model and the data. A permanent rise in inequality leads to a permanent Keynesian recession whose magnitude depends on the elasticity of aggregate savings to idiosyncratic uncertainty—a potentially much larger effect. Economic slumps create endogenous redistribution and give rise to an *inequality multiplier*. By reducing the marginal product of capital, they also lead to declines in investment that further amplify the recession. Government spending and public debt issuances are expansionary and crowd capital in. Our methodology separates sufficient statistics from general equilibrium multipliers and is applicable to the study of all macroeconomic models of aggregate demand.

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1 Introduction

There is an old idea[1] that the distribution of income is an important determinant of aggregate economic activity, with higher income inequality reducing aggregate demand and employment. This idea is reflected in modern discussions on the consequences of income inequality. For example, a recent Economic Report of the President (Council Of Economic Advisers 2012) argues that

> some of the recent patterns in aggregate spending and saving behavior—including the sluggish growth in consumer spending—may reflect the sharp rise over the past 30 years in the inequality in the income distribution in the United States. [...] The rise in income inequality may have reduced aggregate demand, because the highest income earners typically spend a lower share of their income—at least over intermediate horizons—than do other income groups.

In partial equilibrium, there are two related reasons why higher income inequality can reduce aggregate consumption. The first pertains to transitory changes: if poorer households have higher marginal propensities to consume than richer households, then any redistribution of income from the former to the latter will lower consumption in the short run. The second pertains to permanent changes: due to precautionary motives and the presence of borrowing constraints, increases in the volatility of household earnings processes will typically permanently raise aggregate savings. Both of these effects are present in a class of incomplete-market macroeconomic models that are widely used to analyze the interaction between macroeconomics and inequality (Bewley 1980, Huggett 1993, Aiyagari 1994), and both have received empirical support.[2]

To date, however, this mix of theory and empirical evidence has not proved useful to evaluate the general equilibrium consequences of higher income inequality for output. This is because the vast majority of closed-economy equilibrium versions of the models in use in the literature are studied in their *neoclassical regime* and tend to predict an output effect of the opposite sign. When all resources are fully utilized in production, the imbalance resulting from the fall in desired consumption and increase in desired savings is resolved by a decrease in real interest rates and an increase in investment—leading to a subsequent output boom from capital accumulation.

In this paper, we augment an otherwise standard Bewley-Huggett-Aiyagari model with downward nominal wage rigidities and study its *Keynesian regime*. We specify monetary policy in this regime so that real interest rates are fixed, and hence the (rationed) level of employment

[1] Famous proponents include Pigou (1920), Keynes (1936), Kalecki (1954) and Kaldor (1955).

[2] On MPC differences by income group, see Dynan, Skinner and Zeldes (2004), Johnson, Parker and Souleles (2006), Parker, Souleles, Johnson and McClelland (2013), Jappelli and Pistaferri (2014), or Misra and Surico (2014). On the effect of idiosyncratic risk on savings, see Carroll and Samwick (1997) and Browning and Lusardi (1996) for a review.

becomes the main equilibrating variable. "Holding monetary policy fixed" in this way—an assumption that we rationalize with a simple rule for the nominal interest rate over the short run, and with the presence of a binding lower bound on this rate over the long run—draws a clear contrast between our regime and the traditional neoclassical regime.[3] We show that the Keynesian regime restores partial-equilibrium intuitions, allowing us to investigate the transmission mechanism of inequality to output.

In particular, we establish two sets of first order approximations for the percentage change in the level of output $\frac{dY}{Y}$ resulting from a change in inequality. Our formulas have the general form:

$$\frac{dY}{Y} = (\text{General equilibrium multiplier}) \cdot (\text{Partial equilibrium sufficient statistic}) \tag{1}$$

with the sufficient statistic being directly tied to the empirical evidence previously mentioned.

Our first result concerns unexpected transitory changes in inequality, or short-run redistribution. In this case, $\frac{dY}{Y}$ in (1) refers to the immediate effect on output, and the sufficient statistic is the vector of cross-sectional covariances between the direction of redistribution and marginal propensities to consume at different dates. This establishes the precise sense in which the heterogeneity in MPCs between rich and poor matters for the output response. Because all income is spent in the long-run, there can only be a significant short run output change if the multipliers are larger for consumption spending in earlier dates. This is the case in our model.

Our second result concerns permanent changes in inequality. There, $\frac{dY}{Y}$ refers to the steady-state effect on output, and the sufficient statistic is an appropriately-defined average elasticity of individual savings to the change in their earnings process. In other words, under our monetary policy assumption, our model features a permanently depressed economy—an inequality-driven secular stagnation. In this case, MPC differences no longer play a role in the transmission of inequality to output. We demonstrate why it is important to consider the effect on asset markets rather than consumption, in other words why secular stagnation is a phenomenon due to a shortage of stocks rather than flows.

While we stress that the partial-equilibrium reasons for higher desired aggregate savings are very different in the short run and in the long run, we also uncover similarities in the general equilibrium adjustment of output in these two cases. The general equilibrium multipliers entering (1) are in general complex functions that can only be studied numerically, but they are independent of the source of the shock to aggregate demand. They embody a number of key Keynesian adjustment mechanisms that apply to all models in which quantities adjust to clear markets.

First, the output response is affected by the endogenous redistribution that results from labor demand shortfalls—a phenomenon we call the *inequality multiplier*. We parsimoniously capture the notion that such shortfalls may not affect everyone's employment prospects equally

[3] Woodford (2011) makes a similar assumption in his analysis of fiscal multipliers.

by introducing a reduced form incidence function $\gamma_i(L)$, which parametrizes the extent to which individual i's employment is affected when aggregate employment L changes. When a labor demand shortfall hurts poor individuals the most, it exacerbates the adverse demand effects of inequality, and the inequality multiplier is high. In the short term there is a mitigating redistributive effect: since factors of production are paid their marginal product, a fall in employment raises real wages and lowers capital rental rates. This lowers the inequality multiplier to the extent that the MPCs of wage earners are higher than those of shareholders.

Second, investment plays an amplifying role, in contrast to the dampening role it has in the neoclassical regime. From the point of view of households, falls in employment reduce their desired consumption and lowers their desire to save. From the point of view of firms, low future employment reduces future marginal products of labor and lowers their desire to invest. In equilibrium, both savings and investment are depressed. In other words, in our Keynesian regime, an increase in desired savings results in lower equilibrium savings. This formalizes the Keynesian "paradox of thrift" (Keynes 1936).

Third, fiscal policy plays a crucial role. Our benchmark fiscal assumption is that the levels of government debt and spending are held fixed. We show that the general equilibrium multiplier in (8) is reduced if fiscal policy expands either the level of spending or the level of debt in response to falls in employment. Our analysis therefore illustrates the importance of government liquidity supply in affecting the Keynesian equilibrium level of output, just as others (Aiyagari and McGrattan 1998) have highlighted its effect in affecting equilibrium interest rates in the neoclassical regime of the same model. Public debt increases and government spending both have the property that, by raising the level of employment, they crowd in private sector liquidity and mitigate the asset supply shortfall.

We use our model to quantify the potential effects of inequality on output. Our initial steady-state is meant to capture recent US macroeconomic conditions. The two distinctive features of our calibration are 1) a high degree of income and wealth inequality, a consequence of the high degree of variance and kurtosis in earnings changes documented in the administrative data by Guvenen, Ozkan and Song (2014), and 2) a 0% steady-state real free interest rate, consistent with most recent measures. Our steady-state has full employment, but it features 0% inflation and a 0% bound on nominal interest rates, and so it is at the edge of a liquidity trap. This justifies our assumption of a Keynesian regime for monetary policy for the long run as well as the short run.

In the short-run case the output effect of a transitory increase in inequality turns out to be relatively small: the level of output falls by around 0.2% in response to a shock that increases the standard deviation of log earnings by 0.02, a typical year-to-year change in the U.S. We show that the partial equilibrium effect underlying this result is consistent with the MPC differences by income groups computed from Italian survey data (Jappelli and Pistaferri 2014), with a general equilibrium multiplier that is not much above one.

In the long-run case, both our calibration and the literature's cross-sectional estimates of the effects of individual uncertainty on individual savings (Carroll and Samwick 1997) imply a substantial steady-state fall in output from the same moderate rise in inequality. In the transition, the fall is even larger in the short run due to an abrupt fall in investment. We stress that this result is a hypothetical, based on an assumption that monetary policy does not find a way to lower nominal interest rates or raise inflation, and fiscal policy does not intervene by increasing government debt or spending. Hence our analysis highlights inequality-driven secular stagnation as a theoretical possibility, but also stresses that expansionary fiscal and monetary policy interventions can both mitigate it.

Inequality has been rising for decades in the United States and other developed countries. While our core results relate to the output effects of further increases in inequality in a Keynesian regime where monetary policy is contrained because the natural rate of interest is low, one consequence of the previous rise in income inequality may have been to lower this equilibrium rate. Broad macroeconomic trends are consistent with such a causal effect. Figure 1 plots the Laubach-Williams measure of the equilibrium U.S. real interest rate against the standard deviation of male W2 earnings according to the tabulations of Guvenen et al. (2014) from 1978 to 2012. The purple line shows that household income inequality, after rising rapidly in the 1980s and slowing down at the beginning of the 1990s, resumed its increase in the 2000s, and inequality in the last three available years of data (2009-2011) is the highest on record. During the same period, the U.S. real interest rate fell, as shown by the green line, a widely-used measure of the equilibrium rate (Laubach and Williams 2003). Both trends have been widely documented.

In order to investigate the role of inequality in depressing equilibrium interest rates, we ask how much of a fall in real interest rates towards their current level of 0% could be explained by the increase in inequality alone, assuming that we were in a steady-state both in the early 1980s and now. We find that a rise in inequality that matches figure 1's 12 point increase in the standard deviation of log earnings can explain a fall in the equilibrium rate of around 90 basis points. Hence inequality can, according to the model, account for some, but not all of the fall in equilibrium interest rates. By contrast, the fall in the relative price of investment of the magnitude documented by Karabarbounis and Neiman (2014) can only explain about 20 basis points (Thwaites 2015). Clearly both explanations are complementary, as are others that our model cannot evaluate directly, such as demographics and the global savings glut (Rachel and Smith 2015).

Literature review. Our paper relates to several strands of the literature.

First, a very large literature has documented increasing labor income inequality in the United States and other advanced economies (Katz and Autor 1999, Piketty 2003 , Piketty and Saez 2003, Kopczuk, Saez and Song 2010, Heathcote, Perri and Violante 2010a). Many informal arguments have been raised as to its macroeconomic consequences (Stiglitz 2013). We provide

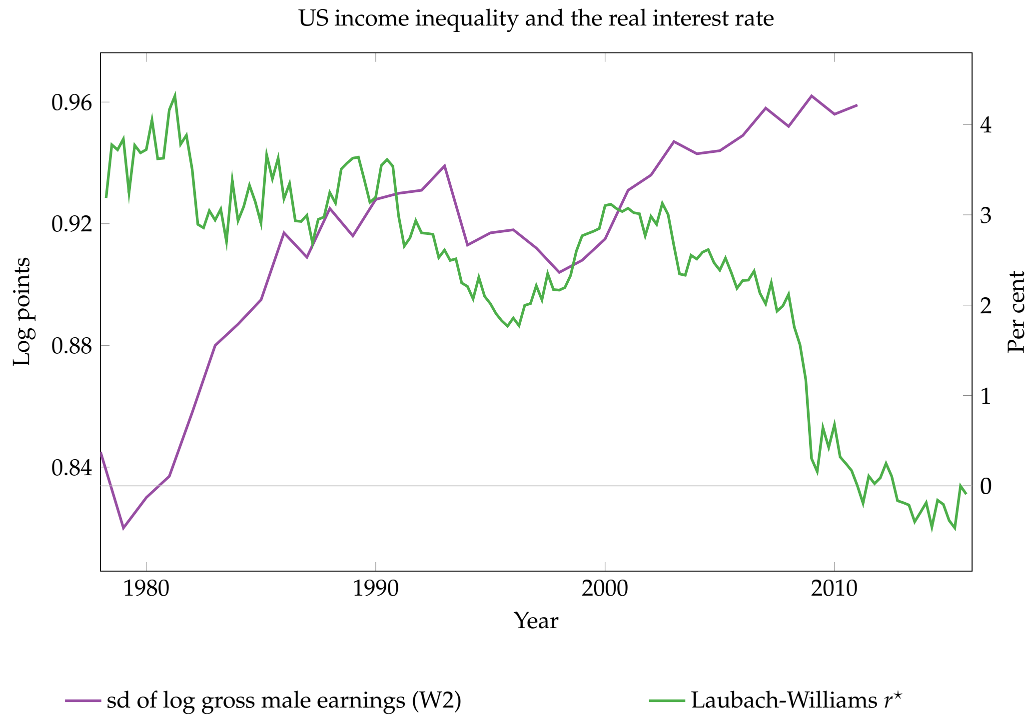

Figure 1: U.S. income inequality and the equilibrium real interest rate

a formalization of these arguments, and a discussion of when partial equilibrium calculations using existing evidence on observed marginal propensities to consume by income group can be appropriate.

Other papers have studied the macroeconomic effects of increasing inequality (see for example Heathcote, Storesletten and Violante 2010b, Iacoviello 2008) and its effect on interest rates (for example Favilukis 2013). All of these papers study neoclassical equilibria. We pave the way for the study of the effects of inequality in Keynesian equilibria of the same models, showing how tractable the wage rigidity assumption can be, demonstrating how to compute general equilibrium multipliers, and pointing out the key forces at play.

Our exploration of the short run properties of the Bewley-Huggett-Aiyagari model in the Keynesian regime is part of a rapidly growing literature that adds nominal rigidities to heterogeneous-agent models to study the effect of exogenous shocks such as tightening of borrowing constraints (Guerrieri and Lorenzoni 2015), or the effects of fiscal policy (McKay and Reis 2016) and monetary policy (Gornemann, Kuester and Nakajima 2014, Auclert 2016, McKay, Nakamura and Steinsson 2015, Kaplan, Moll and Violante 2016).[4] These papers all emphasize the importance of redistribution between agents with different MPCs in affecting output. We fur-

[4] All of these models assume price rigidities and flexible wages, generating either counterfactual wealth effects on labor supply (when preferences are separable) or strong output effects from the complementarities between consumption and labor supply (when they are not). They also make strong assumptions on the distribution of the (countercyclical) profits across agents. Our assumption of wage rigidities sidesteps all of these issues.

ther clarify this mechanism by holding policy constant and studying pure redistribution as an exogenous source of fluctuations.

Our long-run experiments also relate to the literature that models the feedback loop that arises between increases in idiosyncratic risk and the level of economic activity under constrained monetary policy (Ravn and Sterk 2013, den Hann, Rendahl and Riegler 2015, Bayer, Lütticke, Pham-Dao and von Tjaden 2015, Challe, Matheron, Ragot and Rubio-Ramirez 2014, Heathcote and Perri 2016). These papers have more explicit models of the asset and the labor markets than we do. By contrast, we follow Werning (2015) and sidestep the microfoundations of the labor market via our reduced-form γ function. This allows us to illustrate simply the key mechanisms at play in these models. In particular, we show that, depending on the elasticity of γ to employment, the model may be in a region where inequality amplifies exogenous shocks or even a region of multiple steady-states, which shows the continuity between various results in the literature. We also maintain a standard neoclassical formulation for the investment side of our model. In particular, the fall in equilibrium investment we observe is simply due to the effect of unemployment on the marginal product of capital, and not to switch to 'unproductive' investment as in den Hann et al. (2015) and Bayer et al. (2015).

Our exploration of a long run Keynesian regime is part of another growing literature that has taken up the modeling of the Hansen/Summers secular stagnation idea (Hansen 1939, Summers 2013) by assuming permanent price rigidities and constrained monetary policy, in closed economies (Eggertsson and Mehrotra 2014, Benigno and Fornaro 2015, Michau 2015) or in open economies (Caballero, Farhi and Gourinchas 2015, Eggertsson, Mehrotra, Singh and Summers 2016).[5] Our conclusions regarding the role of government spending and liquidity multipliers are related to theirs, though we stress a novel capital crowd-in mechanism which is absent from these models without capital. By stressing the important role of government liquidity supply, our paper also relates to the literature on the optimal level of government liquidity in models with financial frictions (Woodford 1990, Aiyagari and McGrattan 1998) as well as the literature on safety traps (Caballero and Farhi 2015).

Finally, our sufficient statistic approach to the computation of magnitudes relates to a recent literature in trade (Arkolakis, Costinot and Rodríguez-Clare 2012) and macroeconomics (Auclert 2016, Berger, Guerrieri, Lorenzoni and Vavra 2015, Berger et al. 2015, Alvarez, Le Bihan and Lippi 2016, Dávila 2016) that is developing methods to obtain quantitative conclusions in general equilibrium models that are as independent of the details of the underlying structural model as possible. In this paper we explain the general methodology underlying this approach. We view this as a promising way to avance macroeconomic science by improving comparability across increasingly complex numerical models, and making sure their predictions are in line with micro data.

[5] These papers model "demand-side" secular stagnation. See Kozlowski, Veldkamp and Venkateswaran (2015) for an alternative model of secular stagnation relying on an explanation due to technology and belief changes that does not require sticky prices.

2 Model

2.1 Environment

Households. We consider a population made of ex-ante identical households who face idiosyncratic, but no aggregate risk. In each period t, household i is in idiosyncratic state $s_{it} \in \mathcal{S}$. s_{it} follows a Markov process with transition matrix Λ. We assume that at all times, the mass of households in each idiosyncratic state s is equal to the probability $\lambda(s)$ of s in the ergodic distribution induced by Λ.

The household maximizes $\mathbb{E}\left[\sum\left(\prod_{\tau\leq t}\beta_\tau(s_{i\tau})\right)u(c_{it})\right]$, where u is a common period utility function, subject to the period budget constraints

$$\begin{aligned} c_{it} + b_{it} + p_t v_{it} &= y_t(s_{it}) + (1+r_{t-1})\,b_{it-1} + (p_t + d_t)\,v_{it-1} \\ b_{it} + p_t v_{it} &\geq 0 \end{aligned} \tag{2}$$

Two assets are available for intertemporal trade: one-period risk-free bonds b_{it} and shares v_{it} in intermediate-goods firms. There is a mass one of such firms and we will show that they are identical in equilibrium, so we assume without loss of generality that households have an equal investment in each. Each share costs p_t at time t and delivers a stream of dividends $\{d_s\}$ starting at $s = t+1$. Households have perfect foresight over p_t, d_t, and the real interest rate r_t. They may invest any amount in bonds and shares provided that they maintain their net worth $a_{it} \equiv b_{it} + p_t v_{it}$ positive at all times.

Motivated by a large literature on the equity premium, we assume that investing in firms entails earning a premium $\varrho \geq 0$, such that the equation

$$1 + r_t + \varrho = \frac{p_{t+1} + d_{t+1}}{p_t} \tag{3}$$

holds at all times. We make equation (3) compatible with perfect foresight using the following assumption. We specify that, in every period $t+1$, the government levies a tax proportional to the value of shares at the end of period t with rate ϱ, and immediately refunds every household their own tax payments $\varrho p_t v_{it}$. Households rationally perceive the tax, but treat their refund as a lump-sum. Their optimal portfolio choice then implies (3),[6] while their intertemporal consumption choice implies

$$u'(c_t) \geq \beta_{it}(1+r_t)\,\mathbb{E}_t\left[u'(c_{it+1})\right] \tag{4}$$

with strict inequality implying a binding borrowing constraint, with net worth $a_{it} = 0$. In other words, the equity premium affects consumption via an income effect, but no substitution effect.

[6] At time t, unconstrained households are indifferent between investment in bonds yielding return $1 + r_t$ and investment in shares yielding return $\frac{p_{t+1}+d_{t+1}-\varrho p_t}{p_t}$. This yields $\frac{p_{t+1}+d_{t+1}}{p_t} - \varrho = 1 + r_t$.

Under our formulation, households are indifferent between holding bonds and shares, and are effectively discounting the future at rate r. Yet, if $\varrho \neq 0$, their realized returns on assets depend on their portfolio mix, making their asset allocation rule an important determinant of wealth accumulation. We assume that households follow the simple rule of allocating the fraction $\theta(a)$ of their wealth a to shares pv, and in our calibration we will infer $\theta(a)$ directly from the data.[7] Together, these assumptions provide the most tractable way of introducing an equity premium, important to match many aspects of the aggregate macroeconomic data, while preserving the analytical simplicity of a model without risk.

We specify household income in two steps. First, pre-tax labor income z_{it} is given by the product of the real wage $\frac{W_t}{P_t}$ and the amount of endowment that households are able to supply:

$$z_t(s_{it}) = \frac{W_t}{P_t} \cdot \Big(e_t(s_{it}) \cdot L_t \cdot \gamma(s_{it}, L_t)\Big) \tag{5}$$

where the γ function satisfies

$$\gamma(s, 1) = 1 \quad \forall s \tag{6}$$

Households' full idiosyncratic labor endowment is e_{it}, and this is the amount that they supply in case of full employment ($L_t = 1$). As per the standard formulation in the literature, their pre-tax income is then given by $\frac{W_t}{P_t} e_{it}$. We normalize the aggregate endowment of labor to 1: $\mathbb{E}[e_{it}] = 1$.

We think of inequality changes as affecting the way endowments are distributed across individuals in different states s_{it}, through the time-varying function $e_t(s_{it})$. As discussed in section 2.4, this specification is a parsimonious way of capturing a large number of microfounded theories explaining the increase in income inequality in the U.S. and other developed economies.

The economy may experience a labor demand shortfall, $L_t < 1$. In that case, a household in idiosyncratic state s is constrained to supply the fraction $L_t \cdot \gamma(s, L_t)$ of his full endowment, with L_t describing the effect of aggregate employment conditions and γ the distributional impact of these conditions. We make this distinction between the aggregate and distributional effect of employment precise by assuming that γ satisfies

$$\mathbb{E}[e_t(s_{it})\gamma(s_{it}, L)] = 1 \quad \forall L \leq 1,\ \forall t \tag{7}$$

and hence $\mathbb{E}[z_{it}] = \frac{W_t}{P_t} L_t$ at all times. When $\gamma(s, L_t) = 1$, for all s, all households are equally rationed. By contrast, when $\gamma(s, L_t) \neq 1$ for some s, labor demand shortfalls can be a source of endogenous increase in inequality. We will see that this gives rise to a phenomenon we call the *inequality multiplier*.

Our specification of the γ function is similar to Werning (2015)'s, and captures a number of

[7] In our benchmark calibration, the share of assets $\theta(a)$ shares is increasing in wealth. As conjectured by Piketty and Zucman (2014), this mechanism contributes to wealth inequality, albeit modestly in our model.

ways in which an economy might adjust to depressed employment $L < 1$. Adjustment in our model happens along the intensive margin, with the fraction of individuals within each state s remaining constant at $\lambda(s)$. But it can, for example, capture the idea that the recession impact only individuals in certain "underemployment" states $s \in \mathcal{U}$, provided that $\gamma(s, L) = \frac{1}{L}$ for $s \notin \mathcal{U}$. More generally, it can capture broad patterns of cyclicality of income risk, such as those documented in Storesletten, Telmer and Yaron (2004) and Guvenen et al. (2014). Other authors such as den Hann et al. (2015) microfound this γ function with a search and matching model of the labor market; we sidestep this complication while preserving the core insight of their mechanism by directly parametrizing γ directly.

The income y_{it} that enters household's budget constraint is post-tax. We assume that the government runs a tax system with a constant intercept and a linear slop, so that y_{it} is an affine function of z_{it}

$$y_t(s_{it}) = T_t + (1 - \tau_t)(1 - \tau_t^r) z(s_{it})$$

The marginal tax rate on labor income is broken down into a component τ_t^r that is earmarked for redistribution and a component τ_t that is available for general revenue. The government immediately rebates the earmarked revenue, so that $T_t = (1 - \tau_t)\tau_t^r \mathbb{E}[z_{it}]$ and we can rewrite y_{it} as

$$y_t(s_{it}) = (1 - \tau_t)(\tau_t^r \mathbb{E}[z_{it}] + (1 - \tau_t^r) z(s_{it})) \tag{8}$$

Equation (8) shows that τ_t^r quantifies the amount of social insurance embedded in the tax system. Because we assume that labor supply is inelastic, taxes in our model are not distortionary. We abstract away from the efficiency costs of taxes to better focus on their effect on the income distribution and the degree to which households are insured against idiosyncratic income fluctuations.

Final goods firm. The economy has two two types of final goods: consumption goods with price P_t and investment goods with price P_t^I. Both types of final goods are produced by a competitive retail sector that packages intermediate goods x_{jt}. Consumption goods and investment goods are produced with the respective technologies

$$\begin{aligned} Y_t^C &= \left(\int_0^1 \left(x_{jt}^C\right)^{\mu_t}\right)^{\frac{1}{\mu_t}} \\ Y_t^I &= \frac{1}{X_t}\left(\int_0^1 \left(x_{jt}^I\right)^{\mu_t}\right)^{\frac{1}{\mu_t}} \end{aligned}$$

A fall in the variable X_t represents an exogenous improvement in the technology of production of investment goods relative to consumption goods. Such technological improvements lower the relative price of investment. Indeed, our assumptions imply that the price of investment goods is $P_t^I = X_t P_t$, where P_t is the price of consumption goods at date t given by

$$P_t = \left(\int_0^1 \left(p_{jt}\right)^{\frac{1}{\mu_t - 1}} \right)^{\mu_t - 1}$$

Moreover, the demand for intermediate good j at time t is given by

$$\frac{x_{jt}^C}{Y_t^C} = \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu_t}{\mu_t - 1}} \qquad \frac{x_{jt}^I}{Y_t^I} = X_t \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu_t}{\mu_t - 1}} \tag{9}$$

Intermediate goods firms. There exists a continuum $j \in [0,1]$ of monopolistically competitive firms, each producing a quantity x_{jt} of differentiated product j under the constant returns to scale production function:

$$x_{jt} = F_t\left(K_{jt-1}, L_{jt}\right) \tag{10}$$

Each firm owns its capital K_{jt-1} at the beginning of time t, and has a unit mass of shares outstanding. It takes as given the price of consumption goods P_t, the price of investment P_t^I and the nominal wage W_t, as well as the aggregate demand for consumption Y_t^C and investment Y_t^I. Firm j's net investment $K_{jt} - K_{jt-1} = I_{jt} - \delta K_{jt-1}$ is subject to convex adjustment costs worth $\zeta\left(\frac{K_{jt} - K_{jt-1}}{K_{jt-1}}\right) K_{jt-1}$ in units of the investment good, with the ζ function satisfying

$$\zeta(0) = 0 \quad \zeta'(0) = 0 \quad \zeta''(\cdot) \geq 0$$

Each period, the firm chooses its price p_{jt}, investment I_{jt} and employment L_{jt} to maximize the real value of future dividends d_{jt} discounted at the sequence $r_t + \varrho$ of rates inclusive of the equity premium:

$$J_{jt}\left(K_{jt-1}\right) = \max_{d_{jt}, K_{jt}} \left\{ d_{jt} + \frac{1}{1 + r_t + \varrho} J_{jt+1}\left(K_{jt}\right) \right\}$$

where dividends are equal to revenue net of the cost of labor and investment,

$$d_{jt} = \frac{p_{jt}}{P_t} F_t\left(K_{jt-1}, L_{jt}\right) - \frac{W_t}{P_t} L_{jt} - \frac{P_t^I}{P_t} \left(I_{jt} + \zeta\left(\frac{I_{jt}}{K_{jt-1}} - \delta\right) K_{jt-1} \right)$$

taking into account the effect of quantity produced on price according to the demand curve (9). In appendix A we examine the decision of each firm in detail. We show that firm j chooses its investment I_{jt} based on the value of the shadow price of capital q_t, such that

$$\zeta'\left(\frac{I_{jt}}{K_{jt-1}} - \delta\right) = q_t - 1 \tag{11}$$

and determines employment such that the physical marginal product of labor is at a markup

μ_t over the real wage $\frac{W_t}{P_t}$

$$F_{Lt}\left(K_{jt-1}, L_{jt}\right) = \mu_t \frac{W_t}{P_t} \tag{12}$$

Assuming all firms are identical initially, they therefore remain identical at all times. Given the unit mass of shares outstanding overall, the price of shares at time t is given by

$$p_t = \Pi_t + q_t X_t K_t$$

where Π_t is the present discounted value of flow monopoly profits, and $q_t X_t K_t$ is the value of installed capital. In turn, q_t satisfies a first-order difference equation reflecting the way in which firms, in the aggregate, decide to spread their investment plans in response to variations in the cost and the marginal revenue product of capital. In the steady state, where $q = 1$ and capital is a constant K with $I = \delta K$, this equation simplifies to

$$F_K\left(K, L\right) \quad = \quad \mu X \left(r + \varrho + \delta\right) \tag{13}$$

while (12) writes

$$F_L\left(K, L\right) = \mu \frac{W}{P} \tag{14}$$

Equations (13) and (14) determine steady-state labor and capital demand. The steady state capital-labor ratio $\frac{K}{L}$ is reduced with higher monopoly power μ, higher relative price of investment X and higher cost of capital $r + \rho + \delta$. The steady-state level of real wages must then satisfy (14), given $\frac{K}{L}$.[8] The steady state value of monopoly profits is simply

$$\Pi = \frac{\left(1 - \frac{1}{\mu}\right) F\left(\frac{K}{L}, 1\right) L}{r + \varrho} \tag{15}$$

In the past forty years, the share of national income going to labor has been falling (Elsby, Hobijn and Sahin (2013), Karabarbounis and Neiman (2014), Rognlie (2015)), raising questions about the impact of this phenomenon on inequality and aggregate demand. While our main source of change in inequality is within-labor holding the labor share fixed, our specification of production also allows us to address the consequences of a falling labor share. We consider the three major explanations that have been argued to explain such a fall: a change in the production technology (Piketty (2014)), a fall in the relative price of investment combined with a high elasticity of substitution in production (Karabarbounis and Neiman 2014), and an increase in monopoly power (Summers 2016, Krugman 2016). We model these as exogenous changes in F_t, X_t and μ_t, respectively.

[8] Appendix B provides explicit functional forms in the case of a CES production function.

Wage rigidities. We introduce a role for monetary and the possibility for equilibrium slumps in our model by assuming that the nominal wage cannot fall from period to period:

$$W_t \geq W_{t-1} \tag{16}$$

When labor demand falls short of the aggregate endowment because the constraint (16) is binding, households are rationed ($L_t < 1$) and are each constrained to supply the fraction $L_t \gamma(s_{it}, L_t)$ of their labor endowment.

Government. The government runs the redistributive transfer system embedded in (8): each period it taxes the fraction τ_t^r of gross income and rebates it in a lump-sum fashion. It then levies the additional tax rate τ_t on post-redistribution income, supplies a quantity of bonds $\overline{B}_t$ at time t, and spends G_t so as to satisfy its budget constraint

$$\tau_t \frac{W_t}{P_t} L_t + \overline{B}_t = G_t + (1 + r_{t-1}) \overline{B}_t \tag{17}$$

Our benchmark fiscal rule assumes that government maintains spending and debt are constant, while adjusting the tax rate such that (17) holds, in other words

$$G_t = G; \quad \overline{B}_t = \overline{B}; \quad \tau_t = \frac{G + r_{t-1}\overline{B}}{\frac{W_t}{P_t} L_t} \tag{18}$$

Adjusting fiscal instruments following (18) represents a natural way of "holding fiscal policy fixed". In section 4.5, we will consider the effects of alternative rules.

The central bank controls the nominal interest rate i_t on nominal bonds.[9] Perfect foresight implies that the real interest rate is

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{19}$$

where π_{t+1} is the rate of price inflation, $1 + \pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$. We consider a central bank that follows a rule for the real interest rate with a *target* $\widetilde{r}_t$

$$1 + i_t = (1 + \pi_{t+1}) \left(1 + \widetilde{r}_t + \phi \pi_t^W\right)$$

where $1 + \pi_t^W \equiv \frac{W_t}{W_{t-1}}$ is nominal wage inflation, and the response coefficient ϕ is sufficiently above 0.

We consider two types of central banks, each having a different target for the real interest rate. A neoclassical central bank sets its target as $\widetilde{r}_t = r_t^*$ where r_t^* is the equilibrium rate

[9] Nominal bonds can formally be introduced as as are assets in zero net supply that can be traded by households. Condition (19) is then an equation of no arbitrage between nominal and real bonds.

of interest that prevails in the economy without the (16) constraint. This rule achieves the neoclassical equilibrium allocation with r_t, and additionally pins down $\pi_t^W = 0$.

A Keynesian central bank sets its target as $\widetilde{r}_t = \overline{r}$. Under this rule, the central bank maintains the real interest rate at $\overline{r}$ whenever $\pi_t^W = 0$ is binding, tightening policy to maintain $L_t = 1$ whenever $\pi_t^W > 0$. Over the short run, this rule provides both a natural and a simple notion of "holding monetary policy fixed".[10] Over the long run, it is motivated by the literature on secular stagnation (Eggertsson and Mehrotra (2014), Caballero et al. (2015)). This literature posits more directly a lower bound on the nominal interest rate $i_t \geq \underline{i}$, and studies situation in which this lower bound, together with a wage rigidity constraint such as (16), can be permanently binding. In the steady state of this class of model, there is no wage and therefore no price inflation, leading precisely to a constant real interest rate equal to $\overline{r} = \underline{i}$, which may be above the steady state neoclassical rate r^*.

2.2 Equilibrium

Definition 1. Given K_{-1}, a sequence of exogenous shocks $\{e_t(\cdot), \tau_t^r, X_t, \mu_t, F_t\}$, and an initial joint distribution $\Psi_{-1}(s,b,v)$ over idiosyncratic states, bonds and stocks, an equilibrium is a set of aggregate quantities $\{C_t, I_t, K_t, Y_t, L_t, \Pi_t, B_t\}$, prices $\{i_t, r_t, q_t, P_t, R_t, W_t\}$, government policy $\{\tau_t, \overline{B}_t\}$, individual decision rules $\{c_t(s,b,v), b_t(s,b,v), v_t(s,b,v)\}$ and joint distributions $\Psi_t(s,b,v)$, such that households maximize utility subject to their budget constraint, firms maximize profits, the government follows its fiscal rule, the central bank follows its monetary policy rule, the Fisher equation (19) holds, the distribution of households is consistent with the exogenous law of motion and the decision rules, and all markets clear, except possibly for the labor market with complementary slackness in the wage rigidity constraint:

$$\begin{aligned}
L_t &\leq 1 \\
\int v_t(s,b,v)\, d\Psi_{t-1}(s,b,v) &= 1 \\
\int b_t(s,b,v)\, d\Psi_{t-1}(s,b,v) &= \overline{B}_t \\
C_t + X_t I_t + G_t + X_t \zeta \left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) K_{t-1} &= Y_t \\
W_t &\geq W_{t-1} \\
(L_t - 1)(W_t - W_{t-1}) &= 0
\end{aligned}$$

Definition 2. We say that the model operates in a *neoclassical regime* whenever its equilibrium allocation features $L_t = 1$ at all times. We say the model operates in a *Keynesian regime* whenever $r_t = \overline{r}$ at all times and L_t is sometimes strictly below 1.

[10] See Woodford (2011) and McKay et al. (2015) for other specifications of monetary policy as a rule for the real interest rate.

As discussed, the rule followed by the neoclassical central bank places the model in its a neoclassical regime, in which r_t is the only equilibrating variable in response to exogenous shocks. By contrast, the rule followed by the Keynesian central bank in response to negative demand shocks places the model in its Keynesian regime, in which L_t is the only equilibrating variable. The rest of the paper contrasts model outcomes under these two natural opposite specifications of monetary policy.

2.3 Calibration

We now calibrate the steady-state of our economy. We want to capture the recent US macroeconomic condition of very low real interest rates. We choose 2013 as our base year, since this is the last year for which household-level balance sheet data is available from the Survey of Consumer Finances (SCF). Average 10 years TIPS yields over that year were 0.07%. We therefore set $r = 0$. We assume that the economy is at full employment, $L = 1$, and consider shocks that can depress this level, temporarily or permanently. Our full-employment steady state is the benchmark for the large literature that has followed Aiyagari (1994). We defer discussing the properties of our novel Keynesian steady-state to section 4.

Preferences. We adopt a standard specification of preferences, with a constant discount factor for all households $\beta(s) = \beta$ for all s and period utility function with constant elasticity of substitution ν, $u(c) = \frac{c^{1-\nu^{-1}}}{1-\nu^{-1}}$. These preferences are known to generate realistic precautionary savings motives and differences in MPCs. We follow the literature practice of setting $\nu = \frac{1}{2}$ and calibrating β to hit our target for the real interest rate.

Income process. Our process for gross income ν_{it} is the one Kaplan et al. (2016) use to capture the higher-order moments of the distribution of earnings changes for US males documented by Guvenen et al. (2014). This process represents log gross income as the sum of a transitory and a persistent component, and involves substantially more idiosyncratic risk (including a high kurtosis of earnings changes) than typical calibrations based on AR processes with normal innovations.

We calibrate our redistributive tax rate τ^r using data from the Congressional Budget Office (2013). This report shows average market income $\mathbb{E}_J[z_i]$ and average market income plus federal transfers net of taxes $\mathbb{E}_J[y_i]$ at each quintile J of the income distribution of nonelderly households in 2006, the latest year in which the data is available. Running a linear regression, we recover

$$\frac{\mathbb{E}_J[y_i]}{\mathbb{E}[z_i]} = 0.143 + 0.666\frac{\mathbb{E}_J[z_i]}{\mathbb{E}[z_i]} \tag{20}$$

with an R^2 of 0.99, showing that the tax system embedded in (8) is a reasonable description of the federal tax and transfer system for prime-age households. The implied steady-state redistributive tax rate is $\tau^r = \frac{0.143}{0.143+0.666} = 17.7\%$.[11]

Incidence function. To calibrate our incidence function γ, we consider a specification in which the elasticity of the standard deviation log earnings to employment is a constant Γ. This leads us to the choice of

$$\gamma_i(L) = \frac{e_i^{\Gamma \log L}}{\mathbb{E}\left[e^{1+\Gamma \log L}\right]} \tag{21}$$

This function satisfies our normalizations (6) and (7). Under this specification, we have

$$\text{sd}\left(\log z_i(L)\right) = \text{sd}\left(\log z_i(1)\right) + \Gamma \log L$$

and we can conceptually recover Γ as the slope coefficient in an instrumental variables regression of the standard deviation of log gross income on log employment, in which the instrument only affects inequality through its effect on unemployment.

There is a small empirical literature on endogenous inequality which we can try to reinterpret in this light. However, the numbers we are aware of are probably too large to be plausible in the context of our model.[12] As discussed in section 4.4, when Γ is negative enough, the feedbacks from the level of uncertainty to unemployment can be so strong as to generate multiple steady-state equilibria. We remain agnostic as to whether the potential for an inequality trap mechanism could be characterizing the US today, but here we prefer to sidestep the associated issues of equilibrium selection and maintain Γ within a range that delivers unique steady-state equilibrium. This leads us to experiment with a range of values between $\Gamma = -0.3$ and $\Gamma = 0$, with the latter—equal incidence—being our benchmark.

Bonds and shares: micro and macro balance sheets. We choose our calibration to be consistent both with household-level balance sheets from the Survey of Consumer Finances (SCF), and macro-level balance sheets in the Flow of Funds (FoF). We pick our aggregates B, XK and Π to be consistent with both SCF aggregates and FoF data (see appendix C.1). In the individual data, we need to divide individual assets between bonds b and shares v. Even though households are indifferent between holding both in terms of their portfolio choice, this is important for two reasons. First, since we calibrate to a positive equity premium $\varrho > 0$, household's *ex-post* return on wealth depends on the share of assets they have invested in shares.[13] Second,

[11] From (20) we can also recover an estimate of $\tau = 1 - (0.143 + 0.666) = 19.1\%$. This is below the value of $\tau = 29.8\%$ that we derive residually from the government budget constraint. This discrepancy is largely due to the fact that we model the income tax as the only source of government revenue.

[12] Coibion, Gorodnichenko, Kueng and Silvia (2016) report that a monetary policy shock that raises the unemployment rate by 0.25 percentage points results in an increase in the standard deviation of log income of 1 point from a baseline of 100 points. Over their sample period 1969Q3–2008Q4, the average employment rate was 69.3% and fell by 0.8 points for each point increase in unemployment. Assuming that monetary policy shocks are a valid instrument for the regression in (20), these numbers imply an elasticity of $\Gamma \simeq \frac{0.01}{-0.8 \times \frac{0.25}{69.3}} = -3.5$. But, as discussed in section 4.4, our baseline model admits multiple equilibria for $\Gamma \leq -0.3$.

[13] To be precise, the expost return on wealth is $1 + r + \varrho\theta(a)$ if $\theta(a)$ is the fraction of wealth a invested in shares.

unexpected shocks alter the price of shares p and therefore generate portfolio revaluations, making the distribution of share holdings relevant.

A significant advantage of our approach to modeling bonds and shares is that we can take the distribution of capital holdings directly from the data, rather than having to rely on a model-consistent portfolio choice. From the 2013 SCF, we back out a smooth distribution $\theta(a)$ representing the average fraction of wealth invested in shares for individuals with wealth a. As explained in appendix C.1, we do this in two ways. Our benchmark specification, $\theta^b(a)$, interprets capital very broadly to include any wealth that is *not* in the form of deposits or bonds directly held. This implies notably that the premium ϱ applies to all consumer credit including mortgages, as well as to housing. As the blue dots on figure C.2 show, under this interpretation, the distribution of shareholdings is diffuse in the population, except at the very bottom of the wealth distribution where most households tend to mostly have deposits. We also take a stricter interpretation of "shares" as the sum of directly held equity, equity held through mutual funds and the value of closely-held businesses. This implies a distribution $\theta^{eq}(a)$ which rises more gradually and only becomes large at high levels of wealth, as shown by the red squares of figure C.2.

At the macroeconomic level, the value of shares must add up to the value of capital and the capitalized value of monopoly profits $\int \theta(a)\, a d\Psi(a) = \Pi + XK$, while the value of bonds must add up to the stock of outstanding Treasuries, $\int (1-\theta(a))\, a d\Psi(a) = B$. Relative to the patterns in figure C.2, we give the model one degree of freedom, scaling every individual's earnings in the same proportion Θ so that both bonds and equity markets clear at all times. This involves solving for a fixed point in which the scaling factor Θ, individual decision rules, and the wealth distribution Ψ are consistent with each other.

Equity premium and monopoly power. A great advantage of our introduction of an equity premium in our model is that it allows us to simultaneously match the observed labor share and a real interest rate of $r = 0\%$. If $\varrho = 0$, as most macroeconomic models would imply, then a target of $r = 0$ implies that the net capital share is $(r+\varrho)\frac{XK}{Y} = 0$. This is very difficult to reconcile with BEA data on income shares. Moreover, at a steady state of $r = 0$, the capitalized value of constant income streams such as land and monopoly profits is infinity. Some authors have, formally or informally, used these arguments to argue against the possibility of secular stagnation (for example Gomme, Ravikumar and Rupert 2015).

Our equity premium keeps the value of monopoly profits finite even at $r = 0$, and is sufficient to explain the high observed capital share, or alternatively the low observed labor share. Indeed, in our model, the gross labor share is equal to

$$\alpha = \frac{1}{\mu} - (r + \varrho + \delta)\frac{XK}{Y}$$

which is lower when ϱ and μ are higher. We set μ and ϱ jointly to match the observed gross labor share of $\alpha = 0.625$ and our target for the capitalized value of monopoly profits of $\frac{\Pi}{Y} = \frac{1-\frac{1}{\mu}}{r+\varrho} = 0.83$. This implies $\varrho = 7.2\%$ and $\mu = 1.067$, which are quite plausible values. We view the quantitative consistency of our $r = 0$ steady state with all previously-mentioned macroeconomic and microeconomic targets as significant advantages of our model relative to other available models in the literature.

Production function and adjustment costs. We assume that the production function is CES

$$F\left(K_{t-1}, L_t\right) = A\left[\left(1-f\right) K_{t-1}^{\frac{\epsilon-1}{\epsilon}} + f L_t^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{22}$$

We choose a benchmark calibration with a standard Cobb-Douglas production $\epsilon = 1$, and calibrate the remaining production function parameters so as to hit our steady-state targets for the labor share, the equity premium, the real interest rate and the output-labor ratio (see appendix B).

We consider the quadratic adjustment costs function

$$\zeta\left(x\right) = \frac{1}{2\delta\epsilon_I} x^2 \tag{23}$$

implying an elasticity of gross investment to q around the steady-state of ϵ_I.[14] We calibrate to $\epsilon_I = 4$, implying relatively flexible investment, consistent with many prominent business cycle models (for example Bernanke, Gertler and Gilchrist 1999).

Summary. Table 1 summarizes our parameters. As discussed, we calibrate the household discount factor β to hit our target of 0 for the equilibrium real interest rate. Our steady state macroeconomic aggregates are consistent with usual calibrations in the RBC literature, except for $r = 0$ and $\varrho > 0$, which the literature usually lumps together into one positive value for the risk-free rate. We understate the investment ratio in the data, which is natural for a model without growth. Modeling growth is outside the scope of this paper, but our model suggests that secular stagnation should be most naturally thought of as having an effect on the level, not the growth rate of macroeconomic aggregates.

As figure 2 and appendix C.2 illustrate, our benchmark steady-state also achieves a good fit to the distributions of consumption, income, and wealth reported in the 2013 SCF.[15] The fit is much improved over standard calibrations of Aiyagari models (see for example Quadrini and Rıos-Rull (1997)), owing to the richer earnings dynamics in our model. We could improve this

[14] The first-order condition for investment is $\frac{I_t}{\delta K_{t-1}} - 1 = \epsilon_I \left(q_t - 1\right)$, so around the steady state $\frac{dI}{I} = \epsilon_I \frac{dq}{q}$.

[15] We use food consumption in the SCF to approximate the consumption distribution, which is well known to be more equal than the distribution of income, itself more unequal than the distribution of wealth. See appendix C.2 for detailed moment comparisons between data and model.

Parameters	Description	Main calibration	Target
ν	EIS	0.5	Standard calibration
β	Discount factor	0.972	$r = 0$
$\theta(a)$	Asset allocation rule	$\theta^b(a)$	SCF 2013
Γ	Elasticity of inequality to L	0	See main text
ϵ	$K - L$ elasticity	1	Standard calibration
α	Labor share	62.5%	NIPA 2013
δ	Depreciation rate	5.6%	NIPA 2013
r	Eqbm real rate	0%	TIPS yields 2013
$\frac{XK}{Y}$	Capital-output ratio	229%	BEA Fixed Assets 2013
$\frac{\Pi}{Y}$	Capitalized profits	82.0%	FoF hh. net worth 2013
$\frac{I}{Y}$	Investment rate	13.5%	$\delta\frac{XK}{Y}$
μ	Monopoly markup	1.067	$\frac{\Pi}{Y} = \frac{1-\frac{1}{\mu}}{r+\varrho} = 1.5$
ϱ	Equity premium	7.3%	$\frac{\frac{1}{\mu}-\alpha}{r+\varrho+\delta} = \frac{XK}{Y}$
$\frac{B}{Y}$	Govtt debt	55.0%	Domestic holdings 2013
$\frac{G}{Y}$	Govtt spending rate	18.7%	NIPA 2013
τ^r	Redist. rate	17.5%	CBO
τ	Headline tax rate	29.8%	$\frac{G+rB}{\alpha Y}$
$\bar{r}$	Keynesian policy rate	0%	Zero lower bound

Table 1: Calibration parameters

fit of further, by adding heterogeneity in $\beta(s)$ or entrepreneurial risk, but this is not essential for our analysis of aggregate demand. What is essential is that our model matches sufficient statistics for the determinants of consumption. Sections 3.5 and 4.7 show that it successfully does.

2.4 Experiments: inequality changes

As figure 1 illustrates, income inequality has been rising in the United States since at least the beginning of the 1980s. This rise in inequality has been the subject of a very extensive literature. It seems to be attributable to many different factors, including but not limited to: a risking skill premium from skill-biased technological change (Katz and Murphy (1992)), increasing prevalence of superstar pay (Rosen (1981)), improved information technology (Garicano and Rossi-Hansberg (2004)), trade and globalization (Feenstra and Hanson (1999), Autor, Dorn and Hanson (2013)), financial deregulation (Philippon (2015)), rising assortativeness between workers and firms (Card, Heining and Kline (2013)), as well as fundamental changes in labor market institutions. All of these can be argued to have changed the innate earnings ability $e_t(s_t)$ of individuals in different groups s_t. Since our framework is concerned with the change in aggregate consumption and savings patterns induced by this increase in inequality, we do not need to explicitly model the root cause of this change, and can instead focus directly

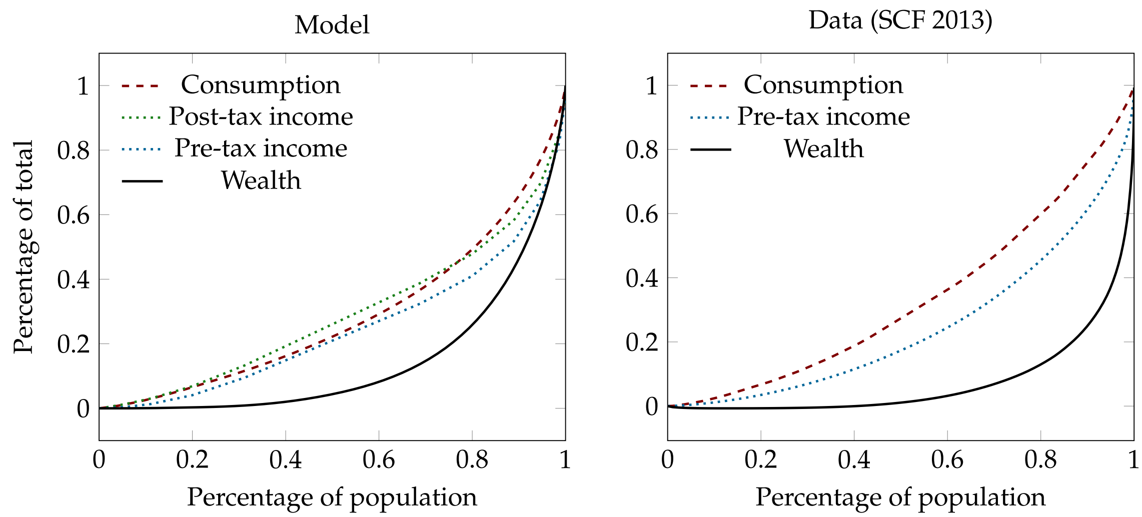

Figure 2: Lorenz curves for consumption, income and wealth at steady-state

on how e_t changed over time for different groups. This approach follows that taken by the large literature on earnings dynamics.

As initially argued by Gottschalk and Moffitt (1994) and since confirmed by Kopczuk et al. (2010) and many others, statistical decompositions of changes in inequality tend to attribute a role for both the the persistent and the transitory component of earnings risk. We follow this approach as well, stretching out the distribution of the log innovations to the persistent and the transitory component of our earnings process until it achieves a given change in cross-sectional earnings inequality.

According to the data from Guvenen et al. (2014) reported in figure 1, the standard deviation of log gross earnings is currently around 0.96, having risen from 0.83 in 1980. In our model, it is 0.92. In the next three sections, we consider three main experiments that each affect this number. Section 3 considers the effect of year-to-year fluctuations in inequality, by assuming the standard deviation of log skills rises by 0.02 for only one year and then reverts to its initial level. This captures the expected effect of a typical change in inequality in the 1990s, where inequality fluctuated without a trend. Section 4 models the effect of a permanent increase in that standard deviation by 0.06, phased in over twenty years. This captures the potential effect, under our monetary and fiscal policy assumptions, of an increase in inequality such as the one we have observed on average between the 1980s and the 2000s. Finally, section 5 asks how much of a decline in r^* can be explained by the observed rise in inequality alone. We consider the maximal impact possible by considering the increase of 0.12 log points that occurred between 1980 and 2010.

Section	Experiment	Regime	sd log e	Timing
3	Temporary	Keynesian	$+0.02$	One year, reversed
4	Permanent	Keynesian	$+0.06$	Phased in over 20 years
5	Retrospective	Neoclassical	-0.12	Phased in over 20 years

Table 2: Experiments

3 Temporary increase in inequality

We start by considering the effect of fluctuating inequality–the experiment in the first row in table 2. We confront the model with a run-of-the mill increase in income inequality and contrast the economic outcomes obtained by neoclassical central bank and a Keynesian central bank. The latter captures a situation where monetary policy is *inattentive* to the change in the income distribution and its effect on the natural interest rate—for example, dismissing it due to its temporary nature—and instead follows a simple rule of raising the nominal interest rate one for one with increases in expected inflation.

3.1 Impulse response

The solid line in figure 3 conducts our numerical exercise.[16] Output falls on impact by a little over 0.2% and recovers slowly, though the effects are concentrated in period 0. Most of this fall reflects a fall in consumption, by about 0.25%. Hence this redistribution of income to low-MPC agents does translate into a fall in aggregate consumption. It does not translate into a rise in aggregate savings, however: investment falls. Capital therefore falls very slightly, and most of the fall in output along the path comes from the employment effect. Finally, factor prices adjust: real wages rise with the marginal product of labor, and dividends fall with the marginal product of capital.

Contrast this response with the dashed lines, which correspond to the response to the same shock under the neoclassical regime. In this case, the central bank reduces the real interest rate in response to the demand shock. This has the effect of both mitigating the fall in consumption and of increasing investment. In equilibrium full employment is reached at all times, implying unchanged output at date zero; the higher level of investment then generates a boom due to capital accumulation.

In order to better shed light on these novel Keynesian effects, as well as to understand the quantitative magnitude of the effects in figure 3, we now dig into the analytical properties of the Keynesian impulse response.

[16]The numbers in this section reflect outcomes under a slightly different calibration relative to that of table 1 and are subject to change.

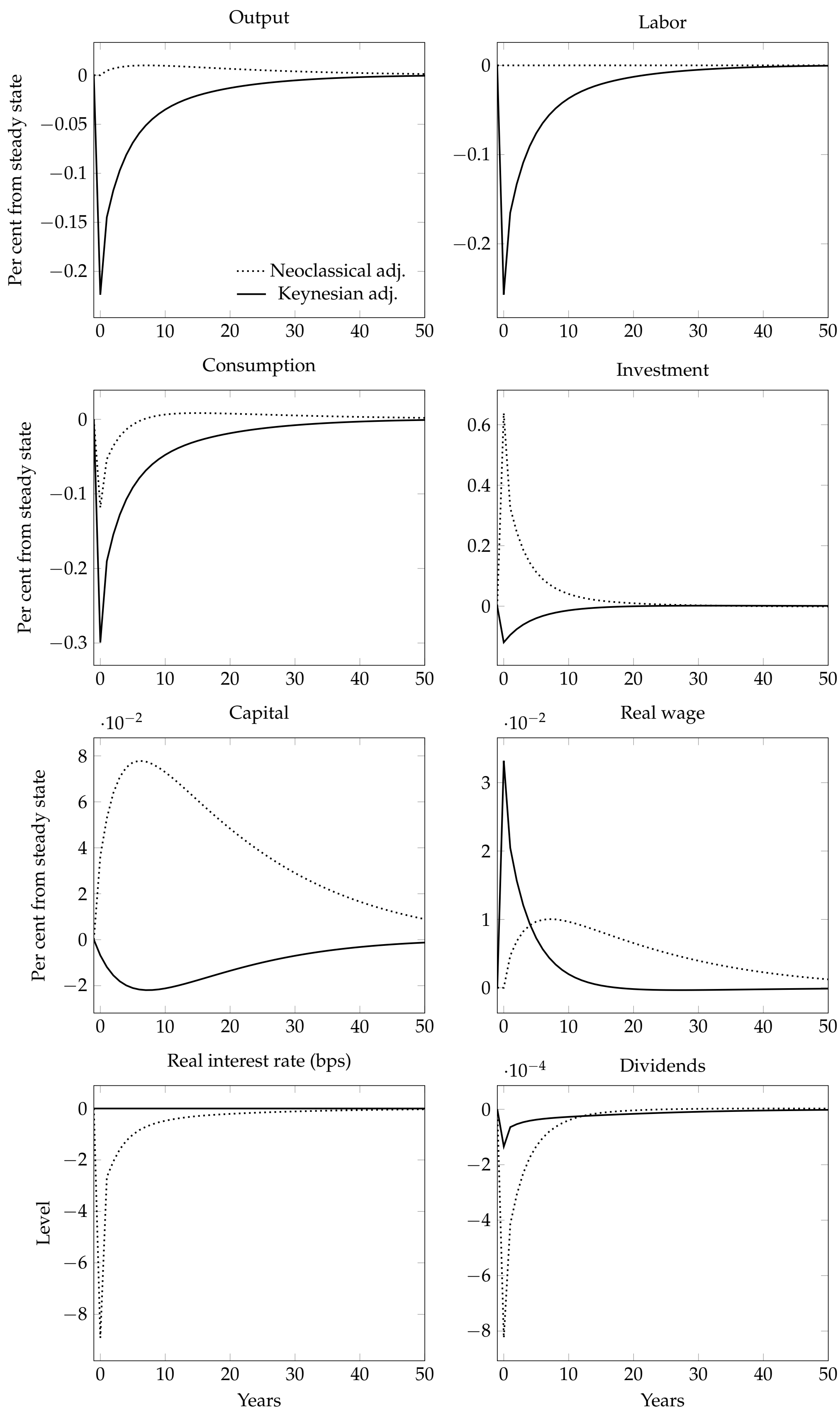

Figure 3: Effect of our baseline transitory shock to labor income inequality under both regimes

3.2 Partial equilibrium effect

Consider a general redistributive shock at date 0: a small perturbation dy_{i0} of individual incomes y_{i0}, satisfying $\mathbb{E}\left[dy_{i0}\right] = 0$. We provide a general decomposition of the aggregate effect of this perturbation into the product of a partial equilibrium and a general equilibrium effect, and explore the determinants of both.

Intuitively, the partial equilibrium effect depends on marginal propensities to consume. In our model, all agents receiving an unexpected unit of income plan to spend it over time at some rate. Define the date-t *MPC* of agent i as this average spending plan:

$$MPC_{it} \equiv \mathbb{E}\left[\frac{\partial c_t\left(y, b, v|\left[L\right], \left[r\right]\right)}{\partial y_0}|y = y_i, b = b_i, v = v_i\right] \tag{24}$$

where, in computing this object, we are holding the path for all macroeconomic aggregates (summarized by $[L] = L_t$ and $[r] = r_t$) fixed. We can then write the partial-equilibrium consumption response of individual i to the redistributive dy_{i0} as

$$dc_{it} = MPC_{it} dy_{i0}$$

Aggregating up these cross-sectional responses, we obtain the vector of partial-equilibrium changes in consumption, which we note $[\partial C]$, where $\partial C_t = \mathbb{E}\left[dc_{it}\right]$. We also define $[\mathcal{R}]$ as the vector of present value factors, with elements

$$\mathcal{R}_t \equiv \prod_{s \leq t}\left(\frac{1}{1+r_s}\right)$$

The following proposition characterizes $[\partial C]$.

Proposition 1. *The elements of the vector of partial equilibrium, per-capita changes in consumption* $[\partial C]$ *are given by*

$$\partial C_t = \mathbb{E}\left[dc_{it}\right] = Cov\left(MPC_{it}, dy_{i0}\right) \tag{25}$$

The vector has net zero present value, i.e. $[\mathcal{R}]' [\partial C] = 0$.

Formula (25) is extremely intuitive. It corresponds to the arithmetic of adding up consumption responses of individuals that experience different changes in income and respond differently to these changes. A covariance term appears naturally due to the redistributive nature of the shock, as in Auclert (2016). It can conceptually be computed for any $t \geq 0$, since—due to perfect foresight and the law of large numbers—the date-0 plan for spending of an individual corresponds to actual average spending over time individuals just like him. Importantly, because everyone's consumption plan exhausts their budget, the overall partial equilibrium consumption response satisfies $[\mathcal{R}]' [\partial C] = 0$. In particular, a fall in aggregate consumption today—as those who are hurt by the redistribution cut back on their consumption more than those who gain increase theirs—must be followed by an increase in consumption at some point in the future, as the savings generated by the initial fall in consumption come on stream. We will see that this property is no longer true in general equilibrium.

Despite its simplicity, formula (25) provides a structured way of thinking about aggregating individual responses to various kinds of redistributive shocks. Two examples provide concreteness.

Redistribution to one group. Consider first a redistributive shock that favors one group J at the expense of all other groups. We can model such as change as the effect of raising a lump sum tax T_0 on all individuals, and rebating this tax to individuals in group J, also in a lump-sum fashion, so that $dy_{i0} = \left(\frac{1_{i\in J}}{\mathbb{E}\left[1_{i\in J}\right]} - 1 \right) dT_0$. Then (25) implies that

$$\partial C_t^{PE} = \left(\overline{MPC}_t^J - \overline{MPC}_t \right) dT$$

where $\overline{MPC}_t^J$ is the average MPC of individuals in group J at time t, and $\overline{MPC}_t$ the corresponding equally-weighted average of MPCs across the population. For example, if group J spends all its income at date 0, then ∂C_0^{PE} will be positive, but ∂C_t^{PE} will be negative for $t \leq 1$ as other agents reduce their consumption in consecutive period in response to their loss of income at date 0.

Linear spread of incomes. Next, consider the effect of an inequality change that spreads apart all incomes linearly around their mean, so that dy_i is proportional to y_i. Since our model features an affine tax system, the simplest example of this is a change in the redistributive rate $d\tau_0^r$. According to (8), $dy_{i0} = (1-\tau_0)\left(\mathbb{E}\left[z_{i0}\right] - z_{i0}\right) d\tau_0^r = -\left(y_{i0} - \mathbb{E}\left[y_{i0}\right]\right) \frac{d\tau_0^r}{1-\tau_0^r}$. Hence in this case

$$\partial C_t^{PE} = -\text{Cov}\left(MPC_{it}, y_{i0}\right) \frac{d\tau_0^r}{1-\tau_0^r} \tag{26}$$

showing that the simple covariance between MPCs and income is relevant to determine the aggregate effect of this particular type of redistribution.

3.3 General equilibrium effect

While formula (25) and its applications provide a useful starting point to evaluate the aggregate effect of an inequality shock, they are not a complete answer. Our impulse responses in figure 3 show the presence of a variety of general equilibrium effect from the initial redistribution: aggregate income effects will make consumption fall further due to the fall in GDP (a Keynesian multiplier effect), real wages rise and dividends fall, creating an additional source of redistribution, and finally these effects are persistent and will affect the behavior of forward looking agents.

We can cut through this complexity, however, using the following proposition which results from a generalized application of the implicit function theorem.

Proposition 2. *There exists a **general equilibrium multiplier matrix** $\mathcal{G}$, depending only on model parameters and policy, mapping any vector of perturbations of consumption $[\partial C]$ satisfying $[\mathcal{R}]' [\partial C] = 0$ to its general equilibrium effect on output $[dY]$ according to*

$$[dY] = \mathcal{G} \cdot [\partial C] \tag{27}$$

Appendix D.1 provides a proof as well as a simple methodology for computing these multipliers.

The entire path for consumption matters, for example, to determine the date-0 output effect, since it affects the path for labor income that consumers foresee—and therefore their initial consumption decision—as well as the path for marginal products of labor which serves as the basis of firms' investment decisions.[17]

Propositions 1 and 2 are important because they provide a general method for understanding and quantifying the equilibrium effects of any redistributive shock on the path for output. Generally, this path results from two independent steps. First, one needs to determine the effect that such a shock has on the path of consumption. This can be done via the use of the sufficient statistics in equation (25). Second, one needs to compute the general equilibrium matrix $\mathcal{G}$. This can be done *independently of the first step*. All the uncertainty surrounding the model and policy are contained in this matrix, so that it summarizes the model's endogenous response to shocks. We now provide illustrate how this decomposition plays out in our benchmark model.

Implementing the decomposition. Figure 4 implements the decomposition of proposition 2 for our baseline inequality shock. The top left panel shows the time path for the consumption response to the shock. In partial equilibrium, aggregate consumption immediately contracts as income is redistributed away from high MPC agents, but it then progressively reverts to its initial level and eventually overshoots. Overall, $[\mathcal{R}]' [\partial C] = 0$ holds as per proposition 1.

The solid line on the top right panel shows the first row of the GE matrix $\mathcal{G}$, which is the model's vector of general equilibrium multipliers for date-0 output. Proposition 2 says that, by multiplying point-by-point the solid lines on the left and the right panels and adding up, we should obtain $dY_0 = \mathcal{G}_0 \partial C$. The bottom panel of figure 4 shows that this is indeed the case numerically. A number of important insights emerge from this decomposition. First, the date-0 effect on consumption is critical to determine the date 0 effect on output, since the multiplier matrix puts by far the heaviest weight on this effect. Second, the fact that the fall in

[17] In appendix D.2 also provide an analytical approximation to the first row of the matrix $\mathcal{G}$ by assuming that it takes the simple form $\mathcal{G} = \begin{pmatrix} g & 0 \end{pmatrix}$—this reduces it to a static multiplier, with all the effects on employment concentrated at date 0. This approximation has the benefit of depending only on observable sufficient statistics, but its quantitative performance is imperfect since it misses out on the rich dynamic feedback effects that our model features.

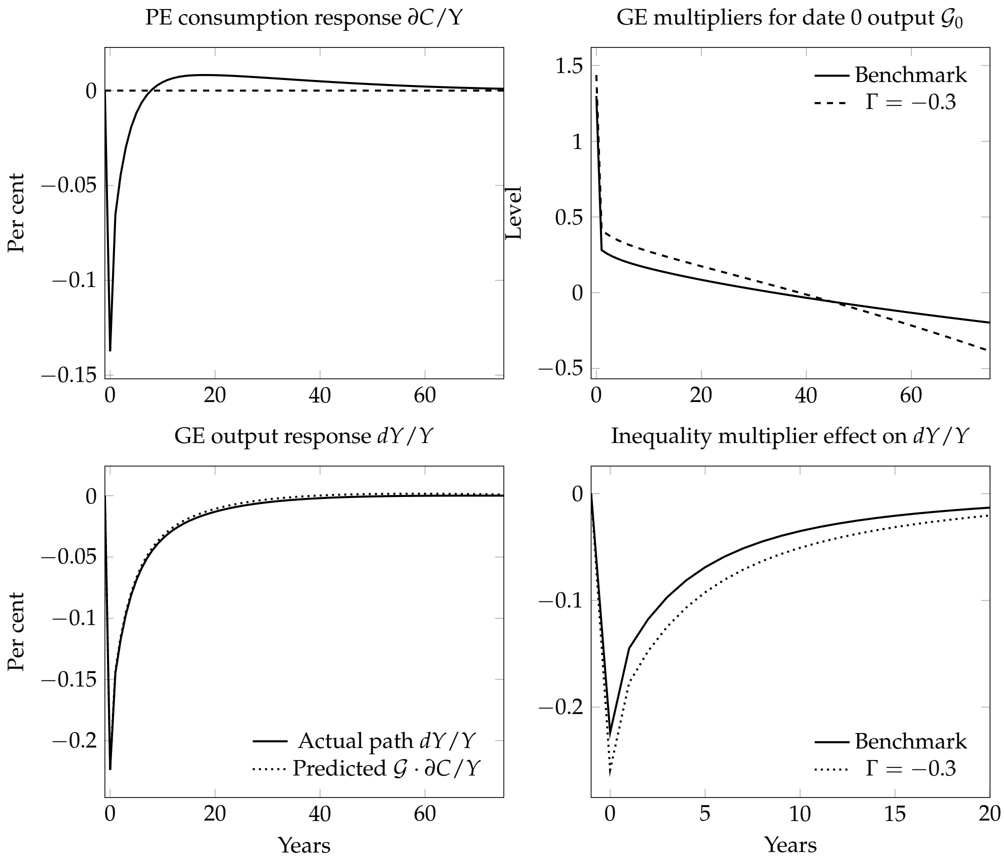

Figure 4: Increase in income inequality: from partial to general equilibrium

consumption persists for a few periods is also important, as the multipliers are still significantly positive for these dates as well. Economically this reflects the fact that, as labor income falls in the near future, today's consumers cut back further on consumption plans while firms, who now foresee a lower marginal product of capital, also cut back on investment.

As figure 4 shows, our methodology allows us not only to approximate the change in output at date 0 dY_0, but also the path for dY_t at every date. The full path of actual and predicted paths are virtually identical, illustrating the success of this methodology and its promise as a way to understand the aggregate effects of redistribution in macroeconomic models.

The inequality multiplier. The analysis of the case where $\Gamma < 0$ fits squarely into the framework of the previous section. We say that there is an *inequality multiplier* in this case, because $\Gamma < 0$ affects the GE multiplier matrix $\mathcal{G}$ that translates the effect of a given exogenous increase in income inequality into general equilibrium output. The dashed lines on the right panel of figure 4 illustrate this claim. Intuitively, a more negative Γ—more unequal incidence of business cycles—should aggravate the fall in output that follows a given change in inequality, and

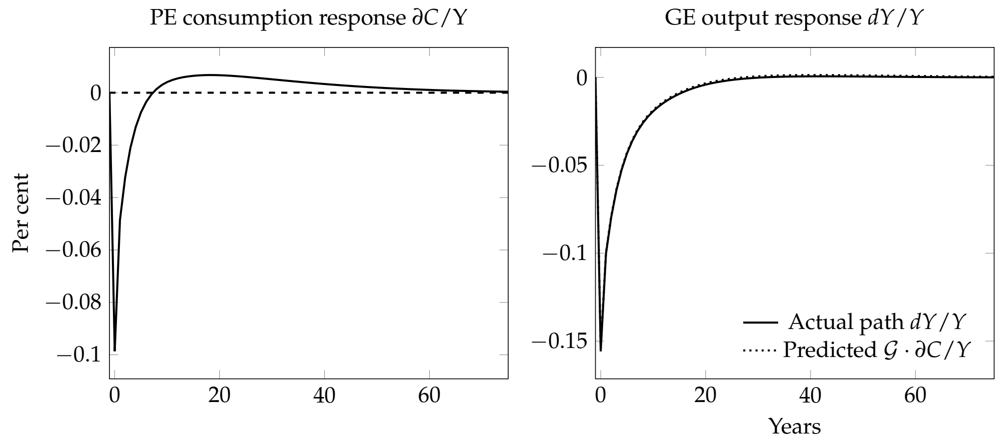

Figure 5: PE and GE effect of a 1pp exogenous change in the labor share

this happens because the same change in inequality now gets amplified by larger mulipliers. This is what we observe when $\Gamma = -0.3$: the multiplier for date-0 consumption is 15% larger, causing output to fall by about 15% more on impact relative to the case without an inequality multiplier. Note, however, that even in this case the overall effect remains relatively small.

3.4 Falling labor share

We now consider the effects of short-run changes in the distribution of capital vs labor. The decline in the US labor share has attracted an great amount of policy attention in the past few years. From the point of view of our model, a transitory fall in the labor share is contractionary for the reason that is often hypothesized: this change redistributes away from agents from high MPC (workers) towards agents with lower MPC (shareholders). This intuition turns out not to be correct in a long run secular stagnation steady state, where falls in the labor share are expansionary as we show in section 4.6.

In our model, the labor share could be changing due to exogenous investment price changes (X_t), exogenous changes in markups (μ_t) or simply exogenous changes in the production function F_t. We analyze this latter simple case, showing how it can be directly handled by our methodology. The case of changing markups is conceptually very similar and we ommit a detailed treatment for brevity.

Exogenous change in the labor share. Consider the effect of an exogenous change to the labor share, modeled as a change in the production function at date 0. Specifically, let us allow the production function F_0 to change but satisfy two restrictions: a) unchanged output for fixed inputs and b) the new labor share is temporarily changed by $d\alpha_0^x$. In partial equilibrium this change does not affect output, and results in total wages falling while total dividends rise in equal and opposite amounts: $d\left(\frac{W_0}{P_0}L_0\right) = Y_0 d\alpha_0^x = -d\left(d_0\right)$. Because total wages fall, the government budget is affected. We make the natural assumption that our concept of partial equilibrium embodies the response of taxes via the government fiscal rule (18). This, in differential form, yields $d\left(\tau_0 \frac{W_0}{P_0}L_0\right) = 0$. Hence i's income changes by

$$dy_{i0} = d\left(\left(1-\tau_0\right)\frac{W_0}{P_0}L_0\right)\frac{y_{i0}}{\mathbb{E}\left[y_{i0}\right]} + d\left(d_0\right)v_i = d\left(\frac{W_0}{P_0}\right)L_0 e_i + d\left(d_0\right)v_i = Y_0\left(e_i - v_i\right)d\alpha_0$$

Applying (25) we finally find

$$\partial C_t^{PE} = \left(\overline{MPC}_t^y - \overline{MPC}_t^v\right)Y_0 d\alpha_0 \tag{28}$$

where $\overline{MPC}_t^y$ is weighted by individuals' date-0 net incomes, while $\overline{MPC}_t^v$ is weighted by their date-0 holdings of shares. Hence, this labor share change is ultimately a simple redistribution between workers and capitalists, whose aggregate effect can be evaluated using the average MPC difference between the former and the latter.

Figure 5 operationalizes our methodology to this case. We conduct an experiment where the labor share falls by 1% from its benchmark level of 62.5% ($d\alpha_0^x = -0.01$)—which is a typical magnitude for the kinds of year-over-year changes in the labor share observed in the US in the post-war period (Rognlie (2015)). Output falls on impact by about 0.15%. It is the consequence of an immediate partial-equilibrium effect on consumption of around 0.1% that is moderately persistent, reflecting the time path of the difference in MPCs between workers and capitalists as captured by (28). Apart from the question of magnitude, the path for the partial-equilibrium response of consumption to this labor share shock is similar to that coming from an increase in income inequality. Our Proposition 2 therefore explains why it translates into a similar path for the final output response: the same economic mechanisms are at play to amplify the initial redistributive shock.

All in all, in our model, plausible temporary changes in labor income inequality or capital-labor inequality translate into relatively small effects. Are our magnitudes plausible? Our results show that this can be gauged by confronting the sufficient statistics produced by our model to those in the data. Moreover, the time path of the GE multipliers for date 0 output suggests that the key determinant of the date-0 effect on output is simply the date-0 effect on consumption—a sufficient statistic on which micro data is directly available.

3.5 Empirical assessment

There is a large empirical literature that estimates instantaneous MPCs—our MPC_0— in cross-sections, and reports them by income group (see for example Johnson et al. 2006, Parker et al. 2013 or Jappelli and Pistaferri 2014). Moreover, researchers have recently started to collect individual-level data by asking for self-reported measures in household surveys. One such

	Value, Data	Value, Model
$\text{Cov}\left(MPC_i, \frac{y_i}{Y}\right)$	-0.018	-0.015
$\overline{MPC}^y - \overline{MPC}^v$	0.029	0.038
$\overline{MPC}^y$	0.436	0.126
$\overline{MPC}^v$	0.407	0.088

Table 3: Comparison between sufficient statistics in the model and the data: transitory shock

survey is the 2010 Italian Survey of Household Income and Wealth (SHIW), analyzed in detail in Jappelli and Pistaferri (2014). Table 3 compares the sufficient statistics produced by our model to those that can be computed from the survey.

As is clear from the table, our model does very well at matching redistributive covariances: the covariance between MPCs and income, which matters for inequality increases in levels somewhat analogous to our baseline experiment, and the difference in average MPCs between workers and capitalists, which matters for the endogenous redistribution due to factor price movements as well as exogenous changes in the labor share. This is a key success of our model, making us confident that the overall magnitudes we find are plausible.

Note, however, that the last two rows of table 3 show that our model does less well at matching the *average* MPC level, a plight that is common to most calibrations of Aiyagari models. This suggests that our model, while correctly capturing the partial equilibrium response to temporary inequality shocks, might understate its general equilibrium effect, since high MPC levels will tend to translate into larger immediate multipliers.

4 Permanent increase in inequality

We now turn to the predicted effect of a permanent increase in inequality–the experiment in the second row in table 2. Recall that the zero lower bound on interest rates provides our motivation for studying the Keynesian regime of our model in response to such a shock. [18] As in section 3, we first provide our numerical estimates before applying our general methodology to break down the steady state effect of inequality on output into partial equilibrium and general equilibrium effects.

4.1 Impulse response

The solid line of figure 6 presents the predicted effect of this permanent inequality shock—an experiment in which inequality keeps rising in the next two decades at the same pace as it did in the past two. Under the Keynesian regime, this shock pushes the economy into a deep

[18] Starting from our 2013 steady state with $r = 0$ and $\pi = 0$, if nominal interest rates cannot fall, the wage rigidity constraint implies that economy features a constant real interest rate $\overline{r} = 0$, even though r^* permanently falls. Note that if wages were allowed to fall at some predetermined rate, the steady state would be associated with price deflation and therefore a strictly positive real rate, moving it further away from r^*.

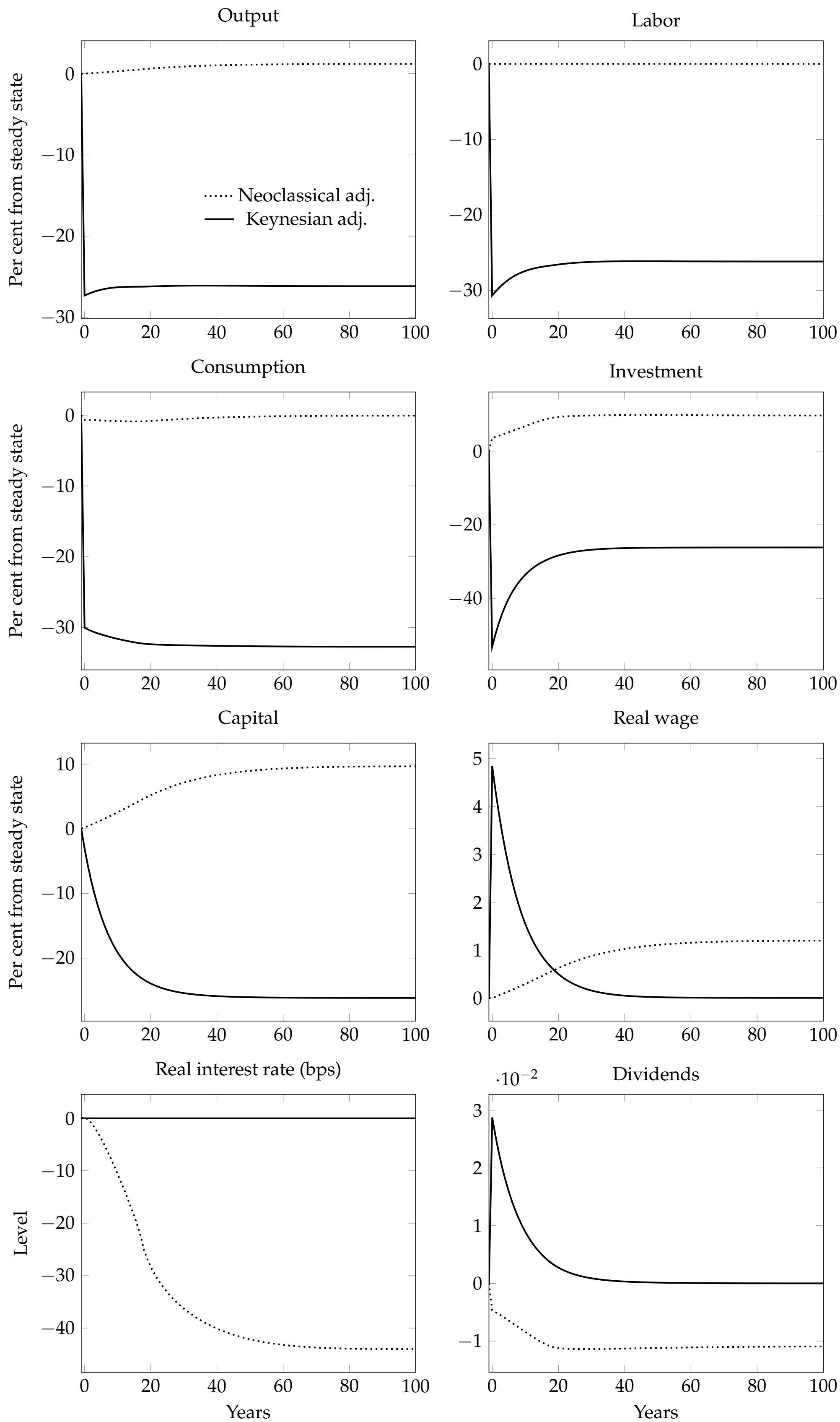

Figure 6: Effect of our baseline permanent shock to labor income inequality under both regimes

recession that lasts forever, eventually reaching a secular stagnation steady-state. This steady state features depressed labor and a capital stock that has shrunk in proportion, so that firm bring their marginal product of capital back in line with its pre-recession level. Firms disinvest in this process, which amplifies the fall in output in the short run relative to the long run. Consumption is only temporarily and mildly sustained by shareholders who spend the extra dividends. This stark outcome should be contrasted with the one that arises under neoclassical central bank policy in the dashed lines, in which the real interest rate falls permanently by around 40bps, generating a steady state increase in the level of capital and therefore an increase in output. Unfortunately, with a zero lower bound constraint on nominal interest rates, this outcome would not be achievable given that nominal interest rates are initially at their floor.

4.2 Steady state sufficient statistic: stocks, not flows

The most striking feature of our impulse responses is their convergence to a new, depressed steady state. We therefore wish to establish a steady state analogue of Proposition 2—a decomposition of the output effects of a rise in inequality into a partial equilibrium sufficient statistic and a general equilibrium multiplier. Throughout this discussion, let us assume for simplicity that there is no equity premium, in other words that

$$\varrho = 0 \tag{29}$$

The case of a positive equity premium is treated in the appendix.

One intuition might be that the relevant sufficient statistic here is the steady state effect on consumption of a permanent change in inequality. This would be a natural extension of our results for the short term case. This intuition, however, turns out not to be correct. One way to understand why is to recall that Proposition 1 states that $\mathcal{R}'\partial C = 0$ in response to redistributive shocks: while redistribution alters the *timing* of aggregate consumption, in the long-run everyone spends their income, so that MPCs are all equal to one in a present value sense, and steady state redistribution need not change consumption.

More formally, suppose that $r = 0$ and consider the implications of aggregating consumer budget constraints. Using (2), (5) and (8), we find

$$C = (1 - \tau)\,\frac{W}{P}L$$

In partial equilibrium, $\frac{W}{P}$, τ and L are unchanged in response to a change in inequality, and therefore aggregate consumption is also unchanged. Hence, we need to look for a sufficient statistic elsewhere. The answer lies in the asset market.

One quantity that we do expect to change in response to a change in steady state inequality is the aggregate amount of desired savings, which we write A. Market clearing for the stocks of assets can be written as

$$A\left(L, \frac{W}{P}\left(X,\mu,\alpha,r,L\right), \tau\left(r,\overline{B},G,L\right), r, \varphi\right) = \overline{B} + XK\left(X,\mu,\alpha,r,L\right) + \Pi\left(X,\mu,\alpha,r,L\right) \tag{30}$$

where we have indicated the dependence of aggregate stocks on relevant exogenous model parameters (markups μ, the price of investment X, the labor share α and an index of inequality φ) and endogenous outcomes (r or L). This equation can be further rewritten as

$$a^{net}\left(r,L;X,\mu,\alpha,\varphi\right) \equiv \frac{A-\left(\overline{B}+XK+\Pi\right)}{\left(1-\tau\right)\frac{W}{P}L} = 0 \tag{31}$$

where we have normalized net asset demand $A^{net} = A - \left(\overline{B}+XK+\Pi\right)$ by post tax wage income $\left(1-\tau\right)\frac{W}{P}L$, for a reason that will become clear in the next section. Suppose we are studying the Keynesian regime, in which r is fixed. Applying the implicit function theorem to (31) with respect to $x = \varphi, X, \mu$ or α results in our main Proposition of this section.

Proposition 3. *The employment response to a change in any of the inequality-inducing exogenous variables $x = \varphi, X, \mu$ or α is given by the product of a general equilibrium multiplier by a sufficient statistic,*

$$\frac{dL}{L} = -\left(\frac{\partial \log a^{net}}{\partial \log L}\right)^{-1}\frac{\partial \log a^{net}}{\partial x}dx \tag{32}$$

where a^{net} is net normalized asset demand, defined in (31).

Hence, our partial equilibrium sufficient statistic of interest is $\frac{\partial \log a^{net}}{\partial x}$, which indicates how net (normalized) asset demand responds to a change in x. The general equilibrium multiplier, instead, determines the way in which net asset demand responds to a change in L. The next sections specialize the study of both terms in equation (32) to various cases of interest.

The discussion in this section underlies the importance of thinking of steady state secular stagnation equilibrium in terms of stocks rather than flows. Why? Because, at $r = 0$, market clearing in flow terms (eg, clearing the goods market) does not imply market clearing in stock terms (eg, clearing the asset market), while the opposite is true. Walras's law fails in steady state because the key price moving between markets is equal to zero ($r = 0$).[19]

4.3 Steady state output response to increasing labor income inequality

In this section, we specialize our framework to the study of increasing labor income inequality, which we formalize as an increase in $x = \varphi$. We therefore fix the production parameters μ and α, fix and normalize $X = 1$, maintain assumption (29) and further shut down the inequality

[19] Away from $r = 0$, even though the equivalence between the goods and asset market clearing is restored, the behavior of steady state consumption remains of limited use, since the aggregation condition $C = \left(1-\tau\right)\frac{W}{P}L + rA$ shows that, if $r > 0$, anything that increases desired savings A also *increases* consumption in partial equilibrium.

multiplier by assuming equal incidence, ie

$$\gamma(s,L) \quad = \quad 1 \quad \forall s,L \tag{33}$$

Under these assumptions, appendix E.1 shows that, as a consequence of homotheticity, normalized asset demand $a\left(r,\varphi\right) = \frac{A\left(L,\frac{W}{P},\tau,r,\varphi\right)}{(1-\tau)\frac{W}{P}L}$ is independent of τ, $\frac{W}{P}$ and L: it is the amount of aggregate savings in an endowment economy with total income normalized to 1. It is quite intuitive that aggregate post-tax wage income should serve as a base for aggregate savings, since all savings initially originate in labor earnings. Under our assumptions here, aggregate savings is exactly proportional to it.

Turning to the production side of the economy, note that the steady state neoclassical conditions (13), (14) and (15) imply that the capital-labor ratio, the real wage, and the profit-labor ratio are all simple functions of r,[20] which we write $\frac{K}{L} \equiv \kappa\left(r\right)$, $\frac{W}{P} = w\left(r\right) \equiv F_L\left(\kappa\left(r\right),1\right)$, and $\frac{\Pi}{L} = \left(1-\frac{1}{\mu}\right)\frac{1}{r+\varrho}F\left(\kappa\left(r\right),1\right) \equiv \pi\left(r\right)$. Using these equations together with the government fiscal rule $\tau\frac{W}{P}L = G + r\overline{B}$, we can rewrite (30) as

$$\left(w\left(r\right)L - \left(G + r\overline{B}\right)\right)\widehat{a}\left(r,\varphi\right) = \overline{B} + \left(\kappa\left(r\right) + \pi\left(r\right)\right)L \tag{34}$$

Equation (34) brings out the nature of the equilibrating adjustment mechanism in our Keynesian regime very clearly. Consider figure 7, which represents steady state equilibrium determination in (A,L) space. The green line shows initial asset demand—the right-hand side of (34)— initially intersecting the black asset supply line at $L = 1$. A permanent shock to φ increases desired savings $\widehat{a}\left(r,\varphi\right)$ at given r, rotating the green curve to the right. Restoring equilibrium requires a fall in L, moving equilibrium down on this asset demand curve. This in turn triggers a fall in asset supply, since firms adjust their level of capital down so as to maintain a constant steady state capital-labor ratio. Equilibrium is restored at the intersection of the black and the red line.

Note that equation (34) was derived under very few assumption about the nature of idiosyncratic risk—it notably accommodates arbitrary earnings processes and heterogeneity in discount factors, which only affect the $\widehat{a}$ function. While assumptions (29)—(33) are restrictive, they explain clearly the role of the employment rate L as an equilibrating variable in the Keynesian regime. The key intuition is that when desired savings go up relative to labor income and monetary policy does not respond, *labor income falls* until desired savings is sated with the supply of available assets. Contrary to the neoclassical case of r adjustment where capital plays an equilibrating role, here instead it acts as an amplification mechanism, so it is only the *outside supply of assets* (ie, government debt) that ends up restoring equilibrium.

Equation (34) shows clearly that the fiscal policy variables $\overline{B}$ and G can play a role in restoring equilibrium. In figure 7, higher G moves the asset demand curve to the left, while increases

[20] Appendix B makes these mappings explicit for the case of our CES production function (22).

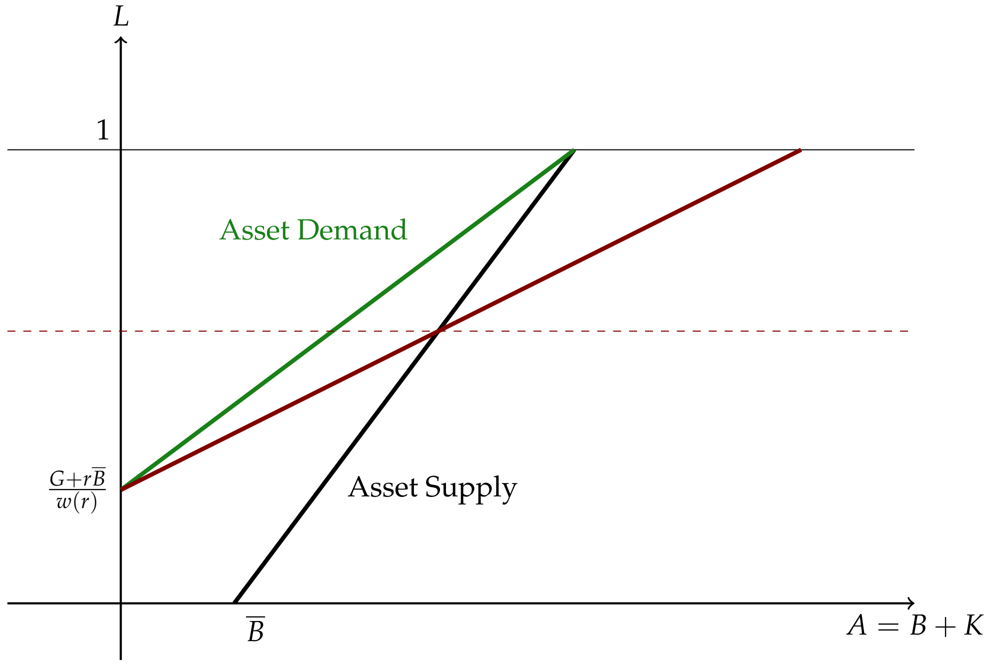

Figure 7: Keynesian equilibrium determination of L

in $\overline{B}$ shift the asset supply curve to the right. In section 4.5, we explore these effects further by relaxing our benchmark fiscal rule (18), which holds $\overline{B}$ and G constant.

We now establish our main statistic formula for the long-run Keynesian equilibrium, as our first application of Proposition 3.

Corollary 1. *To first order, around a steady state with $r = 0$ and $L = 1$, the change in steady state GDP $\frac{dY}{Y}$ is linked to the change in labor income inequality $d\varphi$ through the expression:*

$$\frac{dY}{Y} = -\frac{1}{\omega + \frac{\tau}{1-\tau}} \frac{\partial \log A}{\partial \varphi} d\varphi \tag{35}$$

Where $\omega \equiv \frac{\overline{B}}{\overline{B}+XK+\Pi}$ is the share of bonds in total assets in steady state and $\tau = \frac{G}{\frac{W}{P}L}$ is the tax rate at steady state.

Proof. The proof, in Appendix E.2, takes a log-linear approximation of (34) around an initial steady state with $r = 0$ and $L = 1$, and uses the fact that $\frac{Y}{L} = F(\kappa(r), 1)$ and therefore $\frac{dY}{Y} = \frac{dL}{L}$ in Keynesian steady state with fixed r. $\square$

Collary 1 shows that the steady-state general equilibrium multiplier takes an extremely simple form in the case we are considering. It embodies the two adjustment mechanisms described above, both of which depend entirely on fiscal policy variables, namely the amount of government liquidity and government spending. We now relax our assumptions of equal incidence and constant fiscal policy, before turning to the effect of other forms of inequality changes.

4.4 The inequality multiplier and multiple steady states

We first relax assumption (33), which corresponds to moving away from our benchmark parametrization of $\Gamma = 0$. This breaks the simple homotheticity assumption, because as L changes, this affects the income distribution which in turns affects A though γ. This process is captured by an extra term in the multiplier:

Corollary 2. *To first order, around a steady state with $r = 0$ and $L = 1$, the change in steady state GDP $\frac{dY}{Y}$ is linked to the change in labor income inequality $d\varphi$ through the expression*

$$\frac{dY}{Y} = -\frac{1}{\omega + \frac{\tau}{1-\tau} + \epsilon_{a,\Delta} \cdot \epsilon_{\gamma,L}} \frac{\partial \log A}{\partial \varphi} d\varphi \tag{36}$$

where $\epsilon_{a,\Delta(s)}$ is the elasticity of aggregate savings with respect to income in state s, and $\epsilon_{\gamma(s),L}$ the elasticity of the γ function for state s with respect to employment L. $\epsilon_{a,\Delta} \cdot \epsilon_{\gamma,L}$ is increasing in Γ.

Hence, as was the case in section 3.3, the inequality multiplier contributes to the GE multiplier term, and in general makes it larger. Here nothing restricts the strenght of this effect: as Γ becomes more negative the multiplier in (36) increases and eventually becomes infinity. At that point, the feedbacks from inequality to uncertainty become so strong as to generate multiple steady states. This is illustrated in figure 8 for our baseline calibration. When $\Gamma = 0$, asset demand in the green line is a linear function of employment, as in figure 7. As Γ becomes negative (the blue line), initially a unique steady-state remains, and the inequality multiplier simply amplifies the effect of exogenous inequality shocks. But as Γ falls to levels below -0.3, another steady-state equilibrium appears. This is an *inequality trap* steady-state, in which an anticipation of high unemployment generates an expectation of high inequality and high saving, and (with a constant real interest rate) this in turn results in a high level of unemployment.

Several papers in the recent literature combining incomplete markets and sticky prices have exhibited a related mechanism. In Ravn and Sterk (2013), den Hann et al. (2015) and Bayer et al. (2015), the feedback acts as an amplifier of exogenous shocks as in the case of Γ mildly negative. In Heathcote and Perri (2016), there are multiple steady states as in the case of very negative Γ. Our parametrization of the Γ function and simple specification of monetary policy gives us a very clear illustration of this mechanism, and allows us to tie it to data estimates of Γ. As discussed in section 2.3, one can try to estimate Γ directly, and the estimate available from Coibion et al. (2016) suggests the model may be in a multiple region of multiple equilibria. This discussion highlights the importance of an accurate empirical measurement of Γ before drawing definitive conclusions.

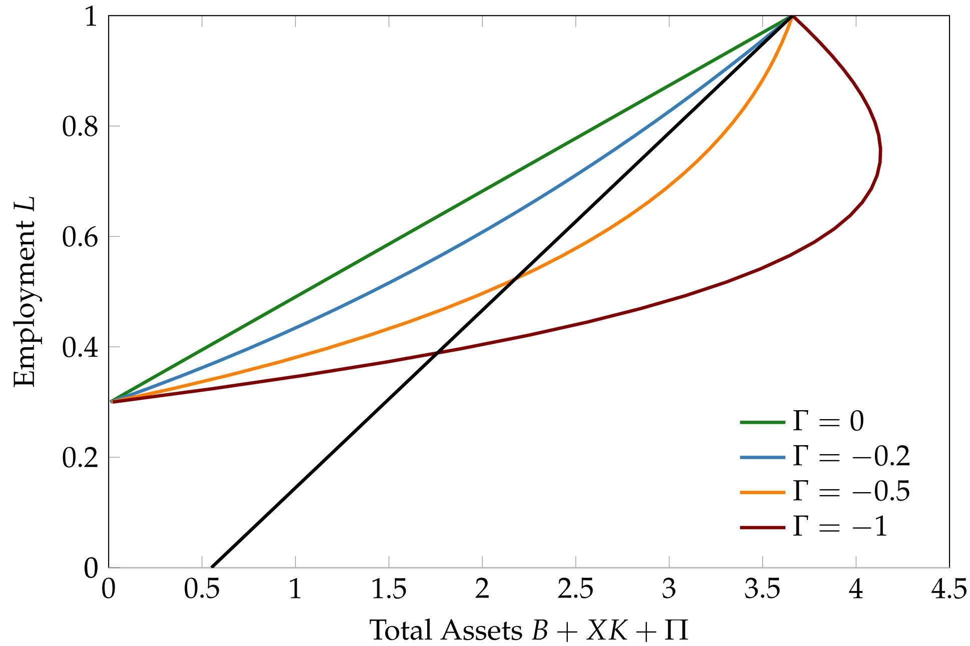

Figure 8: Possibility of multiple equilibria when Γ is negative enough

4.5 Fiscal policy multipliers

We next relax our benchmark fiscal policy assumption (18). Specifically, we assume that the government has the following fiscal rule in place

$$\frac{d\overline{B}}{\overline{B}} = \epsilon_{BL}\frac{dL}{L} \tag{37}$$

$$\frac{dG}{G} = \epsilon_{GL}\frac{dL}{L} \tag{38}$$

The case of countercyclical fiscal policy is the one where $\epsilon_{BL} < 0$ and $\epsilon_{GL} < 0$, so that the government increases both debt and spending in response to falls in employment. We obtain:

Corollary 3. *With the fiscal policy rule (37) and (38) in place, the expression for (35) is modified to*

$$\frac{dY}{Y} = -\frac{1}{\omega\left(1-\epsilon_{BL}\right)+\frac{\tau}{1-\tau}\left(1-\epsilon_{GL}\right)}\epsilon_{a\varphi}d\varphi \tag{39}$$

This proposition shows that countercyclical policies reduce the value of the value of the multiplier and therefore mitigate the Keynesian stagnation. This is intuitive, but note here that government debt plays an independent role from spending. In fact, this model features a government liquidity multiplier (in the traditional Keynesian sense) as well as a government spending multiplier. In the interest of space, we study these multipliers in Appendix E.4.

4.6 Labor share changes

We now study the effect on aggregate output of a falling labor share generated via changes in markups μ, the price of investment X, or the production function. It is often hypothesized that a fall in the labor share could aggravate secular stagnation (Krugman (2016), Summers (2016)). We showed that this intuition was correct in the short run. We now show that it is incorrect in the long run. The intuition is that a falling labor share, *no matter its source*, increases normalized net asset supply (or reduces normalized net asset demand). This plays a stabilizing role in a world where excess demand for assets is the problem.

Changes in technology: the production function and investment prices. In the model, either a change in the production function or a change in investment prices (assuming $\epsilon \neq 1$) can lead to a shift in the long-run labor share. This movement in labor share causes normalized net asset supply to move in the same direction, with a magnitude that depends only on the labor share; conditional on this, the source of the change (the production function or investment prices) is irrelevant.

Proposition 4. *An exogenous change in the production function or investment prices leading to a decrease in the labor share causes a decrease in net normalized asset demand, and an increase in aggregate output in the Keynesian regime.*

The logic behind proposition 4 is that either source of exogenous technological change, if it causes a decline in labor share, will commensurately increase the capital share and the ratio XK/Y of capital to output. There is no direct effect on the other two assets in the economy, bonds B/Y and capitalized profits Π/Y , relative to output. Hence the overall ratio of assets to output increases, and since the after-tax labor share is declining, *normalized* asset supply in (31) increases by an even greater proportion.

Changes in markups. The consequences of markups are somewhat more complex. A permanent increase in markups has, in principle, an ambiguous effect on the labor share. The rise in profits relative to factor earnings tends to push down the labor share, but the markup distortion also leads to substitution away from capital in production—which, if the elasticity of substitution is high enough, can actually lead to an increase in the labor share. This unconventional effect only happens for elasticities of substitution well beyond the levels usually used by economists, however: for higher markups to cause an increase in the labor share, ϵ must exceed the inverse of the capital share, which in our calibration is roughly 3.

The same forces mean that the effect on normalized asset supply can, in principle, also go in either direction. Proposition 5 shows that the result is unambiguous, however, if we know that the labor share declines.

Proposition 5. *If a rise in μ leads to a decrease in the labor share, it also causes a decrease in net normalized asset demand, and an increase in aggregate output in the Keynesian regime.*

If rising markups cause a decline in labor share, that means that the rise in profit share exceeds the (possible) fall in capital share. In the model, profits are capitalized into assets at a higher rate than capital earnings, since capital depreciates at rate delta but claims on monopoly profits do not.[21] The income shift toward profits therefore results in a net increase in asset supply, as stated in proposition 5.

Arguments that rising markups will cause secular stagnation (e.g. Summers (2016)) have often emphasized the role of markups in discouraging capital accumulation. While proposition 5 does not rule out this mechanism entirely, it shows that markups must then also be associated with a rising labor share, contrary to recent experience.

4.7 Empirical assessment

Whichever specification of inequality multipliers and fiscal policy we adopt, the previous sections show that the key determinant of the magnitude of any steady-state effect from an increase in labor income inequality is the semielasticity of aggregate savings to idiosyncratic risk. For our analysis to be credible, we therefore have to show that they compare favorably to empirical estimates of the effect of idiosyncratic risk on savings. This is the subject of a vast literature, an important contribution to which is Carroll and Samwick (1997). These authors use the PSID to obtain group-level measures of the variance of innovations to the permanent component $s^2_{i\eta}$ and the transitory component $s^2_{i\epsilon}$ of income, and then run a regression

$$\log a_i = \alpha_1 s^2_{i\eta} + \alpha_2 s^2_{i\epsilon} + \beta Z_i + u_i \tag{40}$$

where a_i is individual wealth and Z_i are individual-level controls. In (40), α_1 provides a weighted average of individual semielasticities of savings to a change in the variance of the permanent component of earnings, $\varphi = s^2_{i\eta}$, while α_2 provides the equivalent for innovations to the transitory component. For our formula (35) we need a related semielasticity, with weights equal to those of individuals in the stationary distribution. Abstracting away from differences in weighting schemes, we can compute the equivalent semielasticity in the model, shocking separately the transitory and the persistent component of variance.

Table 4 conducts this exercise and shows that the model's semielasticity is very close to that of the data for permanent shocks, while it is actually below that of the data for temporary shocks. Hence, if a semielasticity model is correct, the model if anything understates the partial equilibrium effect on savings of a given increase in inequality. There are two caveats to this result. First, the baseline levels of shock variance in the model are between four and five times above those in the data. If the true model is one of constant elasticities rather than

[21] Even if claims on profits do depreciate in practice, all that is required for the proof is that the rate of depreciation is lower than that of capital—which, given $\delta = 5.6\%$ in this calibration, seems likely.

		Value, Data	Value, Model
	α_1	12.09	11.83
	α_2	7.11	1.11
Baseline levels:	$\mathbb{E}\left[s_{i\eta}^2\right]$	0.022	0.078
	$\mathbb{E}\left[s_{i\epsilon}^2\right]$	0.044	0.199

Table 4: Comparison between sufficient statistics in the model and the data: transitory shock

semielasticities, then our model may be overestimating the effect coming from the persistent component. Second, we stress that the model interprets all of a given increase in inequality as being the translation of increased idiosyncratic risk. While we think this interpretation can be well defended (any residual would have to be due to fixed effects, which should be fixed by definition, or changing population, which are likely altruistically linked as in one interpretation of our model), if it fails in practice, idiosyncratic risk may not in fact be increasing as fast, which would again dampen the savings response.

5 Inequality and the decline in the equilibrium rate

In this section, we finally consider the role of inequality in the decline of r^*, again focusing our estimates on our benchmark experiment—the bottom row of table 2.

5.1 Impulse response

Figure 9 shows that in our model, inequality alone can explain around 90bps of the total decline in r^* from 1980 to today. This is a nontrivial fraction, though—perhaps unsurprisingly—inequality cannot account for all of the r^* decline. We discuss what is behind this 90bp magnitude in the next section.

While our inequality shock is phased in, starting in 1980, over around 20 years, the transition to the new steady state takes nearly 80 years. In other words, the assumption that we could have reached a new steady state already from this slow rise in inequality does not stand. A recent literature on the dynamics of income inequality has pointed out that microfounded models tend to generate too slow a rise in inequality relative to the data (Gabaix, Lasry, Lions and Moll 2015). Our microfounded model suggests that, even conditional on a rapid rise in income inequality, wealth inequality might be slow to respond, and the real interest rate r^* might keep falling even as inequality as stopped increasing.

5.2 Decomposing the r^* effect

The methodology we have introduced throughout this paper of decomposing the endogenous response of an economy to a shock to aggregate demand does not limit itself to studying output

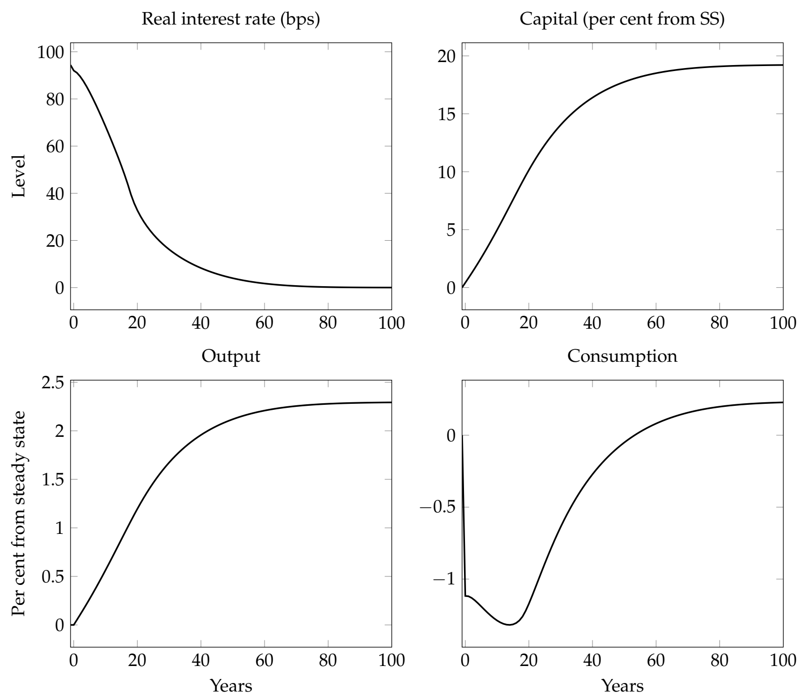

Figure 9: Inequality and the r^* decline

effects. We illustrate this by showing how to decompose the fall in the equilibrium interest rate that follows various sources of increases in inequality. To do this, we go back to equation (34), consider the Keynesian regime $L = 1$, and show

Proposition 6. *The effect on the equilibrium interest rate r^* of any of the inequality-inducing exogenous variables $x = \varphi, X, \mu$ or α is given by*

$$dr^* = -\frac{1}{\frac{\partial \log a}{\partial r} - \frac{\partial \log l}{\partial r}} \frac{\partial \log a^{net}}{\partial x} dx$$

where the semielasticity of normalized liquidity supply with respect to r is given by $\frac{\partial \log l}{\partial r} = \frac{K}{B+XK+\Pi} \frac{\partial \log \frac{K}{L}}{\partial r} - \frac{\partial \log \frac{W}{P}}{\partial r}$.

The last two terms are standard terms from neoclassical theory, and are reported for reference in appendix B. Hence we can again break down the effect into one that comes from the displacement of net normalized saving demand from the inequality shock, and the equilibrium effect which depends the elasticities of asset demand and supply.

As we have argued in section 4.7, our sufficient statistic term appears to be in line with empirical estiamtes: the partial equilibrium effect of a rise in inequality has a plausible magnitude. This focuses the question of the credibility of our estimate on the elasticities of asset demand and supply to interest rates. In our calibration both elastisticites are high, with asset demand contributing roughly twice as much as supply. The elasticity of aggregate savings to interest rates is subject to much controversy. If our model overstates this elasticity, it will understate the decline in r^* attributable to inequality.

A recent literature has been exploring the role of various factors such as demographics in explaining the decline in real interest rates in the context of neoclassical models (see for example Carvalho, Ferrero and Nechio (2016)). We suggest that it would be helpful, going forward, to report both the partial equilibrium effect of the change in net aggregate asset demand being studied, as well as the model's elasticities to interest rates, as a way to help compare estimates and narrow down the range of possibilities.

6 Conclusion

We study a workhorse model of macroeconomics with heterogeneous agents in its Keynesian regime of fixed real interest rates and adjusting labor income in order to evaluate the potential effects of rising income inequality on output. Over the short-run, the magnitude of the effect depends on the covariance between marginal propensities to consume and income and is then amplified by static and dynamic, aggregate and redistributive mechanisms. Over the long run MPCs do not play a role, and the key is instead the extent to which the redistribution perturbs the asset market. Secular stagnation is possible in response to a rise in inequality, but is undone by sufficiently responsive fiscal policy. Debt issuances financing tax cuts raise output and crowd in capital. Investment never rises in response to the increase in desired savings as it is determined by future marginal products of capital, which fall in a depressed economy.

Our model has abstracted from all effects of inequality other than its direct effect on aggregate consumption and its general equilibrium consequences. A more complete analysis would evaluate the supply-side effects of policy instruments to undo redistribution against the effects we studied here. The neoclassical regime of heterogeneous agent models has been the building block of numerous studies of increasing complexity. We hope that our methodology of separating multipliers from partial-equilibrium sufficient statistics effects will help compare across quantitative macroeconomic models of aggregate demand and bring them closer in line with evidence from micro data.

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A The firm problem

Since we focus on the problem of a single firm, we drop the index j for ease of notation. The value of the firm at time t, when its capital is K_{t-1}, is

$$
\begin{aligned}
J_t\left(K_{t-1}\right) \quad &= \quad \max_{p_t, K_t, L_t} \left\{ \frac{p_t}{P_t} F_t\left(K_{t-1}, L_t\right) - \frac{W_t}{P_t} L_t - \frac{P_t^I}{P_t} \left(I_t + \zeta\left(\frac{I_t}{K_{t-1}} - \delta \right) K_{t-1} \right) + \frac{1}{1 + r_t + \varrho} J_{t+1}\left(K_t\right) \right\} \\
&\text{s.t.} \quad I_t = K_t - \left(1 - \delta\right) K_{t-1} \qquad (41) \\
&\qquad \frac{p_t}{P_t} = \left(\frac{F\left(K_{t-1}, L_t\right)}{Y_t} \right)^{\frac{1}{\mu_t} - 1}
\end{aligned}
$$

with $Y_t \equiv Y_t^C + X_t Y_t^I$, the aggregate demand for goods expressed in units of consumption. The last equation is an inverse demand curve derived from the demand of final goods firms in (9).

Labor choice The first-order condition for L_t yields

$$\frac{1}{\mu_t} p_t F_{Lt}\left(K_{t-1}, L_t\right) = W_t \tag{42}$$

The firm equates the marginal revenue product of labor (which is below the value of the physical marginal product due to the effect of additional units produced on price) to its marginal cost.

Capital choice The first order condition for next period capital K_t is

$$\frac{1}{1+r_t+\varrho} \frac{\partial J_{t+1}}{\partial K_t}\left(K_t\right) = \frac{P_t^I}{P_t}\left(1+\zeta'\left(\frac{I_t}{K_{t-1}}-\delta\right)\right) \tag{43}$$

Define

$$q_t \equiv \frac{1}{\frac{P_t^I}{P_t}} \frac{1}{1+r_t+\varrho} \frac{\partial J_{t+1}}{\partial K_t}\left(K_t\right) \tag{44}$$

as the discounted value of a marginal unit of capital next period in units of the investment good. Then (43) is simply the Q-theory relationship

$$\zeta'\left(\frac{I_t}{K_{t-1}}-\delta\right) = q_t - 1 \tag{11}$$

Moreover, the envelope theorem implies that

$$\begin{aligned}
\frac{\partial J_t}{\partial K_{t-1}}\left(K_{t-1}\right) &= \frac{1}{\mu_t}\frac{p_t}{P_t} F_{Kt}\left(K_{t-1}, L_t\right) - \frac{P_t^I}{P_t}\left(-(1-\delta) - \zeta'\left(\frac{K_t}{K_{t-1}}-1\right)\left(\frac{K_t}{K_{t-1}}\right) + \zeta\left(\frac{K_t}{K_{t-1}}-1\right)\right) \\
&= \frac{1}{\mu_t}\frac{p_t}{P_t} F_{Kt}\left(K_{t-1}, L_t\right) - \frac{P_t^I}{P_t}\left(\delta - 1 + \left\{1 - \frac{1}{\frac{P_t^I}{P_t}} \frac{1}{1+r_t+\varrho} \frac{\partial J_{t+1}}{\partial K_t}\left(K_t\right)\right\}\left(\frac{K_t}{K_{t-1}}\right) + \zeta\left(\frac{K_t}{K_{t-1}}-1\right)\right) \\
&= \frac{1}{\mu_t}\frac{p_t}{P_t} F_{Kt}\left(K_{t-1}, L_t\right) - \frac{P_t^I}{P_t}\left(\delta + \left(\frac{K_t}{K_{t-1}}-1\right) + \zeta\left(\frac{K_t}{K_{t-1}}-1\right)\right) + \frac{1}{1+r_t+\varrho}\frac{\partial J_{t+1}}{\partial K_t}\left(K_t\right)\frac{K_t}{K_{t-1}}
\end{aligned}$$

where the second line used (43). Plugging in (44), this implies the following dynamic relationship between q_{t-1} and q_t:

$$\frac{P_{t-1}^I}{P_{t-1}}\left(1+r_{t-1}+\varrho\right) q_{t-1} = \frac{1}{\mu_t}\frac{p_t}{P_t} F_{Kt}\left(K_{t-1}, L_t\right) - \frac{P_t^I}{P_t}\left(\delta + \left(\frac{K_t}{K_{t-1}}-1\right) + \zeta\left(\frac{K_t}{K_{t-1}}-1\right) - \frac{K_t}{K_{t-1}} q_t\right)$$

which we can also write as

$$q_{t-1} = \frac{1}{1+r_{t-1}+\varrho}\left\{\frac{\frac{1}{\mu_t}\frac{p_t}{P_t} F_{Kt}\left(K_{t-1}, L_t\right)}{\frac{P_{t-1}^I}{P_{t-1}}} + \frac{\frac{P_t^I}{P_t}}{\frac{P_{t-1}^I}{P_{t-1}}}\left[\frac{K_t}{K_{t-1}} q_t - \left(\delta + \left(\frac{K_t}{K_{t-1}}-1\right) + \zeta\left(\frac{K_t}{K_{t-1}}-1\right)\right)\right]\right\} \tag{45}$$

Equilibrium value of the firm. In equibrium, the level of technology X_t determines the relative price of investment $\frac{p_t^I}{P_t} = X_t$ at all times. Equation (45) can therefore be rewritten as

$$(1 + r_{t-1} + \varrho)\, q_{t-1} X_{t-1} K_{t-1} = \frac{1}{\mu_t} \frac{p_t}{P_t} F_{Kt}(K_{t-1}, L_t)\, K_{t-1} - X_t \left(I_t + \zeta \left(\frac{K_t}{K_{t-1}} - 1 \right) K_{t-1} \right) + q_t X_t K_t \tag{46}$$

Define also $\Pi_{t-1}(K_{t-1})$ as the capitalized value of present and future pure profits, discounted at the risky rate. It satisfies the recursive equation

$$(1 + r_{t-1} + \varrho)\, \Pi_{t-1}(K_{t-1}) = \frac{p_t}{P_t} \left(1 - \frac{1}{\mu_t} \right) F_t(K_{t-1}, L_t) + \Pi_t(K_t) \tag{47}$$

Note then that, since

$$\left(1 - \frac{1}{\mu_t} \right) \frac{p_t}{P_t} F_t(K_{t-1}, L_t) + \frac{1}{\mu} \frac{p_t}{P_t} F_{Kt}(K_{t-1}, L_t)\, K_{t-1} = \frac{p_t}{P_t} F_t(K_{t-1}, L_t) - \frac{W_t}{P_t} L_t$$

we have, summing (46) and (47),

$$(1 + r_{t-1} + \varrho) \{ \Pi_{t-1}(K_{t-1}) + q_{t-1} X_{t-1} K_{t-1} \} = d_t + \{ \Pi_t(K_t) + q_t X_t K_t \}$$

We recognize the value of the firm J_t, which can therefore be split, assuming no bubbles, into a component of pure profits and a component of installed capital:

$$J_t(K_{t-1}) = (1 + r_{t-1} + \varrho)\, (\Pi_{t-1}(K_{t-1}) + q_{t-1} X_{t-1} K_{t-1})$$

Aggregation. In equilibrium, if all firms begin with the same capital stock, they make the same decisions and remain identical at all times. This implies in particular a constant price for final goods, $P_t = p_{jt}$, and we can assume without loss of generality that there is a unique firm with capital stock K_t. From (42), we obtain in particular (12).

In equilibrium with a unit share outstanding, the price of shares must be equal to the value of the firm. Since the stock price satisfies

$$(1 + r_{t-1} + \varrho)\, p_{t-1} = d_t + p_t$$

and is therefore defined ex-dividend, while the value of the firm $J_t(K_{t-1})$ in (41) is defined cum dividend, we therefore have

$$p_t = \frac{J_{t+1}(K_t)}{1 + r_t + \varrho}$$

implying that the stock price at t is simply

$$p_t = \Pi_t + q_t X_t K_t$$

Steady-state. Consider converge to a steady-state level (r, X, K, L, W, P). We enforce the equilibrium price $p_j = P$ for all j.

Since K is constant in steady state, (45) implies

$$q = \frac{1}{r+\varrho}\left\{\frac{1}{\mu X}F_K(K,L) - \delta\right\}$$

while (11) implies $q = 1$. Combining the two we obtain

$$F_K(K,L) = \mu X(r+\varrho+\delta) \tag{13}$$

The first-order condition for labor (42) in steady state reads

$$F_L(K,L) = \mu\frac{W}{P} \tag{14}$$

Finally, (47) implies

$$\Pi = \frac{\left(1-\frac{1}{\mu}\right)F(K,L)}{r+\varrho} \tag{15}$$

Computation. Consider the partial equilibrium problem of a firm with installed capital K_{-1}, facing the sequence of interest rates $\{r_0, r_1, \ldots\}$, demand elasticity parameters $\{\mu_0, \mu_1, \ldots\}$, investment technology $\{X_0, X_1, \ldots\}$ and economy-wide employment levels $\{L_0, L_1, \ldots\}$. These sequences converge to a steady-state level (r, X, L).

First note that (13) determines the steady-state level of capital K, given by

$$F_K\left(\frac{K}{L}, 1\right) = \mu X(r+\varrho+\delta)$$

In turn this determine the steady-state level of real wages $\frac{W}{P}$, given by $\frac{W}{P} = \frac{1}{\mu}F_L\left(\frac{K}{L}, 1\right)$. This equation determines the price level P consistent with a level of real wages W. The steady-state level of q is 1, and the steady-state profit level Π follows from (15).

The solution to the optimal investment problem consists of the dynamic path $\{K_0, K_1, \ldots\}$, $\{q_0, q_1, \ldots\}$, $\{\Pi_0, \Pi_1, \ldots\}$ that jointly solves (11), (45) and (47), given initial K_{-1} and convergence to the steady-state levels K and $q = 1$. We can rewrite these equations in terms of the path for the *net investment ratio* $\widehat{i}_t \equiv \frac{K_t}{K_{t-1}} - 1 = \frac{I_t}{K_{t-1}} - \delta$, using the fact that $\frac{P_t^I}{P_t} = X_t$ and $p_{jt} = P_t$

at each t. This gives:

$$\begin{aligned}
\zeta'\left(\widehat{i}_t\right) &= q_t - 1 \\
q_{t-1} &= \frac{1}{1+r_{t-1}+\varrho}\left\{\frac{F_K\left(K_{t-1},L_t\right)}{\mu_t X_{t-1}} + \frac{X_t}{X_{t-1}}\left[\left(1+\widehat{i}_t\right)q_t - \left(\delta+\widehat{i}_t+\zeta\left(\widehat{i}_t\right)\right)\right]\right\} \\
\left(1+r_{t-1}+\varrho\right)\Pi_{t-1} &= \left(1-\frac{1}{\mu_t}\right)F\left(K_{t-1},L_t\right)+\Pi_t
\end{aligned}$$

The algorithm is as follows:

- guess an initial value for q_0
- this determines $\widehat{i}_0$ as the solution to $q_0 - 1 = \zeta'\left(\widehat{i}_0\right)$ and therefore $K_0 = K_{-1}\left(1+\widehat{i}_0\right)$.
- next, find $\widehat{i}_1$ that solves

  $$q_0 = \frac{1}{1+r_0+\varrho}\left\{\frac{F_K\left(K_0,L_1\right)}{\mu_0 X_0} + \frac{X_1}{X_0}\left[\left(1+\widehat{i}_1\right)\left(1+\zeta'\left(\widehat{i}_1\right)\right) - \left(\delta+\widehat{i}_1+\zeta\left(\widehat{i}_1\right)\right)\right]\right\}$$

  and obtain $q_1 = 1+\zeta'\left(\widehat{i}_1\right)$ and $K_1 = K_0\left(1+\widehat{i}_1\right)$. Iterating in this way delivers a path for $\widehat{i}_t$, q_t and K_t.
- Check whether this path converges to $q_t \to 1$ and therefore $\widehat{i}_t \to 0$. If so, then the steady-state level of capital is reached and the algorithm terminates. If q_t reaches 0 in finite time, increase the initial guess for q_0. If it diverges to $+\infty$, reduce the initial guess for q_0.
- After convergence has occurred, use equation (42), to determine the level of real wages $\frac{W_t}{P_t} = \frac{1}{\mu_t}F_L\left(K_{t-1},L_t\right)$ at each $t \geq 0$. Use equation

  $$d_t = F\left(K_{t-1},L_t\right) - \frac{W_t}{P_t}L_t - X_t\left(\delta+\widehat{i}_t+\zeta\left(\widehat{i}_t\right)\right)K_{t-1}$$

  to determine the level of dividends paid at each t, and finally use

  $$\left(1+r_{t-1}+\varrho\right)\Pi_{t-1} = \left(1-\frac{1}{\mu_t}\right)F\left(K_{t-1},L_t\right)+\Pi_t$$

  to solve backwards for the level of profits Π_t at each $t \geq 0$ starting from steady-state.
- Under our calibration for ζ in (23), the dynamic equations to solve for are

  $$\widehat{i}_t = \delta\epsilon_I\left(q_t - 1\right)$$

and

$$q_{t-1} = \frac{1}{1+r_{t-1}+\varrho}\left\{\frac{F_K\left(K_{t-1},L_t\right)}{\mu_t X_{t-1}}+\frac{X_t}{X_{t-1}}\left[\left(1+\widehat{i}_t\right)\left(1+\frac{\widehat{i}_t}{\delta\epsilon_I}\right)-\left(\delta+\widehat{i}_t+\frac{1}{2\delta\epsilon_I}\left(\widehat{i}_t\right)^2\right)\right]\right\}$$

whose solution with a gross investment ratio consistent with $K_t > 0$ is

$$\widehat{i}_t = -1+\sqrt{1-2\delta\epsilon_I\left[1-\delta+\frac{X_{t-1}}{X_t}\left(\frac{F_K\left(K_{t-1},L_t\right)}{\mu_t X_{t-1}}-\left(1+r_{t-1}+\varrho\right)q_{t-1}\right)\right]}$$

B Steady-state with CES production function

Consider a CES production function

$$F(K,L) = A\left[(1-f)K^{\frac{\epsilon-1}{\epsilon}}+fL^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{48}$$

where A represents total factor productivity.

Marginal products. We first establish some simple expressions for the marginal product of capital and the marginal product of labor that are important to compute investment dynamics. These are:

$$F_K(K,L) = A(1-f)\left[(1-f)+f\left(\frac{K}{L}\right)^{-\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}}$$

$$F_L(K,L) = Af\left[(1-f)\left(\frac{K}{L}\right)^{\frac{\epsilon-1}{\epsilon}}+f\right]^{\frac{1}{\epsilon-1}}$$

Steady-state relationships. Next, we establish the relationship between the steady state levels for the real interest rate r, the equity premium ϱ, the relative price of investment X, and markups μ on the one hand, and all scaled quantities in table B.1 on the other hand. We treat separately the Cobb-Douglas case where $\epsilon = 1$ and $F(K,L) = AK^{1-f}L^f$.

The first order condition (13) is

$$\mu X(r+\varrho+\delta) = (1-f)AK^{\frac{-1}{\epsilon}}\left[(1-f)K^{\frac{\epsilon-1}{\epsilon}}+fL^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}} = (1-f)A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{K}{Y}\right)^{-\frac{1}{\epsilon}} \tag{49}$$

from which we immediately obtain the capital-output ratio

$$k \equiv \frac{XK}{Y} = A^{\epsilon-1}(1-f)^{\epsilon}X^{1-\epsilon}\mu^{-\epsilon}(r+\varrho+\delta)^{-\epsilon}$$

Table B.1: Quantity ratios for the CES production function in (48)

Notation	Definition	Expression for $\epsilon \neq 1$	Cobb-Douglas case $\epsilon = 1$	Semielasticity $\epsilon_{\cdot r}$
α	$\frac{W}{P}\frac{L}{Y}$	$\frac{1}{\mu} - A^{\epsilon-1}(1-f)^{\epsilon} X^{1-\epsilon}\mu^{-\epsilon}(r+\varrho+\delta)^{1-\epsilon}$	$\frac{f}{\mu}$	$\frac{1-\mu\alpha}{\mu\alpha}(\epsilon-1)\frac{1}{r+\varrho+\delta}$
k	$\frac{XK}{Y}$	$A^{\epsilon-1}(1-f)^{\epsilon} X^{1-\epsilon}\mu^{-\epsilon}(r+\varrho+\delta)^{-\epsilon}$	$\frac{1-f}{\mu(r+\varrho+\delta)}$	$-\epsilon\frac{1}{r+\varrho+\delta}$
κ	$\frac{K}{L}$	$\left(\frac{f}{1-f}\frac{1-\mu\alpha}{\mu\alpha}\right)^{\frac{\epsilon}{\epsilon-1}}$	$\left(\frac{\mu X}{A}\frac{r+\varrho+\delta}{1-a}\right)^{-\frac{1}{f}}$	$-\frac{1}{\mu\alpha}\frac{\epsilon}{r+\varrho+\delta}$
w	$\frac{W}{P}$	$\left(\frac{f}{\mu}\right)^{\frac{\epsilon}{\epsilon-1}} A\alpha^{-\frac{1}{\epsilon-1}}$	$f\left(X\frac{r+\varrho+\delta}{1-f}\right)^{-\frac{1-f}{f}}$	$-\frac{1-\mu\alpha}{\mu\alpha}\frac{1}{r+\varrho+\delta}$
y	$\frac{Y}{L}$	$A\left(\frac{f}{\mu\alpha}\right)^{\frac{\epsilon}{\epsilon-1}}$	$\mu\left(\frac{A}{\mu}\right)^{\frac{1}{f}}\left(X\frac{r+\varrho+\delta}{1-f}\right)^{-\frac{1-f}{f}}$	$-\frac{1-\mu\alpha}{\mu\alpha}\frac{\epsilon}{r+\varrho+\delta}$

Given that output decomposes as

$$Y = F_K K + F_L L = \mu(r+\varrho+\delta)XK + \mu\frac{W}{P}L$$

we next find that the labor share is

$$\alpha \equiv \frac{WL}{PY} = \frac{1}{\mu} - (r+\varrho+\delta)\frac{XK}{Y} = \frac{1}{\mu} - A^{\epsilon-1}(1-f)^{\epsilon}X^{1-\epsilon}\mu^{-\epsilon}(r+\varrho+\delta)^{1-\epsilon} \tag{50}$$

Note that α falls when the relative price of investment X falls or when the real interest rate falls if and only if $\epsilon > 1$. Provided that $\epsilon \neq 1$, from (49) we also have an alternative expression for k as a function of α

$$\frac{1}{\mu} - \alpha = (r+\varrho+\delta)\frac{XK}{Y} = \frac{(1-f)}{\mu}A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{k}{X}\right)^{\frac{\epsilon-1}{\epsilon}}$$

so

$$k = \frac{X}{A}\left(\frac{1-\mu\alpha}{1-f}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{51}$$

Next, since the first order condition for labor (14) is

$$\mu\frac{W}{P} = fA^{\frac{\epsilon-1}{\epsilon}}\left(\frac{L}{Y}\right)^{-\frac{1}{\epsilon}} \tag{52}$$

we also have that

$$\alpha = \frac{1}{\mu}fA^{\frac{\epsilon-1}{\epsilon}}\left(\frac{L}{Y}\right)^{\frac{\epsilon-1}{\epsilon}}$$

so that output per worker solves

$$y \equiv \frac{Y}{L} = A\left(\frac{f}{\mu\alpha}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{53}$$

Note also from (52) that the real wage is

$$w \equiv \frac{W}{P} = \frac{1}{\mu} f A^{\frac{\epsilon-1}{\epsilon}} \left(\frac{Y}{L}\right)^{\frac{1}{\epsilon}} = \left(\frac{f}{\mu}\right)^{\frac{\epsilon}{\epsilon-1}} A\alpha^{-\frac{1}{\epsilon-1}}$$

and finally that the capital-labor ratio is just

$$\kappa \equiv \frac{K}{L} = \frac{k}{X} \times y = \left(\frac{f}{1-f}\frac{1-\mu\alpha}{\mu\alpha}\right)^{\frac{\epsilon}{\epsilon-1}}$$

Cobb-Douglas case. In the Cobb-Douglas case we start from (14), which implies

$$\mu\frac{W}{P} = f\frac{Y}{L}$$

Hence the labor share is $\alpha = \frac{W}{P}\frac{L}{Y} = \frac{f}{\mu}$, or $f = \alpha\mu$. We next use the fact that (13) implies

$$\mu X\left(r+\varrho+\delta\right) = (1-f)\frac{Y}{K} = (1-f)\,A\left(\frac{K}{L}\right)^{-f}$$

to find the capital-labor and the capital-output ratios

$$\begin{aligned} k &= \frac{1-f}{\mu\left(r+\varrho+\delta\right)} = \frac{1-f}{\mu\left(r+\varrho+\delta\right)} \\ \kappa &= \left(\frac{\mu X}{A}\frac{r+\varrho+\delta}{1-f}\right)^{-\frac{1}{f}} \end{aligned}$$

We then use (14) again to find

$$\mu\frac{W}{P} = fA\kappa^{1-a} = fA^{\frac{1}{f}}\left(\mu X\frac{r+\varrho+\delta}{1-f}\right)^{-\frac{1-f}{f}}$$

so

$$w = f\left(\frac{A}{\mu}\right)^{\frac{1}{f}}\left(X\frac{r+\varrho+\delta}{1-f}\right)^{-\frac{1-f}{f}}$$

and, though $\alpha = \frac{W}{P}\frac{L}{Y}$, we finally find

$$y = \frac{1}{\alpha}w$$

Table B.1 collects all of these relationships and reports semielasticities with respect to r, for example $\epsilon_{\frac{K}{L},r} = \frac{\partial \log\left(\frac{K}{L}\right)}{\partial r}$

Calibration given δ, ϵ, μ, X and targets α^{ss}, r^{ss}, y^{ss}, and k^{ss}. From the expression for the capital-output ratio

$$k^{ss} = \frac{\frac{1}{\mu} - \alpha^{ss}}{r^{ss} + \varrho + \delta}$$

we first obtain the equity premium consistent with our targets,

$$\varrho = \frac{k^{ss}}{\frac{1}{\mu} - \alpha^{ss}} - (r^{ss} + \delta)$$

When $\epsilon \neq 1$, we combine (51) and (53) in steady state, $k^{ss} = \frac{X}{A}\left(\frac{1-\mu\alpha^{ss}}{1-a}\right)^{\frac{\epsilon}{\epsilon-1}}$ and $y^* = A\left(\frac{f}{\mu\alpha^{ss}}\right)^{\frac{\epsilon}{\epsilon-1}}$ to get

$$k^{ss}y^{ss} = X\left(\frac{f}{1-f}\frac{1-\mu\alpha^{ss}}{\mu\alpha^{ss}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

which we can use to solve for

$$f = \frac{\mu\alpha^{ss}h}{1-\mu\alpha^{ss}+\mu\alpha^{ss}h}$$

where $h \equiv \left(\frac{k^{ss}y^{ss}}{X}\right)^{\frac{\epsilon-1}{\epsilon}}$. Using (53) again, we obtain

$$A = y^{ss}\left(\frac{\mu\alpha^{ss}}{f}\right)^{\frac{\epsilon}{\epsilon-1}}$$

When $\epsilon = 1$, instead, we have $f = \mu\alpha$ and $y^{ss} = A\left(\frac{k^{ss}y^{ss}}{X}\right)^{1-a}$ so

$$A = (y^{ss})^{\mu\alpha}\left(\frac{k^{ss}}{X}\right)^{\mu\alpha-1}$$

Using these expressions for f and A, we can determine all quantities in terms of our calibration targets.

C Calibration details

C.1 Micro to macro balance sheets and the distribution of shareholdings $\theta(a)$

We start by aggregating the 2013 SCF data according to the broad categories that are comparable to the FoF data, along the lines of Henriques and Hsu (2012). The top panel of figure C.1 shows the 2013 SCF side by side with equivalent categories in the FoF. Once defined benefit pension wealth is taken out of the FoF (since it is not recorded in the SCF), the two sources give a very similar picture of household balance sheets. The major remaining difference is that the SCF seemingly underreports deposits and bonds relative to the Flow of Funds, an issue that

Henriques and Hsu (2012) attribute to the fact that FoF household sector include nonprofits which may hold deposits and bonds in large quantities.

Next, we obtain the 2013 value of the government bond stock B. We include Treasury securities (federal, state and local) as well as municipal securities. Since our model is one of a closed economy, we exclude held by the rest of the world.[22] This gives us a value for B of \$9.2trn in 2013.

We take a highly simplified view of other sectors in the flow of funds so as to provide the simplest possible map from consumer balance sheets to the aggregate capital stock. To this end, we assume that the business sector (nonfinancial and financial sectors combined) are pure passthrough entities. They issue the bonds and equities that are the counterparts of those on household balance sheets, and hold the domestic government bond stock as well as their own capital. We calculate the business capital stock as the residual implied by the value of household net worth under this abstraction. As figure C.1 shows, this calculation gives a business capital stock of \$24.5trn, which is slightly below the \$31.3trn implied by summing all business nonfinancial assets directly.[23] The difference between these two numbers is mostly due to the fact that we are not counting defined benefit pensions in household net worth, which we choose to do because capital in our model is very liquid and DB pensions are completely illiquid. Using this procedure, we obtain the two simplified balance sheets presented in the bottom panel of figure C.1. Combining the balance sheets of the FoF household and business sectors, we obtain a simple aggregate balance sheet dividing household net worth into \$9.2trn in government bonds and \$51.8trn in domestic capital.

We next want to split the capital stock into the value of installed capital XK and pure profits Π. To do this, we take the simple approach of using the value reported in the BEA fixed asset tables, \$38.2trn, as the value of capital XK, and the residual (\$13.6trn) is therefore treated as monopoly profits. One source of discrepancy between the FoF and the BEA numbers is that the latter excludes land. Our model therefore treats land and monopoly profits identically, as assets that generate a permanent stream of income and do not depreciate.

Going back to individual-level balance sheets, we have some degree of freedom as to how we categorize assets into individual bonds b and shares v, given the SCF/FoF discrepancy and the fact that we are not modeling the financial sector in any detail. Our preferred measure takes a broad interpretation of shares as including any wealth that is not in the form of deposits or bonds directly held. For each household, we count all deposits and 48% of mutual fund wealth towards a measure of bonds, and label the remained of net worth as shares. This definition

[22] We sum lines 2 from table L.210 of the FoF and line 1 of table L.212, subtracting the cross-holdings of Federal bonds by state governments (line 20). This gives us total outstanding government securities (\$15.1trn). We then subtract foreign holdings (\$5.7trn) of Treasuries. The rest of the world owns few municipal securities because it does not benefit from its favorable tax treatment.

[23] In the Integrated Macroeconomic Accounts of the 2013 FoF, the capital stock of the nonfinancial noncorporate sector is \$11trn, (table S.4.a, line 79), that of the nonfinancial corporate sector is \$18.5trn (table S.5.a, line 98) and that of the financial sector is \$1.7trn (table S.6.a, line 100)

SCF 2013

Assets		Liabilities and Net Worth	
Real estate	$26.1trn	Consumer credit	$11.2trn
Consumer durables	$2.4trn	Net worth	$54.9trn
Deposits&bonds	$6trn		
Equities&pensions	$31.6trn		

FoF 2013, Households

Assets		Liabilities and Net Worth	
Real estate	$22.4trn	Consumer credit	$13.8trn
Consumer durables	$4.9trn	Net worth	$61.1trn
Deposits&bonds	$13.9trn		
Equity&pensions	$33.6trn		

FoF 2013, Business (simplified)

Assets		Liabilities	
Business capital	$24.5trn	Deposits&Bonds	$13.9trn
Consumer credit	$13.8trn	Equity&pensions	$33.6trn
Govtt Bonds	$9.2trn		

FoF 2013, Government

Assets		Liabilities	
Backing assets	$9.2trn	Govtt Bonds	$9.2trn

Figure C.1: 2013 SCF and FoF data (simplified business and government sectors)

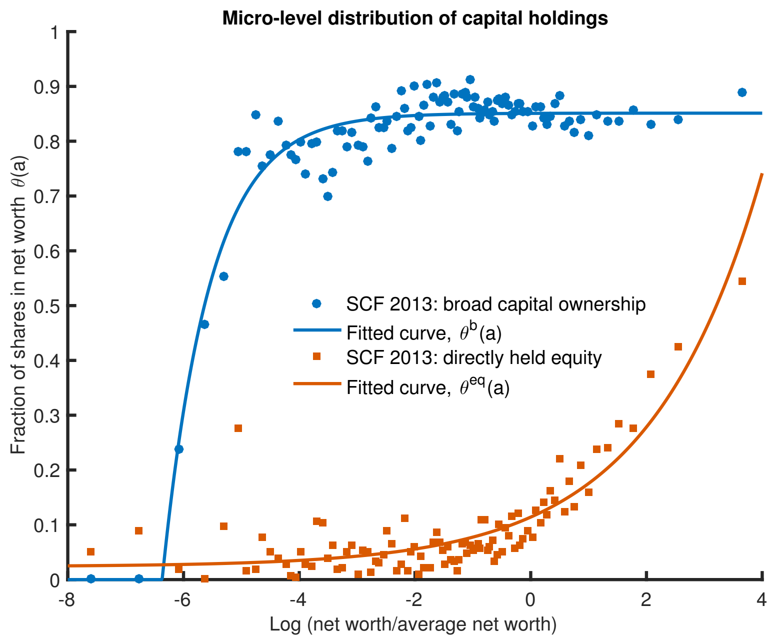

Figure C.2: Individual holdings of equity in the SCF

ensures exact aggregation to our target for B. It implies notably that the premium ϱ applies to all consumer credit including mortgages, as well as to housing. We also take a more narrow definition of equity, in which we include only the total value of directly-held stocks, stocks held through mutual funds, and closely held businesses. This definition is not consistent with aggregation, but we scale up shareholdings in proportion until aggregation obtains.

To be precise, we keep households with at least \$100 in net worth and group them by centiles $i = 1 \ldots 100$ of net worth. We then compute total capital holdings k_i and total net worth a_i in each bin i under both definitions of k_i, and fit a smooth curve f through the relationship between $\frac{k_i}{a_i}$ and $\log a_i$. This allows us to back out $\theta^b(a) = f^b(e^a)$ and $\theta^{eq}(a) = f^{eq}(e^a)$. These two distributions, together with the underlying centile values, are plotted in figure C.2.

Our narrow capital measure reveals a well-known pattern: the share of wealth invested in stocks rises quickly with wealth. Once those positions are scaled up, the richest individuals in the economy own levered equity claims. We verify that $b_{it} + p_t v_{it} \geq 0$ in all our simulations, so that we do not have to deal with cases of bankruptcy.

C.2 Moments of state distributions vs data

		sd. log	CV	Gini	Percentage held by				
					Bottom 40%	Top 20%	Top 10%	Top 5%	Top 1%
Consumption	Model	0.72	1.01	0.43	16%	51%	34%	22%	7%
	Data	*0.65*	*0.65*	*0.33*	*19%*	*40%*	*24%*	*14%*	*4%*
Pretax Income	Model	0.92	1.97	0.51	13%	52%	31%	26%	5%
	Data	*0.95*	*2.26*	*0.51*	*12%*	*55%*	*39%*	*28%*	*13%*
Wealth	Model	2.08	1.81	0.71	2%	74%	54%	36%	12%
	Data	*2.22*	*6.28*	*0.81*	*1%*	*84%*	*72%*	*60%*	*33%*

Table C.1: Steady-state distribution statistics

D Decomposing partial and general equilibrium effects

D.1 A general decomposition

[To be added]

D.2 Approximation to $\mathcal{G}$

Here we derive an approximation to the first row element $\mathcal{G}_0$ of the GE matrix $\mathcal{G}$ as follows. We counterfactually assume that all of the effects on employment take place at date 0: $L_t = 1$ for $t \geq 1$. Recall also that real interest rates are not changing due to our assumption on the monetary policy response.

Effect on investment. Under our assumption of no future change in employment, investment at date 0 would not change:

$$dI_0 = 0$$

The intuition is that investment decisions are forward looking and only based on the path of r_t for $t \geq 0$ and L_t for $t \geq 1$. The effect on current employment L_0 does affect the marginal product of capital (and therefore dividends) at date 0, but this effect is sunk from the point of view of investment decisions.

Factor prices. We next turn to the effect on factor prices, which follow from neoclassical theory. Note first that, since capital is fixed and labor is paid its marginal product, differentiation of the production function yields

$$dY_0 = \frac{W_0}{P_0} dL_0 \tag{54}$$

Since constant returns to scale also implies $Y_0 = \frac{W_0}{P_0} L_0 + \frac{R_0}{P_0} K_{-1}$, we therefore have

$$d\left(\frac{W_0}{P_0}\right) L_0 = -d\left(\frac{R_0}{P_0}\right) K_{-1} = -d\left(d_0\right)$$

showing that the change in factor payments is a redistribution between wage earners to capital owners. Since, by definition of the elasticity of substitution between capital and labor, we have

$$d\left(\frac{W_0}{P_0}\right)\frac{P_0}{W_0} - d\left(\frac{R_0}{P_0}\right)\frac{P_0}{R_0} = -\epsilon^{-1}\frac{dL_0}{L_0}$$

simple algebra then establishes that

$$d\left(\frac{W_0}{P_0}\right) L_0 = -\frac{1-\alpha}{\epsilon} dY_0 \tag{55}$$

Hence as output falls, wages rise to an extent that depends on ratio between the capital share and the elasticity of capital-labor substitution.

Consumption. Moving on to individual consumption, individual i's date-0 consumption change is given by the sum of his gain or loss from the redistribution, the additional effect from adjusting incomes, and the effect from changing dividends:

$$dc_{i0} = MPC_{i0}\left\{dh_{i0} + d\left(y_{i0} - h_{i0}\right) + v_i d\left(d_0\right)\right\}$$

The first term is our impulse and we have already established an expression for the last term. In the proof of proposition 7 below, we show the remaining term to be equal, under our fiscal rule, to

$$d\left(y_{i0} - h_{i0}\right) = \frac{y_i}{\mathbb{E}\left[y_i\right]} dY_0 - \frac{y_i}{\mathbb{E}\left[y_i\right]}\frac{1-\alpha}{\epsilon} dY_0 + \left(1-\tau_r\right)\left(1-\tau_0\right) e_i \gamma_{Li} dY_0 \tag{56}$$

where $\epsilon_{L\gamma i}$ is the elasticity of individual γ_i so L. Hence the endogenous change in post-tax incomes is the sum of three terms: an effect from the fall in the *level* of GDP (affecting the overall level of employment), an effect from the rise in real wages according to (55), and an effect from the endogenous redistribution of income coming from the unequal incidence of employment.

Aggregating up these responses, we obtain a per-capita change in consumption of

$$dC_0 = \mathbb{E}\left[dc_{i0}\right] = \mathrm{Cov}\left(MPC_i, dh_{i0}\right) + \overline{MPC}^y dY_0 - \left(\overline{MPC}^y - \overline{MPC}^v\right)\frac{1-\alpha}{\epsilon} dY_0 + c\Gamma dY_0 \tag{57}$$

where $\overline{MPC}^y$ is the income-weighted MPC, $\overline{MPC}^v$ the share-weighted MPC, with

$$c\Gamma = \left(1-\tau_r\right)\left(1-\tau_0\right)\mathrm{Cov}(MPC_i, \gamma_{iL})$$

the endogenous redistribution term. Under our calibrated functional form for γ_i in (21), it turns out that

$$c = (1-\tau_r)(1-\tau_0)\,\mathrm{Cov}\left(MPC_i, \frac{z_{i0}}{E[z_{i0}]}\log\left(\frac{z_{i0}}{E[z_{i0}]}\right)\right)$$

Hence, to the extent that Γ is negative (falls in employment affect agents with low incomes the most) and c is negative (agents with low incomes have higher MPCs), the term of the right-hand side of (57) has the same sign as dY and generates amplification.

Putting all responses together. With no change in investment or government spending, the change in GDP is $dY = dC$. We can therefore solve (57) to obtain

$$dC_0 = \frac{\mathrm{Cov}(MPC_i, dh_{i0})}{1-\overline{MPC}^y + \left(\overline{MPC}^y - \overline{MPC}^v\right)\frac{1-\alpha}{\epsilon} - c\Gamma} \tag{58}$$

For clarity we separate the aggregate effect and these redistributive effects and collect our results in the following proposition.

Proposition 7. *Assuming that the change in employment beyond $t=1$ is negligible, the impact effect on output of an arbitrary redistributive change dh_i is given by*

$$dY_0^{GEs} = \underbrace{\frac{1}{1-\frac{c\Gamma - \left(\overline{MPC}^y - \overline{MPC}^v\right)\frac{1-\alpha}{\epsilon}}{1-\overline{MPC}^y}}}_{\text{Inequality multiplier}} \times \underbrace{\frac{1}{1-\overline{MPC}^y}}_{\text{Keynesian multiplier}} \times \underbrace{\mathrm{Cov}(MPC_i, dh_{i0})}_{\text{Redistributive impulse}} \tag{59}$$

in particular, for a change $d\tau_r$ in the redistributive tax rate, $\mathrm{Cov}(MPC_i, dh_{i0}) = -\mathrm{Cov}(MPC_i, y_i)\frac{d\tau_r}{1-d\tau_r}$.

The term c is positive when incomes and MPCs are positively correlated. This formula shows that, in this static general equilibrium exercise, the effect of the partial equilibrium consumption response gets amplified through Keynesian income channels (falls in GDP reduce aggregate income and lower consumption) as well as through two sources of endogenous inequality: a redistribution from capitalists to workers, which mitigates the fall in consumption if $\overline{MPC}^y > \overline{MPC}^v$, and a redistribution term due to unequal incidence, which accentuates the fall in GDP if $c\Gamma > 0$.

This approximation sheds light on the main transmission mechanism of transitory redistributive shocks to output, assuming unresponsive monetary policy. Absent a response of labor income, the exogenous redistribution affects consumption through the covariance between MPCs and the direction of the redistribution (26). As labor incomes adjust, two additional static redistributive mechanisms come into play: a redistribution between workers and share owners due to factor price adjustment, and a redistribution between workers with different exposures to falls in labor income.

This approximation, however, misses dynamic feedback effects that contribute to amplifying the initial falls in consumption: falls in future labor income reduce consumption today. Moreover, and contrary to the neoclassical intuition, there is no reason to expect investment to rise in response to the increase in overall desired savings after a redistribution. With the real interest rate fixed at r, the main determinant of investment is *future employment*, through its effect on future marginal products of capital. Lower future employment lowers the future marginal product of capital and makes investment fall today, an effect visible in figure 3. All of these effects are captured by the dynamic GE multipliers discussed in section 3.3.

Proof of equation (55). Write

$$y_i - h_i = (1-\tau_0)\frac{W_0}{P_0}L_0\left[\tau_r + (1-\tau_r)e_i\gamma_i(L_0)\right]\Big)$$

We therefore have

$$\begin{aligned}\frac{d(y_i-h_i)}{y_i} &= \frac{d\left((1-\tau_0)\frac{W_0}{P_0}L_0\right)}{(1-\tau_0)\frac{W_0}{P_0}L_0} + \frac{(1-\tau_r)e_i\gamma_i(1)}{\tau_r+(1-\tau_r)e_i\gamma_i(1)}\frac{\gamma_{Li}(1)}{\gamma_i(1)}\frac{1}{}\frac{dL_0}{L_0} \\ &= \frac{d\left((1-\tau_0)\frac{W_0}{P_0}L_0\right)}{(1-\tau_0)\frac{W_0}{P_0}L_0} + \frac{(1-\tau_r)e_i}{\tau_r+(1-\tau_r)e_i}\gamma_{Li}\frac{dL_0}{L_0} \end{aligned} \tag{60}$$

with $\gamma_{Li}(1)$ the local derivative of the γ function with respect to L for individual i. Next, given our fiscal ule

$$\frac{d\left((1-\tau_0)\frac{W_0}{P_0}L_0\right)}{(1-\tau_0)\frac{W_0}{P_0}L_0} = \frac{d\left(\frac{W_0}{P_0}L_0\right)}{(1-\tau_0)\frac{W_0}{P_0}L_0} = \frac{1}{1-\tau_0}\left(\frac{d\left(\frac{W_0}{P_0}\right)}{\frac{W_0}{P_0}} + \frac{dL_0}{L_0}\right) \tag{61}$$

We now decompose

$$d(y_i-h_i) = \frac{d(y_i-h_i)}{y_i}\frac{y_i}{\mathbb{E}[y_i]}\mathbb{E}[y_i] = \frac{d(y_i-h_i)}{y_i}\frac{y_i}{\mathbb{E}[y_i]}(1-\tau_0)\frac{W_0}{P_0}L_0$$

and insert (60) and (61) to find

$$d(y_i-h_i) = \frac{y_i}{\mathbb{E}[y_i]}\left\{L_0 d\left(\frac{W_0}{P_0}\right) + \frac{W_0}{P_0}dL_0 + \frac{(1-\tau_r)e_i}{\tau_r+(1-\tau_r)e_i}\gamma_{Li}(1-\tau_0)\frac{W_0}{P_0}dL_0\right\}$$

But note that, since

$$y_i = (1-\tau)(\tau_r\mathbb{E}[\nu_i] + (1-\tau_r)\nu_i)$$

we have

$$\frac{y_i}{\mathbb{E}[y_i]} = \tau_r + (1-\tau_r)\frac{z_i}{\mathbb{E}[z_i]}$$

with

$$z_i = \frac{W}{P} L e_i \gamma_i (L)$$

so

$$\frac{z_i}{\mathbb{E}[z_i]} = \frac{e_i \gamma_i (1)}{\mathbb{E}[e_i \gamma_i (1)]} = e_i$$

and finally

$$\frac{y_i}{\mathbb{E}[y_i]} = \tau_r + (1 - \tau_r) e_i$$

Hence,

$$d(y_i - h_i) = \frac{y_i}{\mathbb{E}[y_i]} L_0 d\left(\frac{W_0}{P_0}\right) + \frac{y_i}{\mathbb{E}[y_i]} \frac{W_0}{P_0} dL_0 + (1 - \tau_r)(1 - \tau_0) e_i \gamma_{Li} \frac{W_0}{P_0} dL_0$$

and using the relationships (54) and (56), we obtain (55) as claimed. □

E Additional proofs

E.1 Homotheticity

Under (29)-(33), the following lemma allows us to considerably simplify the analysis of steady state equilibria.

Lemma 1 (Homotheticity of policy functions)**.** *In any steady state, the policy functions scale with post-tax labor income; in particular*

$$a'\left(s, b, v; L, \frac{W}{P}, \tau, r\right) \quad = \quad (1 - \tau) \frac{W}{P} L \tilde{a}' (s, b, v; r)$$

Proof. Direct consequence of our assumptions that preferences are homothetic, the fact that the borrowing limit scales with income, proportional taxes, and our assumption of proportional distribution of aggregate income to individual incomes. □

E.2 Proof of corollaries 1 and 3

Take the total log-differential of (34) around $r = 0$ and $L = 1$ to find

$$-\frac{\tau}{1 - \tau}\left(\widehat{G} - \widehat{L}\right) + \epsilon_{ar} dr + \epsilon_{a\varphi} d\varphi = \omega\left(\widehat{\overline{B}} - \widehat{L}\right)$$

where $\omega \equiv \frac{\overline{B}}{A}$ is the share of bonds in total assets in the initial steady state and $\tau = \frac{G}{w}$ the tax rate at steady state. Note that all are observable quantities except for the key object $\epsilon_{a\varphi}$ which depend on the particular model of consumption, but not on any general equilibrium mechanism. Grouping terms, and using $\widehat{Y} = \widehat{L}$ and the appropriate fiscal policy assumptions, we obtain equations (35).

E.3 Proof of propositions 4 and 5

Asset supply normalized by output is

$$\frac{B}{Y}+\frac{XK}{Y}+\frac{\Pi}{Y}$$

which can be rewritten as

$$b(L)+\frac{\mu^{-1}-\alpha}{r+\rho+\delta}+\frac{1}{r+\rho}(1-\mu^{-1})$$

Since labor taxes τwL are equal to steady-state government spending $G+rB$, after-tax labor income as a share of output is

$$\alpha-g(L)-rb(L)$$

where $g(L)$ and $b(L)$ are the policy rules for G/Y and B/Y as functions of L.

Hence we can write normalized asset supply as

$$\frac{b(L)+\frac{\mu^{-1}-\alpha}{r+\rho+\delta}+\frac{1}{r+\rho}(1-\mu^{-1})}{(\alpha-g(L)-rb(L))}$$

Note that a shock to either investment prices X or the production function can only show up here in a single place: the gross labor share α. Hence, we can say that the labor share is a *sufficient statistic* for either of these shocks. Log-linearizing with respect to α gives

$$\left(-\frac{\alpha}{\mu^{-1}-\alpha}\frac{XK}{A}-\frac{1}{1-\tau}\right)\hat{\alpha}$$

where the term in parentheses is the elasticity of normalized asset supply with respect to $\hat{\alpha}$, and is unambiguously negative. This establishes proposition 4.

For markup shocks and proposition 5, we instead write XK/Y as inversely proportional to μ^{ϵ} due to the CES assumption, obtaining an expression for normalized asset supply as

$$\frac{b(L)+C_K\mu^{-\epsilon}X^{1-\epsilon}(r+\rho+\delta)^{-\epsilon}+\frac{1}{r+\rho}(1-\mu^{-1})}{\alpha-g(L)-rb(L)}$$

Log-linearizing with respect to α and μ gives

$$\left(\frac{\Pi}{A}\frac{1}{\mu-1}-\epsilon\frac{XK}{A}\right)\hat{\mu}-\frac{1}{1-\tau}\hat{\alpha} \tag{62}$$

There are two potential sources of ambiguity in sign here: the effect of $\hat{\mu}$ on $\hat{\alpha}$, and the two opposite-sign terms in parentheses multiplying $\hat{\mu}$. We will deal with them in turn—and they will turn out to be closely related.

Effect of markups on labor share. First, we should derive an expression for $\hat{\alpha}$ in terms of $\hat{\mu}$. We start with the observation that

$$\frac{wL}{Y} + (r+\rho+\delta)\frac{XK}{Y} = \mu^{-1}$$

Furthermore, as already discussed, the elasticity of XK/Y with respect to μ is $-\epsilon$. It follows that

$$\alpha\hat{\alpha} - \epsilon(\mu^{-1}-\alpha)\hat{\mu} = -\mu^{-1}\hat{\mu}$$

We conclude that

$$\hat{\alpha} = \left(\epsilon(\alpha^{-1}\mu^{-1}-1) - \alpha^{-1}\mu^{-1}\right)\hat{\mu} \tag{63}$$

Since $\alpha^{-1}\mu^{-1} > 1$, the elasticity in (63) has ambiguous sign in general. For $\epsilon \leq 1$, however, it is always strictly negative.

Brief discussion of when this could have the other sign. For high enough elasticity of substitution ϵ, the elasticity in (63) will become positive. At what point does it cross zero? We write

$$\begin{aligned} \epsilon(\alpha^{-1}\mu^{-1}-1) - \alpha^{-1}\mu^{-1} &= 0 \\ \epsilon &= \frac{1}{1-\alpha\mu} \end{aligned} \tag{64}$$

$1-\alpha\mu$ is capital's share of costs, which in our benchmark calibration is almost exactly $1/3$. (64) tells us that the key threshold for ϵ is at the inverse of this share—in our case, an elasticity of substitution of 3, well beyond the values generally assumed in the literature.

Direct effect of markups on asset supply. Now, we will examine the coefficient on $\hat{\mu}$ in (62). Write

$$\begin{aligned} \frac{\Pi}{A}\frac{1}{\mu-1} - \epsilon\frac{XK}{A} &= \frac{Y}{A}\left(\frac{\Pi}{Y}\frac{1}{\mu-1} - \epsilon\frac{XK}{Y}\right) \\ &= \frac{Y}{A}\left(\frac{\mu^{-1}}{r+\rho} - \frac{\epsilon(\mu^{-1}-\alpha)}{r+\rho+\delta}\right) \\ &\geq \frac{Y}{A}\frac{1}{r+\rho}(\mu^{-1} - \epsilon(\mu^{-1}-\alpha)) && (65) \\ &= -\frac{Y}{A}\frac{\alpha}{r+\rho}\left(\epsilon(\alpha^{-1}\mu^{-1}-1) - \alpha^{-1}\mu^{-1}\right) && (66) \end{aligned}$$

Note that the expression in parentheses is exactly the elasticity of $\hat{\alpha}$ with respect to $\hat{\mu}$ in (63). Hence if an increase in markups causes a decrease in the labor share, then the right side of (66) is strictly positive, and we can conclude that it also causes an increase in the ratio of assets to

GDP.

Putting it all together and intuition. As stated earlier, we are interested in signing (62) as a multiple of $\hat{\mu}$. If $\hat{\alpha}$ is a negative multiple of $\hat{\mu}$, then we showed that the term in parentheses is strictly positive, so that this entire expression is strictly positive. It follows that an increase in $\hat{\mu}$ causes an increase in normalized asset demand.

The inequality in (65) holds with equality when $\delta = 0$, in which case our result is if and only if: $\hat{\mu}$ causes a decline in $\hat{L}$ if and only if it causes a decline in $\hat{\alpha}$. When $\delta > 0$, however, then the inequality in (65) is strict, and it is possible for $\hat{\mu}$ to lead to a decline in $\hat{L}$ even if $\hat{\alpha}$ increases.

E.4 Government spending and liquidity multipliers

Our framework has implications for the role of fiscal policy in Keynesian slumps. Manipulating equation (34) we obtain:

$$\left(w(r) - \frac{\kappa(r)}{\widehat{a}(r)}\right) L = G + \overline{B}\frac{c(r)}{\widehat{a}(r)} \tag{67}$$

where $c(r) \equiv 1 + r\widehat{a}(r)$ is aggregate consumption normalized by post-tax wage income. Recall that in a Keynesian slump, r is fixed and $L < 1$. An increase in government debt pays entirely for itself since output and therefore tax revenue increase in proportion. This crowding-in effect of government liquidity on output and capital has been documented in models of financial frictions (Woodford 1990). Importantly, here, liquidity does not facilitate production: it facilitates consumption. This is an effect that has been known to exist in the context of Bewley-Huggett-Aiyagari models, with a literature examining the benefits of public liquidity on welfare (Aiyagari and McGrattan 1998). Relative to this literature, our Keynesian equilibria feature a direct link between consumption and output, so that the increase in liquidity can be purely self-sustaining. At the same time, we abstract away from features such as distortionary taxation, which limit the extent to which higher public debt could be beneficial.

We can go further and characterize the government spending multiplier in this model, as well as the *government liquidity multiplier*. Indeed, combining (67) with the fact that $Y = (w(r) + (r+\delta)\kappa(r))L$ we obtain:

$$Y = \frac{w(r) + (r+\delta)\kappa(r)}{w(r) - \frac{\kappa(r)}{\widehat{a}(r)}} \left(G + \overline{B}\frac{c(r)}{\widehat{a}(r)}\right) \tag{68}$$

Equation (68) shows clearly the symmetric role of government spending G and government debt $\overline{B}$ on steady state output while r is fixed. In particular, we have

Proposition 8. *Throughout the Keynesian regime, holding government debt $\overline{B}$ fixed, each unit of government spending raises output by $\frac{dY}{dG} \geq 1$, with a crowding-in effect ($\frac{dY}{dG} > 1$) in any economy with capital ($\kappa > 0$). Holding government debt spending fixed at G, each extra unit of government debt raises* *output by $\frac{C}{A}\frac{dY}{dG}$, where $\frac{C}{A}$ is the consumption-to-asset ratio in the initial steady state.*

In this economy, the increase in labor income from government spending *crowds in* capital, and the spending multiplier is above 1. The new steady state features higher consumption, higher government spending and higher investment. These results contrast with neoclassical analyses of the steady state government spending multiplier (eg Baxter and King 1993), where the spending multiplier is typically below one, and complement those in the New-Keynesian literature with a zero lower bound (Christiano, Eichenbaum and Rebelo 2011, Farhi and Werning 2013) by highlighting a different mechanism that works through the endogenous creation of liquid assets.

Proposition 8 also uncovers the *government liquidity multiplier* that the model features: in the Keynesian regime, increases in government debt increase steady state output by providing liquidity, relaxing financial frictions, and crowding in additional liquid assets. This works much like a government spending multiplier except for the direct effect that government purchases have on output.